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Smirnov M. A.

Methodology for evaluating effectiveness of air defense missile system (ADMS) information resources when detecting a hypersonic cruise missile, with account for the dynamic target scattering crossover

The study introduces a method that evaluates the effectiveness of information resources of an air defense missile system when detecting a hypersonic cruise missile. The method takes into account the dynamic target scattering crossover. The efficiency is evaluated by conducting statistical tests on a simulation model and constructing a vertical section of the detection zone. In this case, the effective target scattering crossover of a hypersonic cruise missile is calculated depending on the parameters of its movement around the center of mass relative to the ground-based detection radar and its line of sight.

Keywords: efficiency, air defense missile system, information resources, hypersonic cruise missile, dynamic target scattering crossover, target scattering

One of the promising development trends within the framework of the Prompt Global Strike concept in the U.S. is development of hypersonic cruise missiles (HSCM). The high speed of HSCM V = 6...8 M and their capability to fly at altitudes H = 30...70 km may considerably hinder their detection by the existing information resources of an air defence missile system (ADMS).

At the same time, the process of making decisions on selection of the optimal performance characteristics of prospective information resources includes a stage of efficiency evaluation when comparing alternative options.

At present, a vast scientific and methodical knowledge base [1, 2] has been accumulated for carrying out such studies, according to which evaluation of the efficiency of both particular weapon specimens and groupings of the anti-aircraft missile troops (AAMT) is performed with the use of a set of analytical and simulation models enabling to evaluate the efficiency of ADMS against different air attack weapons (AAW) and for different options of AAMT detachment composition as per various integral and particular performance criteria. In so doing, an efficiency criterion for the information resources is determined by the coverable zones where aerodynamic and ballistic targets can be detected by the ADMS radio electronic facilities [1].

At the same time, staging an experiment in real conditions for detection zone assessment requires substantial time, material, and labour expenditures; besides, an aerial vehicle simulating HSCM flight is required, while application of the known analytical methods is only possible under assumption that target is a material point with a mean or median value of the radar cross-section (RCS) in a wide range of expected target illumination directions.

However, in a real situation of HSCM flight, its centre of mass is moving at high speed relative to the detection radar, with its attitude relative to the radar line of sight ever changing. As a result, target scattering properties are changing continuously. To describe the dynamics of target scattering capability, a notion of dynamic radar cross-section is used, representing a dependence of target's RCS on time.

In view of the above, a methodology has been developed for evaluating effectiveness of ADMS information resources when detecting an HSCM, with account for target's dynamic RCS, whose flowchart is given in Fig. 1.

1. At the first stage of the method, the initial data for simulation are input, namely:

- initial position of HSCM centre of mass in the geocentric coordinate system (r, λ, φ), its spatial attitude, specifications, and target backscattering diagram $\sigma(\varphi_n, \lambda_n, \gamma_n)$;

- hit object coordinates in the geocentric coordinate system $(r_ц, \lambda_ц, \varphi_ц)$;
- coordinates of ground-based detection radar in the geocentric coordinate system $(r_0, \varphi_0, \lambda_0)$ and its specifications.

2. The position of HSCM centre of mass is specified by radius r and angles λ and φ (geocentric longitude and latitude), which determine mutual orientation of the axes of Greenwich geocentric rectangular and local geographic coordinate systems.

The kinematic and dynamic equations of motion have the following form [3]:

$$\frac{dr}{dt} = V \sin\theta; \tag{1}$$

$$\frac{d\varphi}{dt} = \frac{V}{r} \cos\psi \cos\theta; \tag{2}$$

$$\frac{d\lambda}{dt} = \frac{V}{r} \frac{\cos\psi \cos\theta}{\cos\varphi}; \tag{3}$$

$$\begin{gathered} \frac{dV}{dt} = \frac{X}{m} - g_r \sin\theta - \\ -g_\omega (\cos\varphi \cos\psi \cos\theta + \sin\varphi \sin\theta); \end{gathered} \tag{4}$$

$$\begin{gathered} \frac{d\psi}{dt} = -\frac{Z}{mV\cos\theta} + \frac{g_\omega \cos\varphi \sin\psi}{V\cos\theta} + \\ + \frac{V}{r} \operatorname{tg}\varphi \sin\psi \cos\theta + \\ + 2\omega_з (\cos\varphi \cos\psi \operatorname{tg}\theta - \sin\varphi); \end{gathered} \tag{5}$$

$$\begin{gathered} \frac{d\theta}{dt} = \frac{Y}{mV} - \frac{g_r}{V} \cos\theta - \\ -\frac{g_\omega}{V} (-\cos\varphi \cos\psi \sin\theta + \sin\varphi \cos\theta) + \\ + \frac{V}{r} \cos\theta - 2\omega_з \cos\varphi \sin\psi, \end{gathered} \tag{6}$$

