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Investigation of dynamic characteristics of three-linear flow regulator

The study focuses on the dynamic model of the three-linear spool flow regulator for various solutions of the spool geometry, and for two variants of mathematical description of hydraulic damping devices. The paper describes the process of small deflection linearization of the obtained mathematical models. As a result of Laplace transformation of the mathematical models, we obtained a block diagram of the spool flow regulator operation. By using Nyquist criterion, we analyzed the spool flow regulator stability. As a result, we draw conclusions on the spool flow regulator stability, and on the various types of damping devices affecting it.

Keywords: hydraulics, spool regulators, hydraulic regulators, characteristics of spool devices, flow regulator, stability, operation modeling, linearization, Laplace transformations

Introduction

Three-linear spool-type flow regulators (hereinafter – SFR) are widely used in throttle controlled hydraulic drives to limit the volumetric flow rate supplied to the actuators in a wide load range. Depending on the hydraulic drive intended use and the SFR loading conditions, different stability and response time requirements may be specified for SFR. In most cases, if SFR is used in hoisting machine hydraulic drives, no special requirements to the response time are specified, however, the regulator operation stability must be ensured in this case.

The SFR in question is applied in a high-capacity hydraulic drive not requiring fast response time. During operation of the drive with the installed SFR, in some cases abnormal noise was observed, caused by oscillations of the SFR movable elements. The acoustic noise level and pressure fluctuations observed in this case enable speaking of resonance phenomena in the hydraulic system and SFR.

Unstable operation of SFR can be due to the resonance phenomena resulting from:

• erroneously selected geometry and physical characteristics of the SFR elements (spring rigidity, throttle orifice closure conditions in the spool and sleeve, hydraulic damping, etc.);

• fluctuations of volumetric flow rate and discharge line pressure caused by pump operation;

• load fluctuations on the hydraulic drive actuator;

• action on the actuator – spool valve – of internal perturbing forces (flow turbulence, cavitation, conditions of friction in the spool and sleeve, etc.).

This article provides the results of the work developing study [1] on the search of the zones of SFR resistance to pressure fluctuations with the spools of different geometry.

Dynamic model description

Fig. 1 shows the analytical diagram of the SFR operation dynamic model. The following assumptions were made during its building:

• hydrodynamic forces and friction forces in the spool and sleeve affecting the spool are small compared to other forces;

• the fluid is incompressible.

As the mathematical description of D1 and D2 dampers, two models were considered [2]:

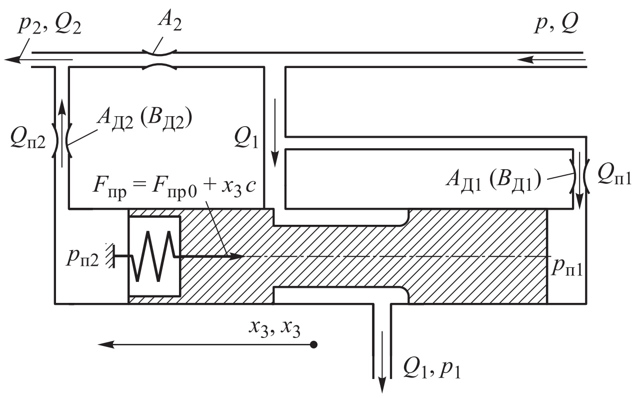

Fig. 1. Analytical diagram of the three-linear SFR dynamic model

1) quadratic – for the opening in the thin wall:

$$Q_{\text{п}1} = \sqrt{\frac{p_{\text{п}1} - p}{A_{\text{Д}1}}},\quad Q_{\text{п}2} = \sqrt{\frac{p_2 - p_{\text{п}2}}{A_{\text{Д}2}}}; \qquad (1)$$

2) linear – for the long crevice flow:

$$Q_{\text{п}1} = \frac{p_{\text{п}1} - p}{B_{\text{Д}1}},\quad Q_{\text{п}2} = \frac{p_2 - p_{\text{п}2}}{B_{\text{Д}2}}. \qquad (2)$$

The SFR stability study was conducted for three geometry options of the spool shown in Fig. 2 [1].

