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Algorithms for optimizing the aircraft flight route

The study introduces the algorithms for optimizing the flight routes of an aircraft when it is bypassing a stationary dangerous zone, which is considered to be the destruction envelope of the air defense missile system, and when it is intercepting an air target. Optimization of the bypass route involves the procedures for finding a safe corridor and building a bypass route with a minimum length. Optimization of the interception route is based on determining the analytical dependence of the penetration distance of the intercepted target on the averaged values of the altitude-velocity characteristics of each of the route segments, and on finding the conditions for the minimum distance of target penetration into the protected airspace.

Keywords: route optimization, flight safety, danger zone, penetration limit, interception of air targets

Introduction

Optimization of aircraft (A/C) flight route with regard to the real environment in the flight area comes down to a variational problem and, if generally formulated, does not have any comprehensive solution. Constant development of aviation engineering and technologies and extension of aviation equipment applications justify the need for finding new algorithmic solutions in order to improve the operational air force effectiveness.

Analysis of recent experience gained in military exercises and conflicts proves the relevance of the problem related to plotting the aircraft flight route with regard to danger arising when an aircraft has to fly in an anti-aircraft weapon engagement zone (friendly and enemy missile engagement zones), as well as the need to reduce the penetration range for a target flying into the prohibited airspace.

Danger area avoidance route calculation algorithm

Flight risk assessment for an aircraft flying in the operational area of surface-to-air missile complexes (SAMC) shows that the longer an aircraft stays in the kill zone, the more dramatically the density of aircraft destruction probability increases [1]. Besides, with well-organized information exchange between anti-aircraft missile systems deployed as a formation, an aircraft which stayed for a short time in the kill zone controlled by one SAMC, may be intercepted by a missile right after entering the kill zone controlled by another missile complex.

Thus, the risk of shooting down an aircraft during flight in the kill zone cannot be assessed properly by means of the probability theory. The most efficient way to reduce the risk is to bypass the danger area along its outer boundary. The corresponding algorithms to calculate a route of flying around a stationary kill zone with a convex smooth boundary are described in [2–4]. However, there is a significant loss of efficiency of known solutions in case of more complex geometry of the danger area boundary, which is formed when a SAMC formation is deployed in the flight area (hereinafter referred to as the area of joint responsibility).

The developed algorithm can be used to fly around either a separate kill zone or a kill zone within the area of joint responsibility under surveillance of several missile complexes. This algorithm is based on procedures intended for searching a safe corridor and plotting the shortest flight route.

Fig. 1 shows variants of safe corridors to fly around a formation consisting of three surface-to-air missile complexes deployed at points O_1, O_2 and O_3. The geometry of the local kill zone's boundary is represented as an approximate circle.

The dotted line designates a safe corridor with a gap between the local zones AB_1B (Fig. 1, a).

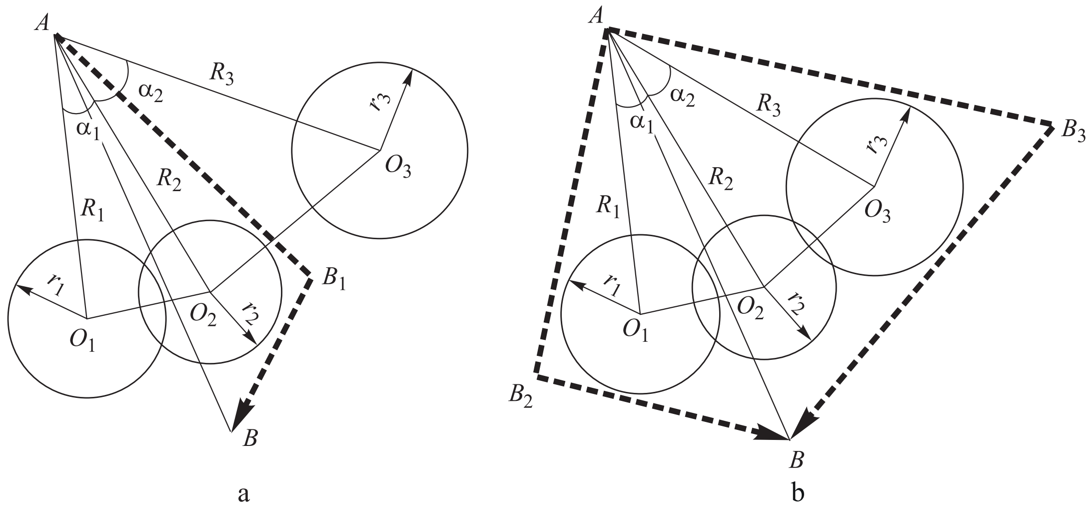

Fig. 1. Variants of safe corridors to fly around a kill zone in the area of joint responsibility if there is a gap (a) or no gap (b)

The following inequality shall be satisfied as the condition for a safe corridor:

$$\sqrt{R_i^2 + R_{i+1}^2 - 2R_i R_{i+1}\cos\alpha_i} \geq r_i(H) + r_{i+1}(H) + b_0, \quad (1)$$

where R_i – distance from the centre of the kill zone of the i-th SAMC to point A;

α_i – azimuth angle between segments AO_i and AO_{i+1};

$r_i(H)$ – radius of the horizontal section of the kill zone of the i-th SAMC at altitude H;

b_0 – selected constant determining the width of the safe flight corridor.