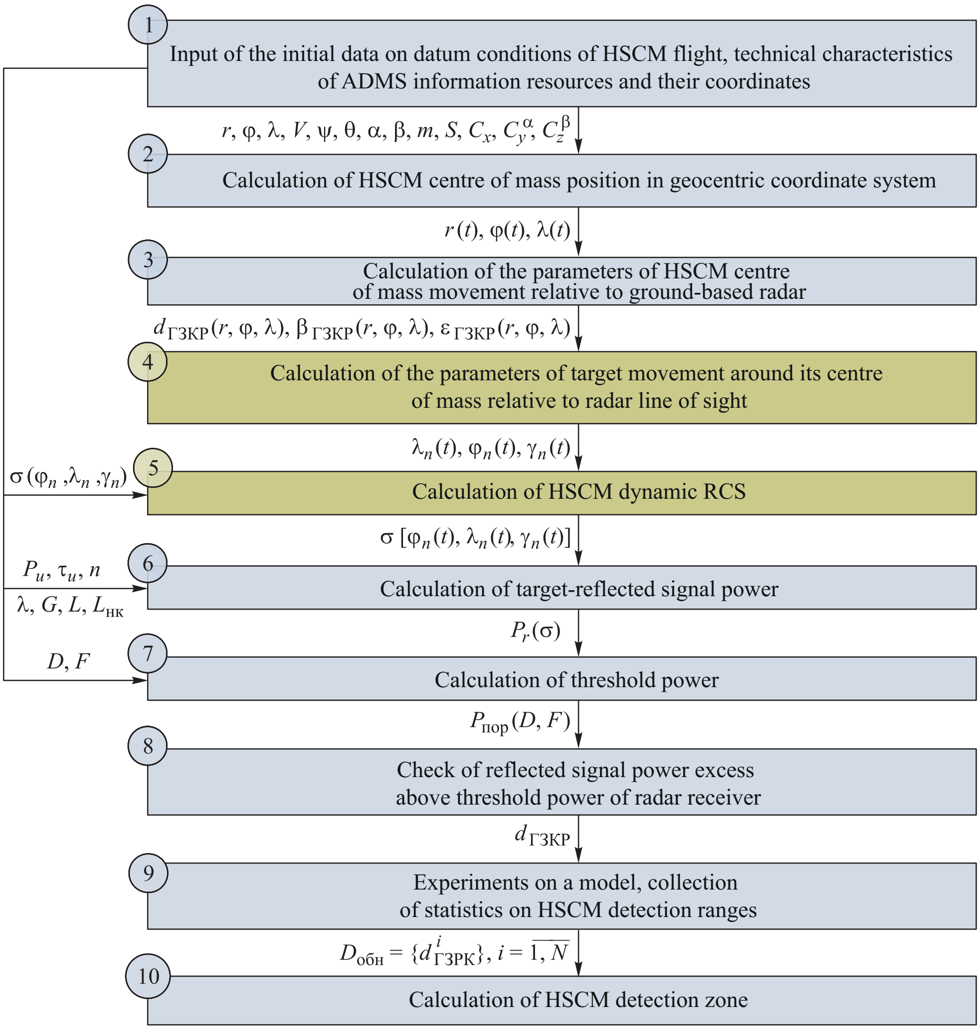

Fig. 1. Flowchart of the proposed methodology

where g_r and g_ω – projections of Earth's gravity acceleration on radius-vector $\bar{r}$ and vector $\bar{\omega}_з$;

$\omega_з$ – Earth's angular velocity;

X, Y, Z – total aerodynamic force components in projections on the axes of half-speed coordinate system:

$$X = -C_x S \frac{\rho V^2}{2};$$

$$Y = C_y^\alpha \alpha S \frac{\rho V^2}{2};$$

$$Z = C_z^\beta \beta S \frac{\rho V^2}{2}.$$

Here, $C_x, C_y^\alpha, C_z^\beta$ – aerodynamic coefficients;

S – mid-section area;

α – incidence angle;

β – sliding angle.

Numerical integration of equations (1)–(6) by the known methods, with selected constant time-step, allows to calculate motion parameters of the HSCM centre of mass in the geocentric coordinate system.

