UDC 621.396.96

Kiryakmasov A. K.

Equivalence of some schemes for constructing adaptive spatial filtering in phased antenna arrays

The study proves that with adaptive spatial filtering in the space of elements in the scheme with a dedicated main channel, the filtering results do not change depending on whether auxiliary elements are included in the main channel or not. It is also proved that in the ray space a homogeneous scheme and a scheme with a dedicated main channel are equivalent.

Keywords: adaptive spatial filtering, scheme with a dedicated main channel, ray space

Introduction

The most well-known implementation of an adaptive spatial filtering (ASF) scheme is a homogeneous scheme in a space of elements (Fig. 1), in which, in the path of each element, there is an adaptive adjustable weighting factor w_i, and the output signal of the entire array y is a weighted sum of the input signals [1]:

$$y = \mathbf{w}^H \mathbf{x}, \tag{1}$$

where $\mathbf{w}$ – vector of weighting factors, $\mathbf{w} = (w_1, \ldots, w_N)$, defined by the relationship (not considering normalisation):

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{s}; \tag{2}$$

$\mathbf{x}$ – N-dimensional (by the number of array elements) vector of signals from element outputs, $\mathbf{x} = (x_1, x_2, \ldots, x_N)^T$.

Here, $\mathbf{R} = \overline{\mathbf{x}\mathbf{x}^H}$ – correlation matrix (CM) of the input signals;

$\mathbf{s}$ – reference vector, consisting of ones[1];

$(\ )^H$ – sign of Hermitian conjugation;

$\overline{(\ )}$ – sign of statistical averaging.

If the number of array elements N is considerably high, the amount of computations by (2), proportional to N^3, turns out to be too large, so a problem arises requiring to decrease the computation amount by reducing dimensionality of the task.

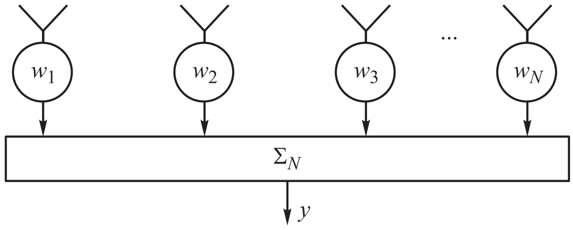

Fig. 1. Homogeneous scheme of ASF in element space

Possible options for reducing task dimensionality can be switching to the scheme with a dedicated main channel or to ASF in the beam-space. In either case, two options for constructing respective ASF schemes are possible. The objective of this paper is to prove equivalence of the two options in each one of these cases.

ASF in the scheme with a dedicated main channel

In the scheme with a dedicated main channel, the adaptive weighting factors are included in paths L of the array elements $(L < N)$, which can be called auxiliary channels. In this case, two construction options are possible: in the first case, in an N-element antenna array, $N - L$ elements belong to the main channel (Fig. 2, a), and in the second case, all N array elements belong to the main channel (Fig. 2, b), with L of them remaining adaptive.

In the scheme with a dedicated main channel, the weighting factor is defined by the following relationship [2]:

$$\mathbf{w}_L = \mathbf{R}_L^{-1}\mathbf{a}, \tag{3}$$

where $\mathbf{R}_L$ – CM of the signals from outputs L of the adaptive elements, $\mathbf{R}_L = \overline{\mathbf{x}_L \mathbf{x}_L^H}$;

$\mathbf{a}$ – correlation vector (CV), $\mathbf{a} = \overline{y_0^* \mathbf{x}_L}$;

y_0 – signal from the output of the main channel;

[1] In a general case, the reference vector is determined by the direction of wanted signal reception, wavelength, and array geometry. However, if phased antenna array (PAA) has a pre-phasing system focusing the array in the direction of the expected wanted signal, then the elements of vector **s** are equal to 1, which does not compromise the commonality of results, but rather simplifies computations.

$(\ )^*$ – sign of complex conjugation;

$\mathbf{x}_L$ – L-dimensional vector of signals from the outputs of the adaptive channels.