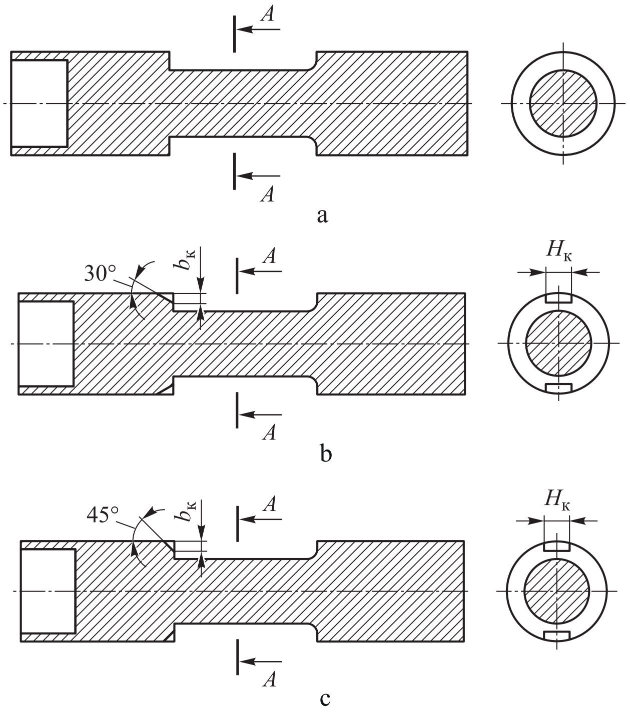

Fig. 2. Spool geometry options under consideration:
a – the first option; b – the second option;
c – the third option

Dependence of the throttle orifice local drag coefficient in the spool and sleeve on the spool position $A_1(x)$ for the first option:

$$A_1(x) = \frac{\zeta\rho}{2(\pi D)^2}\frac{1}{x^2} = k_1 x^{-2};$$

for the second option:

$$A_1(x) = \frac{\zeta\rho}{2(H_\text{к} b_\text{к} \sin(\alpha_2))^2}\frac{1}{x^2} = k_2 x^{-2};$$

for the third option:

$$A_1(x) = \frac{\zeta\rho}{2(H_\text{к} b_\text{к} \sin(\alpha_3))^2}\frac{1}{x^2} = k_3 x^{-2};$$

in general form:

$$A_1(x) = k_i x^{-2},$$

where ζ – flow coefficient for calculations of $\zeta = 2$ [2];

ρ – working fluid density for calculations of $\rho = 860$ kg/m^3;

α_i – spool groove angle, for the second option $\alpha_2 = 30°$, for the third option – $\alpha_3 = 45°$;

i – option number.

The SFR dynamic mathematical model corresponding to the analytical diagram described is a system of differential algebraic equations:

$$\begin{cases} m\ddot{x} = -F_{\text{пр}0} - xc - p_{\text{п}2} f_3 + p_{\text{п}1} f_3; \\ Q = Q_1 + Q_2 + Q_{\text{п}1}; \\ Q_2 = \sqrt{\dfrac{p - p_2}{A_2}}; \\ Q_1 = \sqrt{\dfrac{p - p_1}{k_i x^{-2}}}; \\ p_{\text{п}2} - p_2 = A_{\text{Д}2} Q_{\text{п}2}^2; \\ p_{\text{п}1} - p = A_{\text{Д}1} Q_{\text{п}1}^2; \\ Q_{\text{п}1} = f_3 \dot{x}; \\ Q_{\text{п}2} = f_3 \dot{x}; \end{cases} \qquad (3)$$

or

$$\begin{cases} m\ddot{x} = -F_{\text{пр}0} - xc - p_{\text{п}2} f_3 + p_{\text{п}1} f_3; \\ Q = Q_1 + Q_2 + Q_{\text{п}1}; \\ Q_2 = \sqrt{\dfrac{p - p_2}{A_2}}; \\ Q_1 = \sqrt{\dfrac{p - p_1}{k_i x^{-2}}}; \\ p_{\text{п}2} - p_2 = B_{\text{Д}2} Q_{\text{п}2}; \\ p_{\text{п}1} - p = B_{\text{Д}1} Q_{\text{п}1}; \\ Q_{\text{п}1} = f_3 \dot{x}; \\ Q_{\text{п}2} = f_3 \dot{x}, \end{cases} \qquad (4)$$

where m – spool weight;