3. For calculation of the HSCM centre of mass motion parameters relative to a ground-based radar, recalculation of coordinates r, λ, φ into the Greenwich rectangular coordinate system is performed at each simulation step:

$$X_{\text{ГЗКР}} = r\cos\varphi\cos\lambda; \quad (7)$$

$$Y_{\text{ГЗКР}} = r\cos\varphi\sin\lambda; \quad (8)$$

$$Z_{\text{ГЗКР}} = r\sin\varphi. \quad (9)$$

Then the coordinates of target centre of mass are calculated in the radar topocentric (measuring) coordinate system, first rectangular by formulas (10)–(12), then spherical by formulas (13)–(15):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{N} \begin{pmatrix} X_{\text{ГЗКР}} - X_{\text{РЛС}} \\ Y_{\text{ГЗКР}} - Y_{\text{РЛС}} \\ Z_{\text{ГЗКР}} - Z_{\text{РЛС}} \end{pmatrix}; \quad (10)$$

$$\mathbf{N} = \begin{pmatrix} -\sin B_0 \cos L_0 & -\sin B_0 \sin L_0 & \cos B_0 \\ \cos B_0 \cos L_0 & \cos B_0 \sin L_0 & \sin L_0 \\ -\sin L_0 & \cos L_0 & 0 \end{pmatrix}; \quad (11)$$

$$B_0 = \operatorname{arctg} \frac{\operatorname{tg}\varphi_0}{[1-(2\tilde{a}-\tilde{a}^2)]}; \quad (12)$$

$$d_{\text{ГЗКР}} = \sqrt{x^2+y^2+z^2}; \quad (13)$$

$$\beta_{\text{ГЗКР}} = \operatorname{arctg} \frac{z}{x}; \quad (14)$$

$$\varepsilon_{\text{ГЗКР}} = \operatorname{arctg} \frac{y}{\sqrt{x^2+z^2}}, \quad (15)$$

where **N** – matrix of transition from the Greenwich geocentric rectangular coordinate system to the topocentric system;

$X_{\text{РЛС}}, Y_{\text{РЛС}}, Z_{\text{РЛС}}$ – coordinates of radar standing point in the Greenwich geocentric rectangular coordinate system (calculated similarly to formulas (7)–(9) in accordance with $r_0, \varphi_0, \lambda_0$ specified in the initial data);

B_0 – geodetic latitude of radar standing point;

L_0 – geodetic longitude of radar standing point, $L_0 = \lambda_0$;

$\tilde{a}$ – ellipticity of Earth;

$d_{\text{ГЗКР}}$ – slant range to target (HSCM centre of mass);

$\beta_{\text{ГЗКР}}$ – target azimuth;

$\varepsilon_{\text{ГЗКР}}$ – target elevation angle.

4. The parameters of target movement around its centre of mass relative to the radar and its line of sight are calculated, i. e. angles of nutation $\varphi_n(t)$, precession $\lambda_n(t)$, and self-rotation $\gamma_n(t)$.

To do that, it is necessary to calculate the matrix of guiding cosines between the bound and the sighting coordinate systems:

$$\mathbf{M} = \mathbf{C}^T \mathbf{N} \mathbf{P}^T \mathbf{L}^T \mathbf{S}^T,$$

where **C** – matrix of transition from topocentric coordinate system to the sighting system;

N – matrix of transition from Greenwich geocentric rectangular coordinate system to the topocentric system;

P – matrix of transition from local geographic coordinate system to the Greenwich geocentric rectangular system;

L – matrix of transition from local geographic coordinate system to the half-speed system;

S – matrix of transition from half-speed coordinate system to the bound system.

Angles φ_n, λ_n and γ_n are calculated from matrix **M** [4]:

$$\varphi_n = \arccos(m_{11}),\ \ \varphi_n \in [0, 2\pi],$$

$$\lambda_n = \arcsin\frac{m_{21}}{\sqrt{1-m_{11}^2}},\ \ \lambda_n \in [0, 2\pi],$$

$$\gamma_n = \arcsin\frac{m_{12}}{\sqrt{1-m_{11}^2}},\ \ \gamma_n \in [0, 2\pi],$$

where m_{11}, m_{12}, m_{21} – respective elements of matrix **M**.

5. In accordance with HSCM backscattering diagram $\sigma(\varphi_n, \lambda_n, \gamma_n)$ given in the initial data, the current RCS value is calculated, depending on target attitude relative to the radar line of sight $\sigma[\varphi_n(t), \lambda_n(t), \gamma_n(t)]$ (dynamic RCS).