Then the resultant weighting vector for the entire array in the first option of ASF scheme construction (see Fig. 2, a) looks as follows:

$$\mathbf{w}_1 = \left(\mathbf{w}_0, -\mathbf{w}_{L_1}\right), \tag{4}$$

and in the second option (see Fig. 2, b):

$$\mathbf{w}_2 = \left(\mathbf{w}_0, \mathbf{s}_L - \mathbf{w}_{L_2}\right), \tag{5}$$

where $\mathbf{w}_0$ – fixed weighting vector for $N-L$ non-adaptive elements;

$\mathbf{s}_L$ – L-element reference vector, consisting of ones.

Let us show that $\mathbf{w}_1 = \mathbf{w}_2$. Apparently, CM $\mathbf{R}_L$ and its opposite $\mathbf{R}_L^{-1}$ will be similar for both options, and in case of influence of a single interference source, they appear as:

$$\mathbf{R}_{L_1} = \sigma_1^2 \breve{\mathbf{x}}_{L_1} \breve{\mathbf{x}}_{L_1}^H + \sigma_0^2 \mathbf{I},$$

$$\mathbf{R}_{L_1}^{-1} = \frac{1}{\sigma_0^2}\mathbf{I} - \frac{\sigma_1^2 \breve{\mathbf{x}}_{L_1} \breve{\mathbf{x}}_{L_1}^H}{\sigma_0^2\left(L\sigma_1^2 + \sigma_0^2\right)}; \tag{6}$$

under the influence of M sources:

$$\mathbf{R}_{LM} = \sum_{i=1}^{M} \sigma_i^2 \breve{\mathbf{x}}_{L_i} \breve{\mathbf{x}}_{L_i}^H + \sigma_0^2 \mathbf{I},$$

$$\mathbf{R}_{L_i}^{-1} = \mathbf{R}_{L(i-1)}^{-1} - \frac{\mathbf{R}_{L_{(i-1)}}^{-1} \sigma_i^2 \breve{\mathbf{x}}_{L_i} \breve{\mathbf{x}}_{L_i}^H \mathbf{R}_{L_{(i-1)}}^{-1}}{1 + \sigma_i^2 \breve{\mathbf{x}}_{L_i}^H \mathbf{R}_{L_{(i-1)}}^{-1} \breve{\mathbf{x}}_{L_i}}, \tag{7}$$

where σ_0^2 – power of intrinsic noise in array element;

σ_i^2 – power of the i-th interference source in array element;

$\breve{\mathbf{x}}_{L_i}$ – signal vector from adaptive elements, corresponding to the i-th interference source;

$\mathbf{I}$ – unity matrix.

The last equation in formula (7) represents a recurrent algorithm for finding an inverse CM.

For the two scheme options, the CV will be different, due to correlation or non-correlation of intrinsic noise in the main and auxiliary channels. For the case of influence of a single interference source, the CV for the first and second options is equal to, respectively:

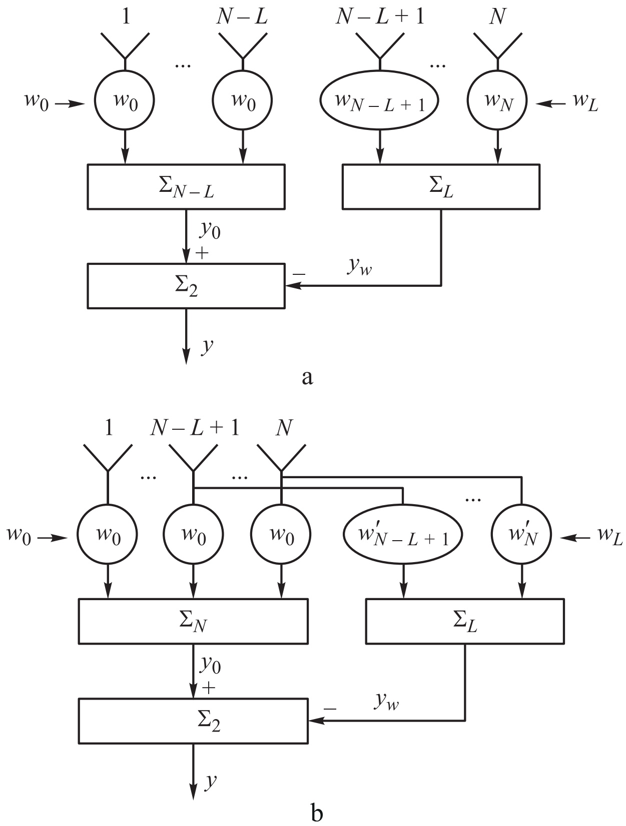

Fig. 2. Two options for constructing ASF scheme with the dedicated main channel:

a – N–L elements in the main channel;
b – N elements in the main channel

$$\mathbf{a}_1 = \sigma_1^2 \breve{\mathbf{x}}_{01}^H \mathbf{s}_0 \breve{\mathbf{x}}_{L_1}; \tag{8}$$

$$\mathbf{a}_2 = \sigma_1^2 \breve{\mathbf{x}}_{01}^H \mathbf{s}_0 \breve{\mathbf{x}}_{L1} + \sigma_0^2 \mathbf{s}_L; \tag{9}$$

under the influence of M sources:

$$\mathbf{a}_1 = \sum_{i=1}^{M} \sigma_i^2 \breve{\mathbf{x}}_{0i}^H s_0 \breve{\mathbf{x}}_{L_i}; \tag{10}$$

$$\mathbf{a}_2 = \sum_{i=1}^{M} \sigma_i^2 \breve{\mathbf{x}}_{0i}^H \mathbf{s}_0 \breve{\mathbf{x}}_{L_i} + \sigma_0^2 \mathbf{s}_L, \tag{11}$$

where $\breve{\mathbf{x}}_{0i}$ – signal vector from non-adaptive elements, corresponding to the i-th interference source;

$\mathbf{s}_0$ – (N–L)-element reference vector, consisting of ones.

Substituting formulas (6), (8), (9) in equation (3), we obtain for the first option

$$\mathbf{w}_{L_1} = \frac{\sigma_1^2}{\sigma_0^2} \breve{\mathbf{x}}_{01}^H \mathbf{s}_0 \breve{\mathbf{x}}_{L_1} \left(1 - \frac{\sigma_1^2 L}{\sigma_1^2 L + \sigma_0^2}\right) = \frac{\sigma_1^2 \breve{\mathbf{x}}_{01}^H \mathbf{s}_0 \breve{\mathbf{x}}_{L_1}}{L\sigma_1^2 + \sigma_0^2}, \tag{12}$$

for the second option

$$\mathbf{w}_{L_2}=\mathbf{s}_L+\frac{\sigma_1^2}{\sigma_0^2}\breve{\mathbf{x}}_{L_1}\left(\breve{\mathbf{x}}_{01}^H\mathbf{s}_0-\frac{\sigma_1^2 L\breve{\mathbf{x}}_{01}^H\mathbf{s}_0}{\sigma_1^2 L+\sigma_0^2}-\frac{\sigma_0^2\breve{\mathbf{x}}_{L_1}^H\mathbf{s}_L}{\sigma_1^2 L+\sigma_0^2}\right), \quad (13)$$

in the first case, vectors $\breve{\mathbf{x}}_{01}^H$ and $\mathbf{s}_0$ are (*N*–*L*)-element, and in the second case, *N*-element ones. Transforming the expression in parentheses and reducing similar terms, relationship (13) can be represented as

$$\mathbf{w}_{L_2}=\mathbf{s}_L+\frac{\sigma_1^2}{\sigma_0^2}\breve{\mathbf{x}}_{01}^H\mathbf{s}_0\breve{\mathbf{x}}_{L_1}\left(1-\frac{\sigma_1^2 L}{\sigma_1^2 L+\sigma_0^2}\right)= \\ =\mathbf{s}_L+\frac{\sigma_1^2\breve{\mathbf{x}}_{01}^H\mathbf{s}_0\breve{\mathbf{x}}_{L_1}}{L\sigma_1^2+\sigma_0^2}, \quad (14)$$

with $\breve{\mathbf{x}}_{01}^H$ and $\mathbf{s}_0$ in expression (14) being already (*N*–*L*)-element vectors.

