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The method of exact solution of the problem of short-range guidance with two turns

The task of determining the guidance parameters by the “two-turn manoeuvre” method is related to numerical solution of an equation defining the trajectory of the fighter-interceptor. We propose an easy-to-implement in software approach which is based on the exact solution of the problem at each step of the iterative process constructed by one of the parameters.

Keywords: fighter interceptor, close-in zone, turning radius.

The problems of providing automated control of fighter aviation, solving navigation tasks and tasks of fighter interceptors’ (FI) guidance control have been dealt with in a number of papers [1–3].

To solve the problem of selecting parameters of FI guidance trajectory with a single turn, different methods have been proposed [3], including those based on the linear programming method.

At the same time, the FI guidance trajectories with a single turn in some cases do not allow to perform target interception from the best engagement position, and at times yield no solution at all [3, 4].

Trajectories with two turns provide more possibilities. However, as can be seen from papers [1, 2, 4], to calculate FI flight trajectory parameters, various presumptions and simplifications are applied, including a premise about the smallness of the angles of turn and discreteness of turning radii change.

To reduce expenditure of computing resources in the automated calculation of FI flight trajectory parameters, in the most elementary case, two levels of the turning radius are considered.

First, a solution for the recommended turning radius is sought. If it is not found, a solution is sought for the minimum turning radius. If no solution is available again, it is assumed that guidance by a given method is impossible.

Such a procedure of FI flight trajectory parameters calculation entails an increase of task solving time; therewith, the more turning radius gradations are used, the higher the expenditure of computing resources.

Moreover, in certain conditions such presumptions lead to ill-founded selection of the minimum turning radius, which is achieved by increasing aircraft roll angle with increased g-load, which is extremely undesirable.

This paper introduces a method that makes it possible to simplify problem solution and is free from the said drawbacks.

The problem under consideration is that of FI short-range guidance to target. The short-range guidance is understood as the final stage of ground-controlled guidance, when FI is brought into a point located at a specified distance from target, ensuring that the specified FI heading relative to target heading is maintained.

For FI guidance, it is necessary to determine its flight trajectory parameters. The trajectory of FI motion under the “two-turn manoeuvre” method consists of the first turn, straight-line flight segment, and second turn.

The initial data for solving the problem of determining FI flight trajectory parameters are the initial positions, headings, and velocities of target and FI, position of the final point of FI motion trajectory relative to target, FI final velocity, aspect of target engagement (FI heading relative to target in the final point).

In the above statement, the problem of determining FI motion trajectory parameters belongs to the subfield of mechanics called kinematics. Kinematics describes motion of bodies, without investigating its causes.

Normally, motion is considered in some or other coordinate system. Paper [5] provides basic information on the coordinate systems used in calculations associated with motion trajectories of aerial vehicles.

The tasks of carrying out navigation calculations and guidance in computer-aided systems are more convenient to solve in a target-related coordinate system [1, 3].

The origin of coordinates in this system coincides with the target position. Axis П of the rectangular coordinate system is directed along the target flight heading. The positive direction of axis П coincides with the direction of target motion. Axis Б is directed perpendicular to the target heading. The positive direction of axis Б – to the right of the direction of target motion.

We shall consider only positive values of Б, as the case of $Б<0$ is reduced to the case of $Б>0$ by respective change of the coordinates and heading.

The initial data on the target and FI are given, as a rule, in the local rectangular coordinate system of one of the controlling command posts (CP). We shall further assume that axis X of the CP coordinate system is directed to the north and axis Y – to the east.

Taking into account the turning angles of axes and the offsets of coordinate origins, recalculation of data from the local rectangular coordinate system of the CP (X, Y) into coordinate system of the target $(Б, П)$ is performed by means of expressions

$$Б=(Y-Y_ц)\cos Q_ц-(X-X_ц)\sin Q_ц;$$

$$П=(Y-Y_ц)\sin Q_ц+(X-X_ц)\cos Q_ц;$$

$$\gamma=-(Q-Q_ц),$$

where Q – FI heading relative to axis X of the CP coordinate system;

X, Y – FI coordinates in the CP coordinate system;

$X_ц, Y_ц$ – target coordinates in the CP coordinate system;

$Б, П$ – FI coordinates in the target coordinate system;

$Q_ц$ – target heading relative to axis X of the CP coordinate system;

γ – FI heading relative to target heading (counter-clockwise).