6. The target backscattered signal power is measured at the radar receiver input:

$$P_r[\sigma(t)] = \frac{P_t G(\varepsilon,\beta)^2 \lambda^2 \sigma(\varphi_n(t), \lambda_n(t), \gamma_n(t)) F_t^2 F_r^2}{(4\pi)^3 d_{\text{ГЗКР}}^4},$$

where P_t – power of signal radiated by the transmitting antenna;

$G(\varepsilon,\beta)$ – antenna gain factor;

λ – radar wavelength;

F_t – pattern propagation factor of the Earth and troposphere (interference multiplier) along the path transmitting antenna – target;

F_r – similar factor along the path target – receiving antenna [5].

7. Then the minimum signal power at the receiver input is calculated, at which the signal can be detected with specified probabilities of correct detection D and false alarm F (threshold power) [6]:

$$P_{\text{пор}} = q_{\text{пор}} \frac{k_{\text{ш}} kTLL_{\text{нк}}}{n\tau_{\text{и}}};$$

$$q_{\text{пор}} = \frac{1}{n}\left(\frac{\ln F}{\ln D} - 1\right),$$

where $k_{\text{ш}}$ – receiver noise factor;

k – Boltzmann’s constant;

T – temperature;

L – factor of losses due to non-optimal receiving conditions;

$L_{\text{нк}}$ – losses for non-coherent integration;

n – number of pulses.

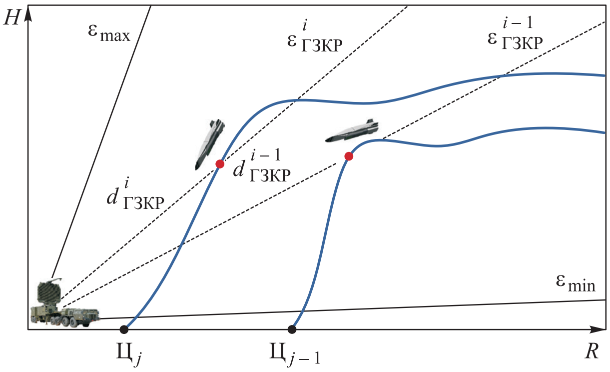

Fig. 2. Calculation of HSCM detection ranges under different values of target elevation angle

8. As the next step, the condition of power excess of target-reflected threshold-power signal at the radar receiver input is checked:

$$P_r[\sigma(t)] \geq P_{\text{пор}}. \tag{16}$$

If condition (16) is satisfied, HSCM is considered detected at range $d_{\text{ГЗКР}}$, with elevation angle $\varepsilon_{\text{ГЗКР}}$ and specified detection quality indices D and F.

9. Using statistical simulation method [7], a required number of tests N are performed on the model (stages 1–8 of the method) for obtaining statistical data of HSCM detection ranges $D_{\text{обн}} = \{d^i_{\text{ГЗКР}}\}$, $i = \overline{1, N}$ under different values of target elevation angle (Fig. 2). In so doing, target coordinates $Ц_j = (r^j_{\text{ц}}, \lambda^j_{\text{ц}}, \varphi^j_{\text{ц}})$ and the initial conditions of simulation (HSCM centre of mass position, orientation relative to velocity vector, target speed and velocity vector orientation) must vary randomly within a specified range.

10. Based on the obtained statistical data, the HSCM detection zone is calculated.

For that, estimates of mathematical expectation and dispersion of HSCM detection range are calculated for required directions by elevation angle ($\varepsilon^k_{\text{ГЗКР}}$, $k = \overline{\varepsilon_{\min}, \varepsilon_{\max}}$) with specified constant increment $\Delta\varepsilon$:

$$\tilde{m}_d^k = \frac{\sum_{i=1}^{N} d^i_{\text{ГЗКР}}}{N};$$

$$\tilde{D}_k = \frac{\sum_{i=1}^{N} (d^i_{\text{ГЗКР}} - \tilde{m}_d^k)^2}{n-1},$$

where $d^i_{\text{ГЗКР}}$ – HSCM detection range obtained in the i-th test, corresponding to elevation angle $\varepsilon^k_{\text{ГЗКР}}$;

N – total number of tests on the model.

Calculated next is horizontal range and altitude of HSCM detection, corresponding to the k-th value of target elevation angle with account of the spherical Earth:

$$\tilde{R}_k = \tilde{m}^k_d \cos(\varepsilon^k_{\text{ГЗКР}});$$

$$\tilde{H}_k = \tilde{R}_k \sin \varepsilon^k_{\text{ГЗКР}} + \frac{\tilde{R}_k^{\,2}}{2R_э},$$

where $R_э$ – effective Earth's radius.

Further, vertical section of HSCM detection zone is calculated by plotting on the coordinate grid the obtained values of $\tilde{R}_k$, $\tilde{H}_k$ by each elevation direction $\varepsilon^k_{\text{ГЗКР}}$, and a closed curve, bounding the detection zone, is constructed (Fig. 3).