Substituting formulas (12) and (14) in equations (4) and (5), respectively, we obtain similar expressions for $\mathbf{w}_1$ and $\mathbf{w}_2$.

For the case of influence of M sources, we shall prove equality of vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ by the method of mathematical induction. For the number of interferences *M*–1, the weighting vector in the first and second options of scheme construction will be equal to, respectively

$$\mathbf{w}_1(M-1)=\left(\mathbf{w}_0,-\mathbf{w}_{L_1}(M-1)\right),$$

$$\mathbf{w}_2(M-1)=\left(\mathbf{w}_0,\mathbf{s}_L-\mathbf{w}_{L_2}(M-1)\right).$$

Let us proceed from assumption that $\mathbf{w}_1(M-1)=\mathbf{w}_2(M-1)$. Then

$$\mathbf{w}_{L_1}(M-1)=\mathbf{w}_{L_2}(M-1)-\mathbf{s}_L. \quad (15)$$

Now let us rewrite expression (15) with consideration of formula (3), as

$$\mathbf{R}_{L(M-1)}^{-1}\mathbf{a}_{1(M-1)}=\mathbf{R}_{L(M-1)}^{-1}\mathbf{a}_{2(M-1)}-\mathbf{s}_L,$$

or, multiplying both parts by $R_L(M-1)$, as

$$\mathbf{a}_{1(M-1)}=\mathbf{a}_{2(M-1)}-\mathbf{R}_{L(M-1)}\mathbf{s}_L. \quad (16)$$

It remains to be shown that in case of *M* interferences, relationship (16) is likewise fair, i. e.

$$\mathbf{a}_{1(M)}=\mathbf{a}_{2(M)}-\mathbf{R}_{L(M)}\mathbf{s}_L. \quad (17)$$

It follows from expressions (7), (10), (11) that

$$\mathbf{a}_{1(M)}=\mathbf{a}_{1(M-1)}+\sigma_M^2\breve{\mathbf{x}}_{0M}^H\mathbf{s}_0\breve{\mathbf{x}}_{LM}; \quad (18)$$

$$\mathbf{a}_{2(M)}=\mathbf{a}_{2(M-1)}+\sigma_M^2\breve{\mathbf{x}}_{0M}^H\mathbf{s}_0\breve{\mathbf{x}}_{LM}; \quad (19)$$

$$\mathbf{R}_{L(M)}\mathbf{s}_L=\left(\mathbf{R}_{L(M-1)}+\sigma_M^2\breve{\mathbf{x}}_{LM}\breve{\mathbf{x}}_{LM}^H\right)\mathbf{s}_L. \quad (20)$$

Substituting formulas (18)–(20) in (17), we have

$$\sigma_M^2\breve{\mathbf{x}}_{0M}^H\mathbf{s}_0\breve{\mathbf{x}}_{LM}=\sigma_M^2\breve{\mathbf{x}}_{0M}^H\mathbf{s}_0\breve{\mathbf{x}}_{LM}-\sigma_M^2\breve{\mathbf{x}}_{LM}\breve{\mathbf{x}}_{LM}^H\mathbf{s}_L.$$

In the last expression, in the left-hand part, vectors $\breve{\mathbf{x}}_{0M}^H$ and $\mathbf{s}_0$ are $(N-L)$-element, and in the right-hand part, N-element ones, which in fact proves the correctness of relationship (17).

Beamspace

Under ASF in the beamspace, transition from element space to the beamspace is provided by means of transformation matrix **B**, whose columns are orthonormalised vectors of amplitude-and-phase distribution across the antenna aperture for formation of respective beams. In other words, *L*-element vector of signals from the outputs of beams $\mathbf{x}_л$, CM $\mathbf{R}_л$, and reference vector $\mathbf{s}_л$ in the beamspace are formed as follows:

$$\mathbf{x}_л=\mathbf{B}^H\mathbf{x},\ \mathbf{R}_л=\overline{\mathbf{x}_л\mathbf{x}_л^H}=\mathbf{B}^H\mathbf{R}\mathbf{B},\ \mathbf{s}_л=\mathbf{B}^H\mathbf{s},$$

where x – *N*-dimensional vector of signals from element outputs;

R – CM of the input signals;

s – reference vector in the signal space.