$\ddot{x}$ – spool acceleration;

$p_{\text{п}1}$, $p_{\text{п}2}$ – pressure in spool piston underside cavities;

$Q_{\text{п}1}$, $Q_{\text{п}2}$ – volumetric flow from spool piston underside cavities caused by spool motion;

$A_{Д1}, A_{Д2}, B_{Д1}, B_{Д2}$ – drag coefficients of the quadratic and linear damping throttles. In the original SFR design the linear damping throttle Д2 is not provided; however, $A_{п2}$ and $B_{п2}$ coefficients are introduced in the mathematical model to evaluate the impact of potential installation thereof;

$\dot{x}$ – spool speed.

Model linearization

Systems (1) and (2) are the systems of non-linear differential equations. To investigate the stability they must be linearized. In this work a small-deflection method was applied for linearization of the system of equations.

Let us introduce the conventions: p_2, p – input variables; x – output variable; $p_{2.0}, p_0, x_0$ – variable values in the linearization point; p_2', p', x' – small deflections in the area of the linearization point. Further expressions are written as applicable to the system of equations (1).

According to study [3], if we decompose a non-linear function $F(u,y,t)$ into Taylor's series about the values u_0, y_0 and omit the addends containing deflections u' and y' to the power higher than one, we obtain:

$$F(u,y,t) = F(u_0,y_0,t_0) + \left.\frac{\partial F}{\partial u}\right|_{\substack{u=u_0\\ y=y_0}} u' + \left.\frac{\partial F}{\partial y}\right|_{\substack{u=u_0\\ y=y_0}} y'. \quad (5)$$

The first equation of the system (1), considering equations for dampers Д1, Д2 and flow rates $Q_{п1}$ and $Q_{п2}$ will look as follows:

$$m\ddot{x} = -F_{пр0} - xc - A_{Д2}(\dot{x}f_3)^2 f_3 -$$
$$-p_2 f_3 + p f_3 - A_{Д2}(\dot{x}f_3)^2 f_3$$

or

$$m\ddot{x} + (A_{Д2} + A_{Д1}) f_3^3 \dot{x}^2 + cx + F_{пр0} + p_2 f_3 - p f_3 = 0.$$

Then after linearization

$$m\ddot{x}_0 + (A_{Д2} + A_{Д1}) f_3^3 \dot{x}_0^2 + cx_0 + F_{пр0} + p_{2.0} f_3 - p_0 f_3 +$$
$$+ m\dot{x}' + (A_{Д2} + A_{Д1}) f_3^3 \dot{x}_0 \dot{x}' + cx' + p_2' f_3 - p' f_3 = 0.$$

Because the linearization point determines the equilibrium position of the system, then $\ddot{x}_0 = 0$, $\dot{x}_0 = 0$, and the sum of forces acting on the spool in the linearization point is also equal to zero, then

$$m\ddot{x}_0 + (A_{Д2} + A_{Д1}) f_3^3 \dot{x}_0^2 + cx_0 + F_{пр0} + p_{2.0} f_3 - p_0 f_3 = 0.$$

Then the equation will be as follows

$$m'\ddot{x}' + cx' + p_2' f_3 - p' f_3 = 0. \quad (6)$$

The second equation of the system (1), the equation of flow rates, considering expressions for Q_1, Q_2, and $Q_{п1}$, will be as follows:

$$K_2\sqrt{p - p_2} + k_i^{-0,5} x\sqrt{p - p_1} + f_3\dot{x} - Q = 0,$$

where $K_2 = \dfrac{1}{\sqrt{A_1}}$.