If the calculated value $\gamma<0$, then it is assumed that $\gamma^*=\gamma+2\pi$.

If the calculated value $Б<0$, then it is assumed that $Б^*=-Б,\ \gamma^*=2\pi-\gamma$.

A reverse calculation is done by the formulas

$$X=X_ц-Б\sin Q_ц+П\cos Q_ц;$$

$$Y=Y_ц+Б\cos Q_ц+П\sin Q_ц.$$

Let us consider FI flight trajectory in the target coordinate system when implementing the “two-turn manoeuvre” method.

We shall use the following denotations:

$$a_1=\frac{V_ц}{V_п},\quad a_2=\frac{2V_ц}{V_п+V_к};$$

$V_ц$ – target velocity;

$V_п$ – programmed velocity of FI flight;

α_1 – first turn angle;

R_1 – first turn radius;

α_2 – second turn angle;

R_2 – second turn radius;

$V_к$ – FI velocity in the final point of guidance trajectory;

$Б$, $П$ – FI initial coordinates in the target coordinate system ($Б$ – lateral component, $П$ – longitudinal component);

C – length of straight-line segment of FI trajectory;

$S_к$ – distance from FI to target in the final point of guidance trajectory;

p – FI aspect relative to target (FI heading relative to target heading) in the final point of guidance;

m_1 – 1st turn direction marker;

m_2 – 2nd turn direction marker;

γ_1 – FI heading on the straight-line segment of flight trajectory;

γ_0 – FI heading in the initial point of guidance trajectory;

L – target location at the moment of FI entering the final point of guidance trajectory.

Fig. 1 shows FI flight trajectory in the target coordinate system.

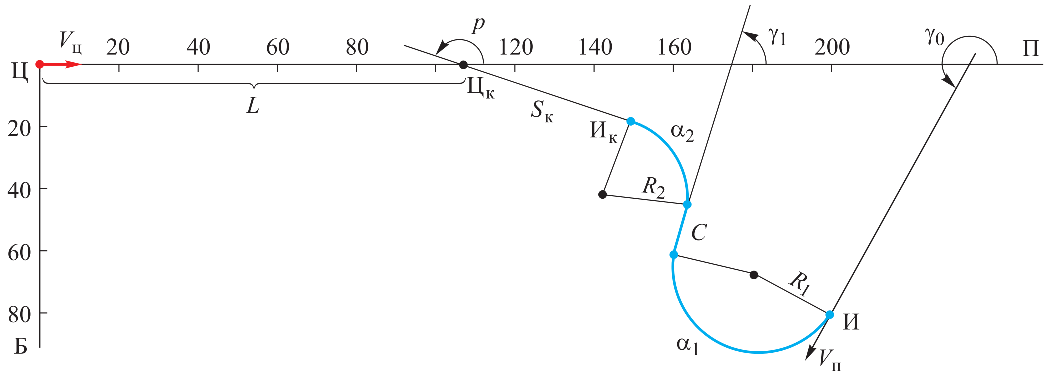

Fig. 1. FI flight trajectory:
Ц – target at the moment of FI being in the initial guidance point; $Ц_к$ – target at the moment of FI being in the final guidance point; И – FI in the initial guidance point; $И_к$ – FI in the final guidance point

Each of the turn direction markers m_1, m_2 takes the value of +1 if FI makes a clockwise turn (right turn) and –1 if it makes a counter-clockwise turn (left turn).

The FI velocity is assumed constant from the initial point till the second turn start, and equal to the programmed velocity of fighter-interceptor flight $V_п$.

On the second turn segment, FI velocity changes uniformly from programmed flight velocity $V_п$ to specified velocity in the final point $V_к$.