Due to orthogonality of the auxiliary beams, reference vector in the beamspace will take the following view (for definiteness, the element relating to the signal beam is placed in the first position):

$$\mathbf{s}_л=\left(\sqrt{N},\ \ldots,0,0\right)^T. \quad (21)$$

The formed beams correspond to *L* processing channels, i.e. their number is less than the number of elements *N* in the array (*L*<*N*). Normally, one of the channels is referred to as a signal channel, and the rest *L*–1 channels – compensation, or auxiliary ones. Such scheme is also referred to as a multi-channel autocompensator [3, 4]. In the beamspace too, two options of ASF scheme construction are possible: a homogeneous scheme (all beams in Fig. 3, a are adaptive) and a scheme with a dedicated main channel, where the main (non-adaptive) is considered to be a single selected beam (beam 1 in Fig. 3, b).

An optimal solution in a homogeneous scheme in the beamspace looks as follows [5]:

$$\mathbf{w}_{\text{л}} = \mathbf{R}_{\text{л}}^{-1}\mathbf{s}_{\text{л}}. \qquad (22)$$

In a scheme with the dedicated main channel, similarly to the ASF in the element space, the weighting factor in the beamspace can be written as

$$\mathbf{w}_L = -\mathbf{R}_L^{-1}\mathbf{a}_L, \qquad (23)$$

where $\mathbf{R}_L$ – correlation matrix of signals from L adaptive beams;

$\mathbf{a}_L$ – correlation vector, $\mathbf{a}_L = \overline{y_0^* \mathbf{x}_L}$;

y_0 – signal from the output of the main beam;

$\mathbf{x}_L$ – vector of signals from the outputs of L adaptive beams.

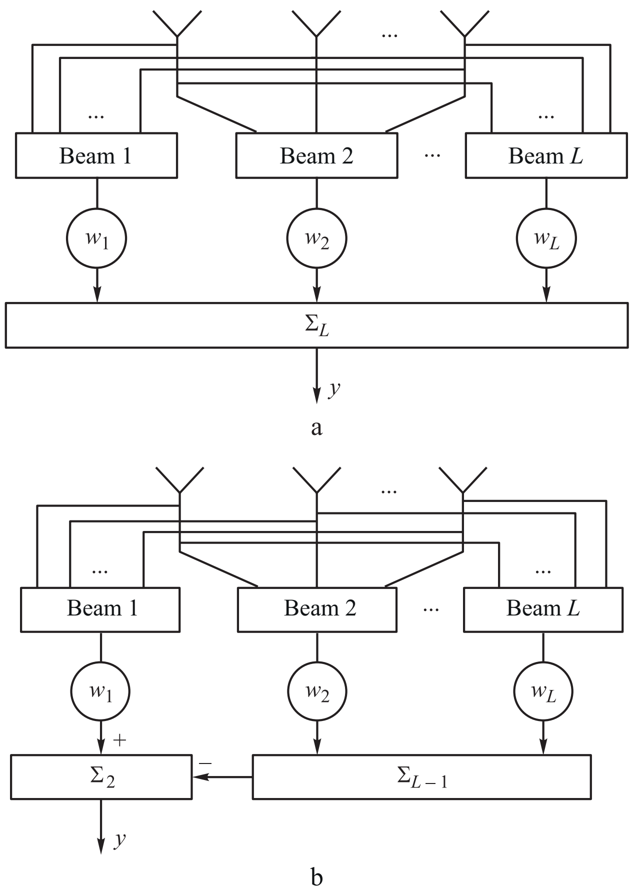

Fig. 3. Two options for constructing ASF scheme in the beamspace:
a – homogeneous scheme;
b – scheme with the dedicated main channel