Then after application of equation (5)

$$K_2\sqrt{p_0 - p_{2.0}} + k_i\sqrt{p_0 - p_1}\,x_0 + f_3\dot{x}_0 - Q +$$
$$+ 0{,}5K_2(p_0 - p_{2.0})^{-0,5} p' - 0{,}5K_2(p_0 - p_{2.0})^{-0,5} p_2' +$$
$$+ 0{,}5k_i x_0 (p_0 - p_1)^{-0,5} p' + k_i\sqrt{p_0 - p_1}\,x' + f_3\dot{x}' = 0.$$

Because the linearization point determines the system equilibrium position, then

$$K_2\sqrt{p_0 - p_{2.0}} + k_i\sqrt{p_0 - p_1}\,x_0 + f_{п1}\dot{x}_0 - Q = 0.$$

Let us write down the linearized equation

$$0{,}5K_2(p_0 - p_{2.0})^{-0,5} p' - 0{,}5K_2(p_0 - p_{2.0})^{-0,5} p_2' +$$
$$+ 0{,}5k_i x_0 (p_0 - p_1)^{-0,5} p' + k_i\sqrt{p_0 - p_1}\,x' + f_3\dot{x}' = 0.$$

Having expressed p' from the equation, we obtain

$$p' = \frac{0{,}5K_2(p_0 - p_{2.0})^{-0,5}}{0{,}5\left(K_2(p_0 - p_{2.0})^{-0,5} + k_i x_0 (p_0 - p_1)^{-0,5}\right)} p_2' -$$
$$- \frac{k_i\sqrt{p_0 - p_1}}{0{,}5\left(K_2(p_0 - p_{2.0})^{-0,5} + k_i x_0 (p_0 - p_1)^{-0,5}\right)} x' - \quad (7)$$
$$- \frac{f_3}{0{,}5\left(K_2(p_0 - p_{2.0})^{-0,5} + k_i x_0 (p_0 - p_1)^{-0,5}\right)} \dot{x}' = 0.$$

Based on equation (7), expression (6) will be as follows

$$m\ddot{x}' + cx' + p_2' f_3 - \frac{0{,}5 f_3 K_2 (p_0 - p_{2.0})^{-0,5}}{0{,}5\left(K_2(p_0 - p_{2.0})^{-0,5} + k_i x_0 (p_0 - p_1)^{-0,5}\right)} \times$$
$$\times p_2' + \frac{f_3 k_i \sqrt{p_0 - p_1}}{0{,}5\left(K_2(p_0 - p_{2.0})^{-0,5} + k_i x_0 (p_0 - p_1)^{-0,5}\right)} x' +$$
$$+ \frac{f_3^2}{0{,}5\left(K_2(p_0 - p_{2.0})^{-0,5} + k_i x_0 (p_0 - p_1)^{-0,5}\right)} \dot{x}' = 0$$

or

$$m\ddot{x}'+\frac{f_3^2}{0,5(K_2(p_0-p_{2.0})^{-0,5}+k_i x_0(p_0-p_1)^{-0,5})}\dot{x}'+$$
$$+\left(\frac{f_3 k_i\sqrt{p_0-p_1}}{0,5(K_2(p_0-p_{2.0})^{-0,5}+k_i x_0(p_0-p_1)^{-0,5})}+c\right)x'+ \quad (8)$$
$$+1-\left(\frac{0,5K_2(p_0-p_{2.0})^{-0,5}}{0,5(K_2(p_0-p_{2.0})^{-0,5}+k_i x_0(p_0-p_1)^{-0,5})}\right)\times$$
$$\times f_3 p_2' = 0.$$

Having standardized equation (8), we obtain

$$a_1\ddot{x}'+a_2\dot{x}'+a_3x'=bp_2'-k_{o.c}x',$$

where $a_1 = m$,

$$a_2=\frac{f_3^2}{0,5\left(K_2(p_0-p_{2.0})^{-0,5}+k_i x_0(p_0-p_1)^{-0,5}\right)};$$

$$a_3 = c;$$

$$b=\left(\frac{0,5K_2(p_0-p_{2.0})^{-0,5}}{0,5\left(K_2(p_0-p_{2.0})^{-0,5}+k_i x_0(p_0-p_1)^{-0,5}\right)}-1\right)f_3;$$

$$k_{o.c}=\frac{f_3 k_i\sqrt{p_0-p_1}}{0,5\left(K_2(p_0-p_{2.0})^{-0,5}+k_i x_0(p_0-p_1)^{-0,5}\right)}.$$