The FI heading is reckoned counter-clockwise from 0 to $2\neq$.

To determine FI flight trajectory parameters, let us compose balance equations according to flight coordinates and time. For that purpose we shall find projections of the components of FI and target flight trajectories on the coordinate axes Б, П.

The FI initial coordinates are equal to Б, П.

FI displacements along the coordinate axes during FI movement are as follows:

• along circular arc of the first turn:

$$\Delta Б_1 = -m_1 R_1 (\cos\gamma_1 - \cos\gamma_0),$$

$$\Delta П_1 = -m_1 R_1 (\sin\gamma_1 - \sin\gamma_0);$$

• along straight-line segment C:

$$\Delta Б_2 = -C\sin\gamma_1, \ \ \Delta П_2 = -C\cos\gamma_1;$$

• along circular arc of the second turn:

$$\Delta Б_3 = -m_2 R_2 (\cos p - \cos\gamma_1),$$

$$\Delta П_3 = -m_2 R_2 (\sin p - \sin\gamma_1).$$

Target displacements along the coordinate axes over the time of FI movement are as follows:

• along circular arc of the first turn:

$$\Delta Б_{11} = 0, \ \ \Delta П_{11} = R_1 a_1 \alpha_1;$$

• along straight-line segment of the trajectory:

$$\Delta Б_{12} = 0, \ \ \Delta П_{12} = C a_1;$$

• along circular arc of the second turn:

$$\Delta Б_{13} = 0, \ \ \Delta П_{13} = R_2 a_2 \alpha_2.$$

Target offset along the coordinate axes relative to FI position in the final trajectory point:

$$\Delta Б_к = -S_к \sin p,$$

$$\Delta П_к = S_к \cos p.$$

Considering the above components of FI and target flight trajectories, we can write down a system of balance equations as per coordinates and time which ensures FI acquiring the specified position with the specified heading relative to target:

$$\begin{cases} Б + \Delta Б_1 + \Delta Б_2 + \Delta Б_3 + \Delta Б_к = \Delta Б_{11} + \Delta Б_{12} + \Delta Б_{13}; \\ П + \Delta П_1 + \Delta П_2 + \Delta П_3 + \Delta П_к = \Delta П_{11} + \\ + \Delta П_{12} + \Delta П_{13}. \end{cases} \quad (1)$$

After substitution of the found components in the system of equations (1), we have

$$\begin{cases} Б - S_к \sin p - m_2 R_2 (\cos p - \cos\gamma_1) - \\ - m_1 R_1 (\cos\gamma_1 - \cos\gamma_0) - C\sin\gamma_1 = 0; \\ П + S_к \cos p - R_2 [a_2 \alpha_2 + m_2 (\sin p - \sin\gamma_1)] - \\ - R_1 [a_1 \alpha_1 + m_1 (\sin\gamma_1 - \sin\gamma_0)] - C(a_1 - \cos\gamma_1) = 0. \end{cases} \quad (2)$$

The system of equations (2) includes FI turning angles α_1 and α_2. To determine modulus of the first turn angle, we set

$$\Delta\gamma_1 = \gamma_1 - \gamma_0 + 2\pi i, \qquad (3)$$

$$i = \begin{cases} 0 & \text{at} \quad \gamma_1 - \gamma_0 \geqslant 0; \\ 1 & \text{at} \quad \gamma_1 - \gamma_0 < 0. \end{cases}$$

Four cases, as shown in Fig. 2, are possible:
if $m_1 = +1$, $\gamma_1 < \gamma_0$ (Fig. 2, a), then $\alpha_1 = 2\pi - \Delta\gamma_1$;
if $m_1 = +1$, $\gamma_1 > \gamma_0$ (Fig. 2, b), then $\alpha_1 = 2\pi - \Delta\gamma_1$;
if $m_1 = -1$, $\gamma_1 < \gamma_0$ (Fig. 2, c), then $\alpha_1 = \Delta\gamma_1$;
if $m_1 = -1$, $\gamma_1 > \gamma_0$ (Fig. 2, d), then $\alpha_1 = \Delta\gamma_1$.