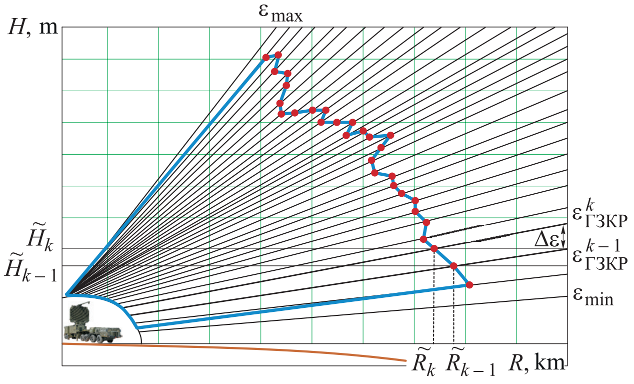

Fig. 3. Detection zone vertical section

Hence, the proposed methodology makes it possible to evaluate the effectiveness of ADMS information resources when detecting an HSCM, with account for target's dynamic RCS, and can be used in forming characteristics and comparing alternative options of prospective information resources with the aim of improving ADMS effectiveness under conditions of HSCM application by the enemy. The line of subsequent investigation is implementation of software support for the simulation model and obtaining assessments according to the methodology.

Bibliography

1. Voennaya kibernetika: metodologiya obosnovaniya napravleniy razvitiya zenitnogo raketnogo vooruzheniya i sinteza zenitnykh raketnykh sistem / pod red. *A. S. Sumina*, *Yu. I. Arepina*. M.: VIMI, 1997. 399 s. (Russian)

2. Imitatsionnoye modelirovaniye boyevykh deistviy: teoriya i praktika / pod red. *P. A. Sozinova*, *I. N. Glushakova*. Tver', 2013. 528 s. (Russian)

3. *Razorenov G. N.*, *Bakhramov E. A.*, *Titov Yu. F.* Sistemy upravleniya letatel'nymi apparatami (ballisticheskimi raketami i ikh golovnymi chastiami): uchebnik dlia vuzov / pod red. *G. N. Razorenova*. M.: Mashinostroyenie, 2003. 584 s. (Russian)

4. Radiolokatsionnye kharakteristiki letatel'nykh apparatov / *M. E. Varganov*, *Yu. S. Zinovyev*, *L. Yu. Astanin* i dr.; pod red. *L. T. Tuchkova*. M.: Radio i svyaz', 1985. 236 s. (Russian)

5. Spravochnik po radiolokatsiyi. V 4 t. T. 1. Osnovy radiolokatsiyi / pod red. *K. N. Trofimova*, *Ya. S. Itskhoki*. M.: Sovetskoye radio, 1976. 456 s. (Russian)

6. *Bakulev P. A.* Radiolokatsionnye sistemy: uchebnik dlia vuzov. M.: Radiotekhnika, 2015. 440 s. (Russian)

7. *Sirota A. A.* Kompyuternoye modelirovanie i otsenka effektivnosti slozhnykh sistem. M.: Tekhnosfera, 2006. 280 s. (Russian)

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Smirnov Mikhail Antonovich – Service Student, Federal State Military Educational Institution of Higher Professional Education "Military Aerospace Defence Academy named after Marshal of the Soviet Union G. K. Zhukov", Russian Federation Ministry of Defence, Tver'.
Science research interests: evaluation of tactical effectiveness, simulation modelling.

Методика оценки эффективности информационных средств ЗРК (ЗРС) при обнаружении ГЗКР с учетом динамической ЭПР цели

Предложена методика, позволяющая оценить эффективность информационных средств зенитного ракетного комплекса (зенитной ракетной системы) при обнаружении гиперзвуковой крылатой ракеты с учетом динамической эффективной площади рассеяния цели. Оценка эффективности осуществляется путем проведения статистических испытаний на имитационной модели и построения вертикального сечения зоны обнаружения. При этом расчет эффективной площади рассеяния гиперзвуковой крылатой ракеты проводится в зависимости от параметров движения цели вокруг центра масс относительно наземной радиолокационной станции обнаружения и ее линии визирования.

Ключевые слова: эффективность, зенитный ракетный комплекс, информационные средства, гиперзвуковая крылатая ракета, динамическая эффективная площадь, рассеяние цели.

Смирнов Михаил Антонович – адъюнкт Федерального государственного казенного военного образовательного учреждения высшего профессионального образования «Военная академия воздушно-космической обороны имени маршала Советского Союза Г. К. Жукова» Министерства обороны Российской Федерации, г. Тверь.
Область научных интересов: оценка эффективности боевых действий, имитационное моделирование.