System (4) may be linearized in a similar way. Then the system of equations (4) can be written as the following equation

$$a_1\ddot{x}'+a_2\dot{x}'+a_3x'=bp_2'-k_{o.c}x'; \quad (9)$$

$$a_1\ddot{x}'+a_{22}\dot{x}'+a_3x'=bp_2'-k_{o.c}x', \quad (10)$$

where $a_{22}=\frac{f_3^2}{0,5\left(K_2(p_0-p_{2.0})^{-0,5}+k_i x_0(p_0-p_1)^{-0,5}\right)}+$

$$+(B_{Д1}+B_{Д2})f_3.$$

Equations (9) and (10) differ from each other only by the constants at variable $\dot{x}'$ (a_2 and a_{22}), consequently, with the admissions made, the quadratic dampers described as the local drag, according to equation (1), have no effect on the system stability.

Considering type similarity of equations (9) and (10), let us then omit the designation of coefficient a_{22} and substitute it for a_2.

To analyze the stability of the systems described using equations (9) and (10), the Nyquist frequency criterion was used.

Let us designate: s – complex variable in the image space, $X(s)$ – image function corresponding to the original function $x(t)$, $P_2(s)$ – image function corresponding to the original function $p_2(t)$. After transition to the image space and standardization, equations (9) and (10) will be as follows:

$$(T_2^2s^2+T_1s+1)X(s)=KP_2(s)-K_{o.c};$$

$$(T_2^2s^2+T_1s+T_0)X(s)=KP_2(s)-K_{o.c}X(s), \quad (11),$$

where $T_2=\sqrt{a_1}$;

$T_1 = a_2$;

$T_0 = a_3$;

$K = b$;

$K_{o.c} = k_{o.c}$.

Fig. 3 (a) shows the block diagram of a negative feedback SFR with the feedback ratio $K_{o.c}$.

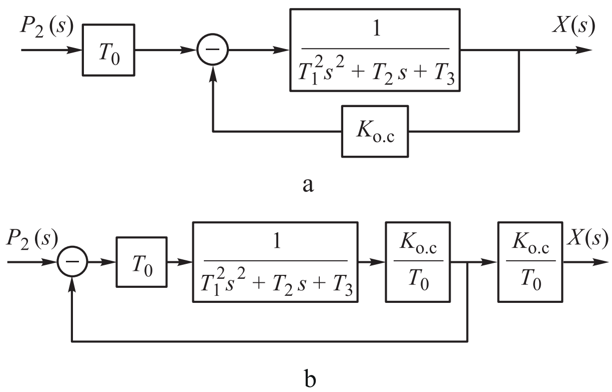

Fig. 3. Block diagrams of closed system and respective unity feedback system:
a – closed block diagram of SFR; b – respective open block diagram of the unity feedback SFR

Fig. 3 (b) shows the SFR block diagram scaled to the unity feedback view. The transfer function of such a system is as follows:

$$W(s)=\frac{Ks}{T_2^2s^2+T_1s+T_0+K_{o.c}}. \quad (12)$$

Stability study

The SFR stability was studied for its operation conditions with the following parameters $p_{2.0}$ = (1, 5, 12, 20) MPa, $p_{1.0}$ = 0 Pa, A_2 = 7.2 × 1013 kg/m^7,

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$B_{Д1} = 0$, $B_{Д2} = 1.379 \cdot 10^{12} \text{kg}/(\text{m}^4 \cdot \text{s})$. The linearization points $(p_{2.0}, p_0, x_0)$, for which the system stability was determined, were calculated using the static model of SFR operation [1]:

$$\begin{cases} p - p_2 = A_2 Q_2^2; \\ Q = Q_1 + Q_2; \\ p - p_1 = k_i x^{-2} Q_1^2; \\ (p - p_2) f_3 = F_{пр0} + xc. \end{cases}$$

According to the Nyquist criterion of stability, the closed system is stable if its equivalent open system is stable and the amplitude-phase-frequency characteristic (APFC) thereof does not envelop point $(-1, j_0)$ [4].