If there is no gap in the kill zone at altitude H, it is reasonable to investigate if the condition (1) is satisfied at a higher altitude. If the condition (1) is not satisfied for the entire flight altitude range, flying around a gapless kill zone along its left or right boundary (lines AB_2B and AB_3B in Fig. 1, b) is the only option to meet flight safety requirements. A high-priority fly-around corridor shall be selected in accordance with the predetermined criteria.

After a safe corridor is selected, we shall calculate parameters of the shortest A/C flight route from point A to point B (Fig. 2), which is based on curve $ACDB$. This curve comprises the segments of two lines tangent to the boundary of the SAMC kill zone and originating from points A and B, plus the arc of the circle adjoint to these segments.

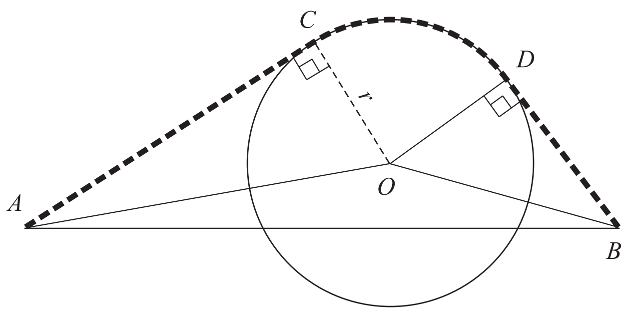

Fig. 2. Geometry of the shortest curve to fly around a danger area

Based on selected properties ΔACO and ΔBDO, let us estimate the coordinates of flight route nodal points $C(x_c, y_c)$ and $D(x_d, y_d)$:

$$x_c = \frac{-a_1^2 x_a + (y_a + y_0)a_1 + x_0}{1 + a_1^2},$$
$$y_c = \frac{a_1^2 y_0 + (x_0 + x_a)a_1 - y_a}{1 + a_1^2}, \quad (2)$$

where x_a, y_a – coordinates of point A;

$$(a_1)_{1,2} = \frac{p_1 p_2 \pm r\sqrt{p_1^2 + p_2^2 - 1}}{p_2^2 - 1};$$

$$p_1 = \frac{y_a + y_0}{r}, \; p_2 = \frac{x_a + x_0}{r}.$$

We will obtain the similar expressions for the coordinates of point D by changing x_a and y_a in the formulae (2) to x_b and y_b, respectively.

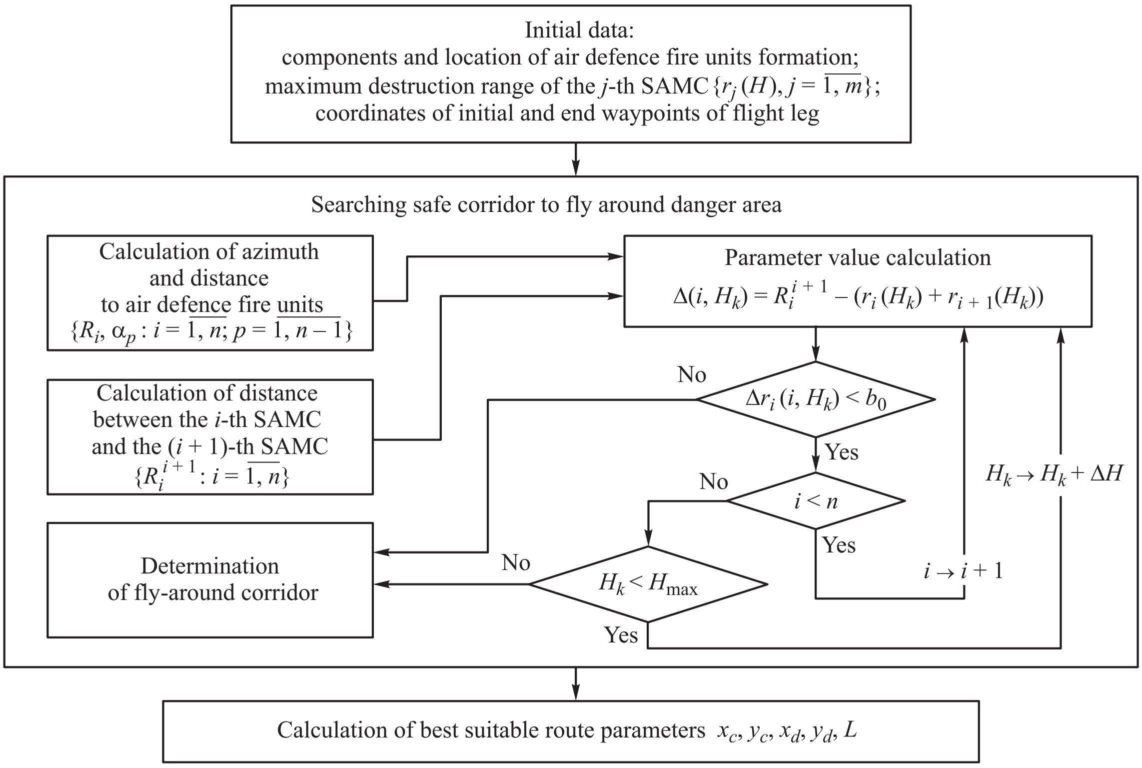

Fig. 3. Block diagram showing the algorithm for calculating the elements of the best suitable route to fly around the kill zone controlled by a SAMC formation