Analysis of the expressions for the first turn angle shows that the sign before increment of heading $\Delta\gamma_1$ is opposite to the sign of turn direction marker m_1. Term 2≠ appears in case of $m_1 = +1$. Therefore,

$$\alpha_1 = \pi + (\pi - \Delta\gamma_1) m_1, \qquad (4)$$

or, considering (3),

$$\alpha_1 = \pi\left[1 + m_1(1 - 2i)\right] - (\gamma_1 - \gamma_0) m_1. \qquad (5)$$

Similarly, having denoted

$$\Delta\gamma_2 = p - \gamma_1 + 2\pi j, \qquad (6)$$

$$j = \begin{cases} 0 & \text{at} \quad p - \gamma_1 \geq 0 \\ 1 & \text{at} \quad p - \gamma_1 < 0, \end{cases}$$

we obtain an expression for the second turn angle:

$$\alpha_2 = \pi + (\pi - \Delta\gamma_2) m_2, \qquad (7)$$

or

$$\alpha_2 = \pi\left[1 + m_2(1 - 2j)\right] - (p - \gamma_1) m_2. \qquad (8)$$

For successful detection and lock-on of target by the on-board radar, with subsequent launch of missiles, FI heading p in the final point must conform to certain limitations.

For tail-chase engagement

$$2\pi - \Delta p \leq p < 0 \quad \text{or} \quad 0 \leq p \leq \Delta p.$$

For head-on engagement

$$\pi - \Delta p < p < \pi + \Delta p.$$

It is usually assumed that $\Delta p = \pi / 6$.

With the known FI initial position $(Б, П)$, velocities of FI $V_п, V_к$ and target $V_ц$, FI initial heading γ_0, radii R_1, R_2 and directions m_1, m_2 of turns, FI final heading p, final distance $S_к$ between FI and target, two unknowns remain in system (2): heading γ_1 on the straight-line trajectory segment and length C of this segment.

As follows from expressions (5), (8), turning angles α_1, α_2 depend on the unknown parameter γ_1.

Substituting in system (2) expressions (5), (8) for α_1, α_2 and excluding parameter C from it, we have

$$\begin{aligned} C &= \big(Б - S_к \sin p + m_1 R_1 \cos\gamma_0 - \\ &- m_2 R_2 \cos p + (m_2 R_2 - m_1 R_1)\cos\gamma_1\big) / \sin\gamma_1 = \\ &= (П + S_к \cos p - R_2 m_2 \sin p + R_1 m_1 \sin\gamma_0 + \\ &+ (R_2 m_2 - R_1 m_1)\sin\gamma_1 - \\ &- R_2 a_2\left\{\pi\left[1 + (1 - 2j)\right] - (p - \gamma_1) m_2\right\} - \\ &- R_1 a_1\left\{\pi\left[1 + (1 - 2i) m_1\right] - (\gamma_1 - \gamma_0) m_1\right\}\big) / (a_1 - \cos\gamma_1). \end{aligned} \qquad (9)$$

Hence, we obtain an equation for determining FI heading γ_1 on the straight-line segment of flight trajectory:

$$f(\gamma_1) = e\cos\gamma_1 + (d_1\gamma_1 + d_2)\sin\gamma_1 + s = 0, \qquad (10)$$

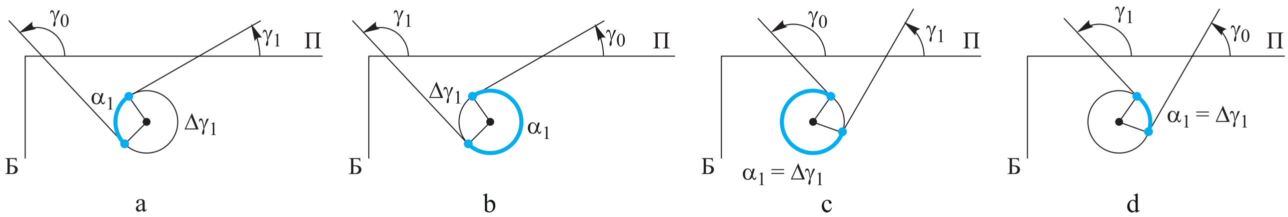

Fig. 2. Angle of the first turn:
a – right turn from greater heading value to smaller one; b – right turn from smaller heading value to greater one;
c – left turn from greater heading value to smaller one; d – left turn from smaller heading value to greater one