The conducted analysis showed stability of the open system with the transfer function (12) on Hurwitz criterion.

The results of build-up of the APFC of a system with transfer function (12) are provided in Fig. 4.

The curves in Fig. 4 indicate that the APFC of the open system 12 envelops point $(-1, j_0)$, in none of the considered cases, which means that the system (11) is stable. Moreover, the largest stability margin both in terms of amplitude and in terms of phase have APFC systems with Case 1 geometry and a linear damper.

Conclusion

The conducted SFR stability analysis enables concluding that with the assumptions made:

- SFR of the design in question retains stability with unsteady pressure performance at load p_2 and at the inlet p;
- none of the considered spool geometry options cause unstable operation of SFR.

The results also showed that in fact the SFR performance stability is not affected by the dampers with the mathematical description similar to that of the local drag (orifice, opening in the thin wall). Herewith, the dampers described by the linear dependence of pressure on flow rate (slot opening) significantly affect the SFR stability margin.

Based on the study results, it is possible to conclude that neither input pressure or flow rate fluctuations, nor SFR geometry cause unstable operation thereof and the reasons should be sought in the internal factors – effect of internal hydrodynamic forces and friction conditions in the spool and sleeve.

During preparation and development of the study methodology the following references [5–7] were also used.

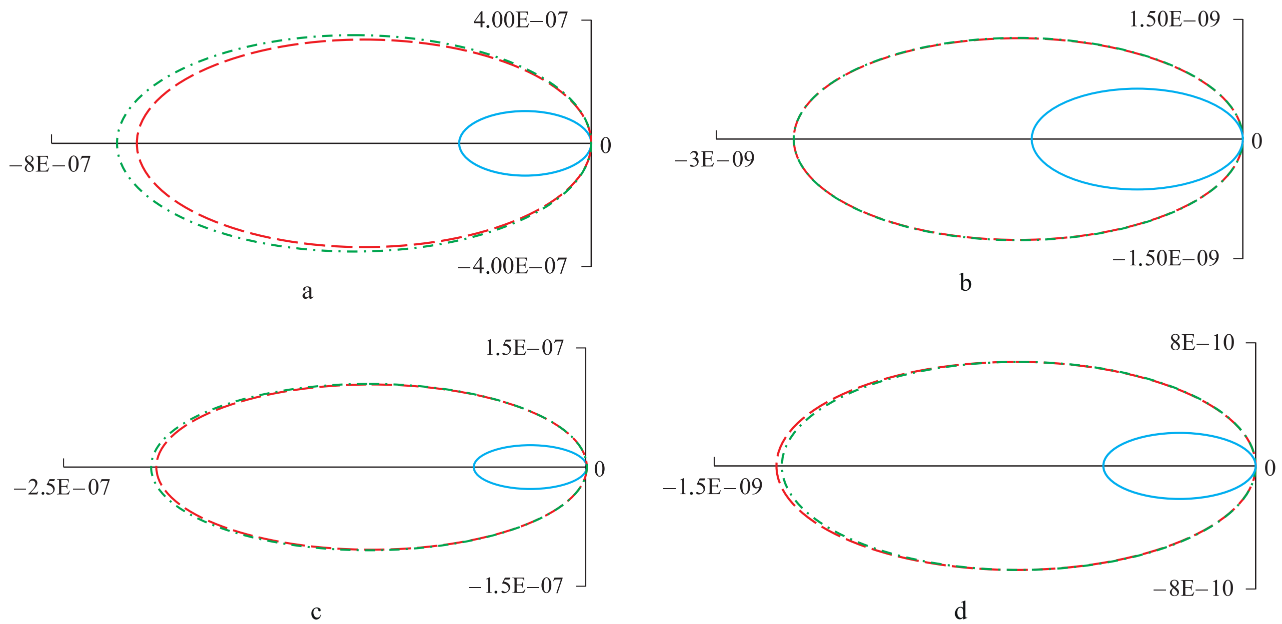

Fig. 4. SFR APFC at various parameters:
—— – the first option; – – – the second option; -·-· – the third option; a – $p_{2.0}$ = 1 MPa, quadratic damper; b – $p_{2.0}$ = 1 MPa, linear damper; c – $p_{2.0}$ = 5 MPa, quadratic damper; d – $p_{2.0}$ = 5 MPa, linear damper