The total (composite) weighting vector in this case will be equal to

$$\mathbf{w}_{\text{л}} = (1, \mathbf{w}_L). \qquad (24)$$

Let us show the equivalence of expressions (22) and (24). In the beamspace, in case of the scheme with the dedicated main channel, the number of adaptive weighting factors is equal to L–1, i. e. one unit less than the total number of beams L. Hence, matrix $\mathbf{R}_{\text{л}}^{-1}$ can be found by the method of bordering matrix $\mathbf{R}_L^{-1}$ [6]. Let us write matrix $\mathbf{R}_{\text{л}}$ as

$$\mathbf{R}_{\text{л}} = \begin{bmatrix} \mathbf{R}_L & \mathbf{a}_L \\ \mathbf{a}_L^H & \alpha \end{bmatrix}, \quad \alpha = y_0 y_0^*.$$

Then the inverse matrix appears as

$$\mathbf{R}_{\text{л}}^{-1} = \frac{1}{\beta}\begin{bmatrix} \mathbf{R}_L^{-1}\beta + \mathbf{R}_L^{-1}\mathbf{a}_L\mathbf{a}_L^H\mathbf{R}_L^{-1} & -\mathbf{R}_L^{-1}\mathbf{a}_L \\ -\mathbf{a}_L^H\mathbf{R}_L^{-1} & 1 \end{bmatrix}, \qquad (25)$$

$$\beta = \alpha - \mathbf{a}_L^H\mathbf{R}_L^{-1}\mathbf{a}_L.$$

Substituting formula (25) in (22) and considering expressions (21) and (23), we have

$$\mathbf{w}_{\text{л}} = \mathbf{R}_{\text{л}}^{-1}\mathbf{s}_{\text{л}} = \left(1, -\mathbf{R}_L^{-1}\mathbf{a}_L\right)^T \frac{\sqrt{N}}{\beta} = (1, \mathbf{w}_L)\frac{\sqrt{N}}{\beta}, \qquad (26)$$

which matches (24) to the accuracy of normalisation.

Conclusion

Hence, the equivalence of two options for constructing an ASF scheme in the beamspace and the equivalence of two options of ASF scheme with a dedicated main channel is proved. It determines a possibility to undertake study for one of the options only, namely, for that one where it is simpler and more convenient in a given situation.

Bibliography

1. *Ratynsky M. V.* Adaptation and Superresolution in Antenna Arrays. M.: Radio i svyaz', 2003. 200 p. (Russian)

2. *Nitzberg R.* Adaptive Signal Processing for Radar. Artech House, 1992. 313 p.

3. *Mervin C.*, *Budge Jr.*, *Shawn R.* German, Basic Radar Analysis. Artech House, 2015. 784 p.

4. *Trees V.*, *Harry L.* Optimum Array Processing. Part 4 of Detection, Estimation, and Modulation Theory. New York: John Willey & Sons, Inc., 2002. 1433 p.

5. *Kiriakmasov A. K.* Interference Coherent Compensation System in Beamspace // Uspekhi sovremennoy radioelektroniki. 2019. No. 1. P. 70–79. (Russian)

6. *Voyevodin V. V., Voyevodin Vl. V.* Entsiklopediya lineinoy algebry. SPb.: BKhV-Peterburg, 2006. 544 s. (Russian)

Submitted on 13.03.2019

Kiryakmasov Aleksandr Konstantinovich – Lead Engineer, Joint Stock Company VNIIRT – All-Russian Scientific Research Institute of Radio Engineering, Moscow.
Science research interests: signal processing.

Эквивалентность некоторых схем построения адаптивной пространственной фильтрации в фазированных антенных решетках

Доказано, что при адаптивной пространственной фильтрации в пространстве элементов в схеме с выделенным основным каналом результаты фильтрации не изменяются от того, включаются ли вспомогательные элементы в основной канал или нет. Доказано также, что в пространстве лучей однородная схема и схема с выделенным основным каналом эквивалентны.

Ключевые слова: адаптивная пространственная фильтрация, схема с выделенным основным каналом, пространство лучей.

Кирякмасов Александр Константинович – ведущий инженер Акционерного общества «Всероссийский научно-исследовательский институт радиотехники», Москва.
Область научных интересов: обработка сигналов.