where $e = (m_2R_2 - m_1R_1)a_1 - Б +$
$+ S_к \sin p - m_1R_1 \cos\gamma_0 + m_2R_2 \cos p;$

$$d_1 = a_2m_2R_2 - a_1m_1R_1;$$

$$d_2 = -П - S_к \cos p + m_2R_2 \sin p - m_1R_1 \sin\gamma_0 + \\ + a_1[\pi + (\pi + \gamma_0 - 2\pi i)m_1]R_1 + \\ + a_2[\pi + (\pi - p - 2\pi j)m_2]R_2;$$

$$s = (Б - S_к \sin p + m_1R_1 \cos\gamma_0 - m_2R_2 \cos p)a_1 - \\ - m_2R_2 + m_1R_1.$$

The length of straight-line segment C is determined by substituting the value of heading γ_1 in one of the expressions (9).

For the problem solution to exist, the length of straight-line trajectory segment C must be a non-negative magnitude.

Selection of the final solution from those possible is done based on the parameter of the best position for target engagement:

$$L = a_1\alpha_1R_1 + a_2\alpha_2R_2 + a_1C. \quad (11)$$

This expression defines target position $Ц_к$ at the moment of FI entering the final point. This is the path covered by target over the time of FI flight from the initial to the final trajectory point.

The turning radii we assume to be equal: $R_1 = R_2 = R$.

In the method being considered, to solve equation (10) it is proposed to use as the initial approximation the FI trajectory with turning radii $R = 0$. In this case equation (10) for the initial approximation has the view

$$e\cos\gamma_1 + d_2 \sin\gamma_1 + s = 0,$$

where $e = -Б + S_к \sin p;$

$$d_1 = 0;$$

$$d_2 = -П - S_к \cos p;$$

$$s = a_1(Б - S_к \sin p).$$

Then the unknown magnitude of γ_1 remains under the sign of trigonometric functions only, and the equation is reduced to a quadratic one.

To obtain solution to the problem with a real turning radius, it is gradually increased from zero to a specified value, with successive determination of FI flight parameters at each step of the turning radius change.

Considering that the process of transition from the initial solution to the final one occurs as a result of gradual deliberate change of an additionally selected variable, the latter can be called a master parameter.

In this paper, the FI turning radius is selected as a master parameter.

A tentative value of the FI heading at any given step is determined using a derivative from the FI heading with regard to the turning radius:

$$\gamma_{1(k+1)} = \gamma_{1k} + \frac{d\gamma_1}{dR}\Delta R, \quad (12)$$

where $\varnothing R$ – step as per the turning radius accepted for calculations.

For computations by formula (12), it is necessary to find an expression of derivative $\frac{d\gamma_1}{dR}$ from function $\gamma_1(R)$, which is implicitly specified by means of equation (10). For that purpose, we perform derivation of equation (10) by parameter R:

$$\frac{de}{dR}\cos\gamma_1 - e\sin\gamma_1\frac{d\gamma_1}{dR} + \frac{dd_1}{dR}\gamma_1\sin\gamma_1 + d_1\frac{d\gamma_1}{dR}\sin\gamma_1 + \\ + d_1\gamma_1\cos\gamma_1\frac{d\gamma_1}{dR} + \frac{dd_2}{dR}\sin\gamma_1 + d_2\cos\gamma_1\frac{d\gamma_1}{dR} + \frac{ds}{dR} = 0,$$