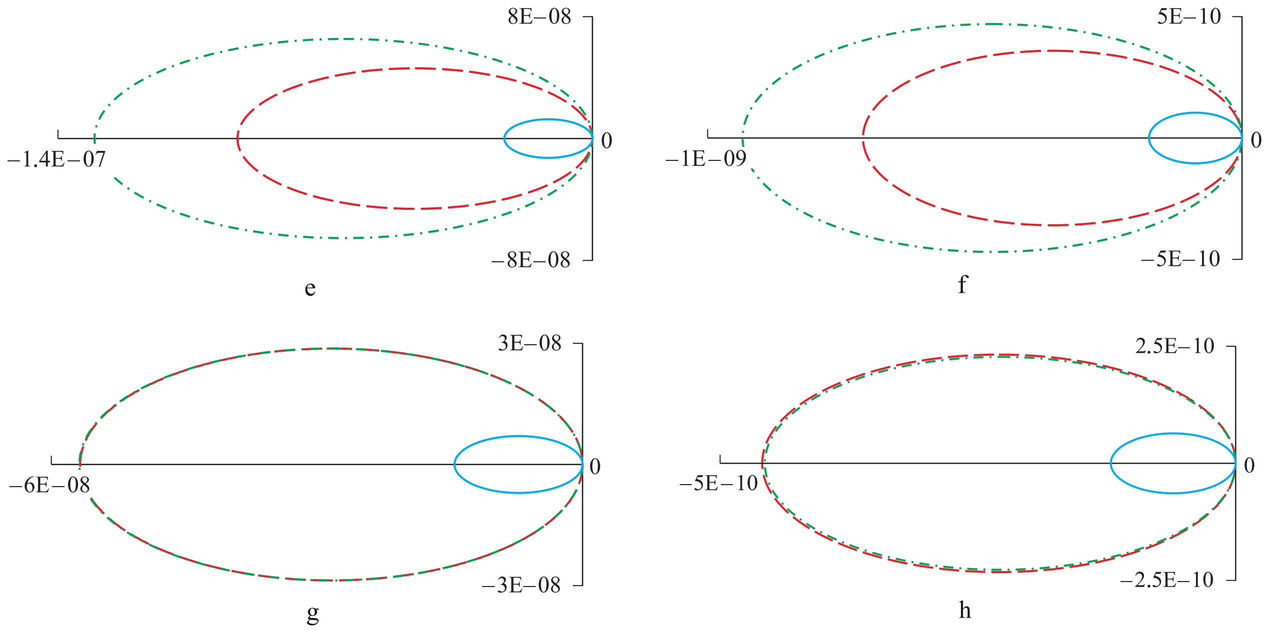

Fig. 4. SFR APFC at various parameters:
—— – the first option; – – – – the second option; -·-·– –the third option; e – $p_{2.0}$ = 12 MPa, linear damper; f – $p_{2.0}$ = 12 MPa, linear damper; g – $p_{2.0}$ = 20 MPa, quadratic damper; h – $p_{2.0}$ = 20 MPa, linear damper

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Исследование динамических характеристик трехлинейного регулятора расхода

Рассмотрена динамическая модель работы трехлинейного золотникового регулятора расхода для различных решений геометрии золотника, а также для двух вариантов математического описания демпфирующих гидравлических устройств. Описан процесс линеаризации полученных математических моделей методом малых отклонений. Приведена структурная схема функционирования золотникового регулятора расхода, полученная в результате преобразований математических моделей по Лапласу. Проведен анализ устойчивости золотникового регулятора расхода с использованием критерия Найквиста. Сделаны выводы об устойчивости золотникового регулятора расхода, а также о влиянии на нее демпфирующих устройств различных типов.

Ключевые слова: гидравлика, золотниковые регуляторы, гидравлические регуляторы, характеристики золотниковых устройств, регулятор расхода, устойчивость, моделирование работы, линеаризация, преобразования по Лапласу.

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