wherefrom

$$\frac{d\gamma_1}{dR} = \frac{\frac{de}{dR}\cos\gamma_1 + \left(\frac{dd_1}{dR}\gamma_1 + \frac{dd_2}{dR}\right)\sin\gamma_1 + \frac{ds}{dR}}{(e - d_1)\sin\gamma_1 - (d_1\gamma_1 + d_2)\cos\gamma_1}, \quad (13)$$

where $\frac{de}{dR} = a_1(m_2 - m_1) - m_1\cos\gamma_0 + m_2\cos p;$

$$\frac{dd_1}{dR} = -m_1a_1 + m_2a_2;$$

$$\frac{dd_2}{dR} = m_2\sin p - m_1\sin\gamma_0 + (\pi + (\pi i + \gamma_0)m_1)a_1 + \\ + (\pi + (\pi j - p)m_2)a_2;$$

$$\frac{ds}{dR} = a_1(m_1\cos\gamma_0 - m_2\cos p) - m_2 + m_1.$$

Since expression (12) considers only the linear part of FI heading γ_1 change, the found value of γ_1 will not be a solution to equation (10). However, equation (10) can be solved relative to the turning radius R.

Considering denotations accepted in equation (10), we have

$$e = \frac{de}{dR}R - Б + S_к \sin p;$$

$$d_1 = \frac{dd_1}{dR}R;$$

$$d_2 = \frac{dd_2}{dR}R - П - S_к \cos p;$$

$$s = \frac{ds}{dR}R + \left(Б - S_к \sin p\right) a_1.$$

Substituting these values in equation (10), we find

$$\left(\frac{de}{dR}R - Б + S_к \sin p\right)\cos\gamma_1 + \\ +\left(\frac{dd_1}{dR}R\gamma_1 + \frac{dd_2}{dR}R - П - S_к \cos p\right)\sin\gamma_1 + \\ +\frac{ds}{dR}R + (Б - S_к \sin p)a_1 = 0.$$

Wherefrom

$$R = R(\gamma_1) = \\ = \frac{(П + S_к \cos p)\sin\gamma_1 + (Б - S_к \sin p)(\cos\gamma_1 - a_1)}{\frac{de}{dR}\cos\gamma_1 + \left(\frac{dd_1}{dR}\gamma_1 + \frac{dd_2}{dR}\right)\sin\gamma_1 + \frac{ds}{dR}}. \quad (14)$$

For the value of heading $\gamma_{1(k+1)}$ found with the help of expression (12), there is a possibility to determine the exact value of turning radius R from expression (14).

The pair (R, γ_1) satisfies equation (10). Successive transition from step to step ensures approximation to the sought solution, corresponding to the specified turning radius $R_з$.

As a result of the described process, we find equation solution

$$F(\gamma_1) = R_з - R(\gamma_1) = 0, \quad (15)$$

where $R_з$ – specified turning radius value;

$R(\gamma_1)$ is determined by formula (14).

Equation (15) is equivalent to equation (10) and provides solution to the task if it exists. If there is no solution to the task for $R_з$, then the iteration process enables to find possible solution options at turning radii R different from $R_з$.

If we use the Newton's method instead of the constant step of $\varnothing R$, then, as demonstrated by numerous examples, it takes 2–7 iterations to determine the sought solution.

In this case, instead of formula (12), to determine the value of heading γ_1 at a given step of the iteration process, it is necessary to apply the expression

$$\gamma_{1(k+1)} = \gamma_{1k} - \frac{F}{\frac{dF}{d\gamma_1}}, \quad (16)$$

where

$$\frac{dF}{d\gamma_1} = -\frac{dR(\gamma_1)}{d\gamma_1} = -\frac{1}{\frac{d\gamma_1}{dR}}. \quad (17)$$

Here the derivative from function $R(\gamma_1)$ is expressed through a derivative determined by formula (13), from function $\gamma_1(R)$ inverse to it.

The turning radius changes at each step of the iteration process, so that the pair (R, γ_1) satisfies expression (10).

To demonstrate specific features and benefits of the method proposed in the paper, let us consider a numerical example.

Let the FI be in point Б = 80 km, П = 200 km, its heading being $\gamma_0 = 250°$ and velocity $V_п$ = 0.35 km/s.

In the final point of its flight trajectory the FI is to be at a distance of $S_к = 50$ km from target, with heading $p = 150°$ (target head-on engagement) and velocity $V_к$ = 0.25 km/s.

The velocity of target to which the FI is guided is $V_ц$ = 0.30 km/s.

Guidance is performed by the "two-turn manoeuvre" method. The first turn is to the right $(m_1 = +1)$, and the second turn – to the left $(m_2 = -1)$. The recommended turning radius is equal to $R_р = 25$ km, the minimum one – $R_{min} = 10$ km.

To compare the processes of computation and the obtained results, let us make calculations by two methods.

New method (master parameter method): the turning radius is a variable value and is accepted as a master parameter in the iteration process. Equation $F(\gamma_1) = R_з - R(\gamma_1) = 0$ is solved.

Old method (constant radius): the turning radius retains a constant value, equal to the specified one, at all iteration process steps. Equation $f(\gamma_1) = e\cos\gamma_1 + (d_1\gamma_1 + d_2)\sin\gamma_1 + s = 0$ is solved.

Fig. 3 shows turning radius dependence on the FI heading on the straight-line segment of flight trajectory, built for the given numerical example with the help of formula (14). With $R_з = R_p =$ 25 km, there is no solution, and with $R_з = R_{min} = 10$ km, a solution exists (see Fig. 3). Given a priori possession of those data, let us assess performance of both of the considered calculation methods.

We summarise the calculation results in tables, which contain calculations of heading γ_1 on the straight-line segment of flight trajectory, turning radius R, straight-line segment length C, turning angles α_1, α_2, location L, functions $f(\gamma_1)$ and $F(\gamma_1)$.

Table 1 gives the data for the newly proposed master parameter method with constant step

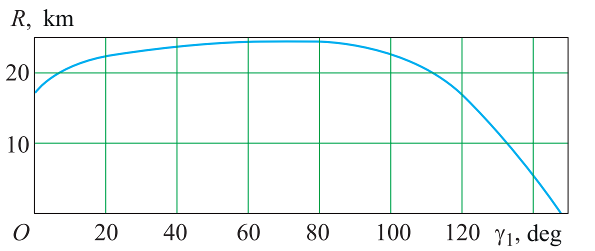

Fig. 3. Dependence of the turning radius on FI heading on the straight-line segment of flight trajectory

$\Delta R = 2.5$ km at $R_з = R_p = 25$ km.

Table 2 gives the data obtained at each step of the iteration process with variable step for $R_з = R_p = 25$ km.

It follows from Tables 1, 2 that the new method, in spite of the absence of solution at $R_з = R_p = 25$ km, makes it possible to accept, as a solution to the task, the results that conform the most to the specified initial data, namely, $\gamma_1 = 74.44°$, $R = 23.47$ km, $C = 14.60$ km, $\alpha_1 = 175.56°$, $\alpha_2 = 75.56°$, $L = 105.09$ km.

The FI flight trajectory corresponding to these data is shown in Fig. 1.

The old method does not yield a solution, as the values of heading γ_1, obtained in the process of iterations, do not satisfy equation (10).

Table 3 gives the data obtained at each step of the iteration process when seeking a solution for $R_з = R_{min} = 10$ km.

Table 3 shows that for $R_з = R_{min} = 10$ km, both methods yield the same result.

This example shows that the new approach makes it possible not just to do without superfluous computations, but to find a more acceptable solution from the viewpoint of g-loads affecting the aircraft.

Table 1

Iteration process with constant step $\Delta R = 2{,}5$ km at $R_з = R_p = 25$ km

Iteration number	Heading γ_1, deg	Turning radius R, km	$f(\gamma_1)$, km	$F(\gamma_1)$, km	C, km	α_1, deg	α_2, deg	L, km
0	144.17	0	0	25.00	93.96	105.83	5.83	80.53
1	141.81	2.39	0	22.61	90.36	108.19	8.19	81.67
2	139.23	4.77	0	20.23	86.46	110.77	10.77	82.90
3	136.37	7.13	0	17.87	82.18	113.63	13.63	84.25
4	133.15	9.47	0	15.53	77.46	116.85	16.85	85.74
5	129.49	11.79	0	13.21	72.24	120.51	20.51	87.39
6	125.21	14.07	0	10.93	66.37	124.79	24.79	89.25
7	120.08	16.31	0	8.69	59.68	129.92	29.92	91.38
8	113.66	18.49	0	6.51	51.86	136.34	36.34	93.89
9	105.10	20.55	0	4.45	42.34	144.90	44.90	96.95
10	92.32	22.39	0	2.61	29.79	157.68	57.68	100.90
11	67.49	23.60	0	1.40	9.12	182.51	82.51	106.22

Table 2

Iteration process with variable step at $R_з = R_p = 25$ km

Method	Iteration number	γ_1, deg	R, km	$f(\gamma_1)$, km	$F(\gamma_1)$, km	C, km	α_1, deg	α_2, deg	L, km
New (master parameter method)	0	144.17	0	0	25.00	93.96	105.83	5.83	80.53
	1	120.61	16.10	0	8.90	60.36	129.39	29.39	91.17
	2	98.30	21.68	0	3.32	35.44	151.70	51.70	99.14
	3	74.44	23.47	0	1.53	14.60	175.56	75.56	105.09
Old (constant radius)	0	144.17	25.00	65.47	0	–	–	–	–
	1	105.26	25.00	19.55	0	–	–	–	–
	2	76.86	25.00	8.12	0	–	–	–	–
	3	27.99	25.00	7.08	0	–	–	–	–

Table 3

Iteration process with variable step at $R_з = R_{min} = 10$ km

Method	Iteration number	γ_1, deg	R, km	$f(\gamma_1)$, km	$F(\gamma_1)$, km	C, km	α_1, deg	α_2, deg	L, km
New (master parameter method)	0	144.17	0	0	10.00	93.96	105.83	5.83	80.53
	1	134.75	8.35	0	1.65	79.79	115.25	15.25	85.00
	2	132.48	9.92	0	0.08	76.49	117.52	17.52	86.04
	3	132.36	10.00	0	0	76.32	117.64	17.64	86.10
Old (constant radius)	0	144.17	10.00	26.19	0	–	–	–	–
	1	132.98	10.00	1.29	0	–	–	–	–
	2	132.36	10.00	0.00	0	76.32	117.64	17.64	86.10

The volume of the program that implements the proposed method is 100 operators of the C language. Therewith, the program part run within a cycle makes 25 operators. The total volume of an executable file, with account of the initial conditions, data preparation, and computation management, is 30 Kbyte, which is by 15 % less than the volume of the previously applied program [4].

Under the proposed method, the average task completion time for a set of tasks is by 30 % less than under the previously applied one, with the errors of FI bringing to the final point of flight trajectory not exceeding 10^{-2} km.

The results of the undertaken study are as follows:

1. A method is developed for solving a system of balance equations for FI and target, which allows to avoid selection of FI flight trajectory parameters with unreasonably small turning radii.

2. The method is based on performing iterations by means of which the FI turning radius and heading on a straight-line trajectory segment are changed in such a way that they conform to the balance equations.

3. If solution for a given turning radius does exist, the method makes it possible to find an exact solution to the task, and if it does not, to choose and exact solution for another turning radius without making any additional computations.

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Метод точного решения задачи ближнего наведения с двумя разворотами

Задача определения параметров наведения методом «маневр с двумя разворотами» связана с решением уравнения, определяющего траекторию движения истребителя-перехватчика, численными методами. Предложен простой в программной реализации подход, основанный на точном решении задачи на каждом шаге итерационного процесса, построенного по одному из параметров.

Ключевые слова: истребитель-перехватчик, ближняя зона, радиус разворота.

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