With the known coordinates of nodal points, we can estimate the entire length of curve *ACDB* (L), using the following expression:

$$L = \sqrt{(x_c - x_a)^2 + (y_c - y_a)^2} + \\ + \sqrt{(x_d - x_b)^2 + (y_d - y_b)^2} + \\ + r\arccos\left(1 - \frac{(x_c - x_d)^2 + (y_c - y_d)^2}{4r^2}\right). \quad (3)$$

The expressions (1)–(3) are the basis for optimizing the aircraft flight route to fly around a local kill zone or a kill zone within the area of joint responsibility. The calculation algorithm block diagram is shown in Fig. 3.

The represented algorithm allows to optimize the aircraft flight route to fly around a danger area irrespective of its boundary geometry. The effective use of a calculated route largely depends on the accuracy of information on the quantity and location of SAMC, maximum flight range of surface-to-air missiles, and on the validity of constant b_0 selection.

The optimization algorithm is also applicable for calculating the flight route to fly around a danger area associated with a man-made disaster or hazardous natural phenomena.

Air target interception route calculation algorithm

As an air-to-surface missile carrying aircraft needs to be intercepted before launching cruise missiles, i. e. at early phases of its flight, this results in the urgent problem related to the reduction of penetration range of an air target entering the protected airspace (hereinafter – penetration range). Technological prerequisites for implementing the early interception concept enable early detection of air targets by air defence information facilities and allow to increase the launch range of domestic air-to-air missiles.

In practice, to accomplish tasks involving route plotting for missile guidance, if high accuracy is not required, the simplified heuristic approach is widely used [5, 6]. This approach is based on the following:

• the most suitable motion of an aircraft in the vertical plane does not depend on the behaviour of aircraft motion in the horizontal plane;

• minimum-energy programs for flight altitude change and speed control on each flight leg for specific engine performance mode and the set aircraft flight profile.

With regard to the approach described above, we have developed an aircraft flight route optimization algorithm based on the following: determination of the analytical dependence of the penetration range of an intercepted target on averaged values of aircraft flight altitude and speed on each flight leg; selection of an option that ensures the minimum range of target penetration into the prohibited airspace with a variable length of the balanced segment and a variable air-to-air missile launch range.

Standard flight route patterns for direct interception, manoeuvre and head-on engagement, as well as for manoeuvre and tail-chase engagement are shown in Fig. 4.

To determine the analytical dependence of the penetration range of an intercepted target, the route includes balanced segment *PM*, variable length of which ensures better flexibility of the interception route parameters.

Representation of the route as a combination of segments with fixed *(AP, MD, DH)* and variable lengths allows to create a system of equations for various target approach algorithms. Such a system of equations is based on formalization of the basic triangle closure condition in order to determine unknown elements of an aircraft flight route.

According to the preliminary analysis, the best suitable aircraft flight route in case of direct interception can be plotted as a line segment, with the length of two segments to be determined depending on aircraft performance, but irrespective of target flight speed and direction [7]. In order to estimate the target penetration range $(r_{\text{пр}})$ in case of direct interception, we will obtain the following expression:

$$r_{\text{пр}} = v_3\left(t_1 + t_2 - \frac{a - \sqrt{a^2 + b\left(v_p^2 - v_3^2\right)}}{v_p^2 - v_3^2}\right), \qquad (4)$$

where $a = \left(v_3^2 - v_1 v_p\right)t_1 + t_2\left(v_3^2 - v_2 v_p\right) - S v_3 \cos\beta;$

$$b = \left(v_3^2 - v_1^2\right)t_1^2 + \left(v_3^2 - v_2^2\right)t_2^2 +$$
$$+ 2\left(v_3^2 - v_1 v_2\right)t_1 t_2 - 2S\left(t_1 + t_2\right)v_3\cos\beta + S^2;$$

v_i, t_i – interceptor’s average speed and time in flight in the *i*-th segment, $i = \overline{1,2}$;

S – initial distance between the target and the interceptor (segment *AC* in Fig. 4);

v_3, v_p – average flight speeds of the target and the missile, respectively.

One of the advantages of the selected route pattern in case of direct interception shown in Fig. 4 (a) is a possibility to find an accurate solution (4). The resulting value of the penetration range is not a comprehensive solution and its application is limited to the condition that allows to implement a straight flight route of the aircraft.

In addition to unknown time and speed characteristics, the equations formalizing triangle closure conditions in case of manoeuvring (Fig. 4, b, c) include both angle φ itself with the unknown value and its trigonometric functions. As a result,

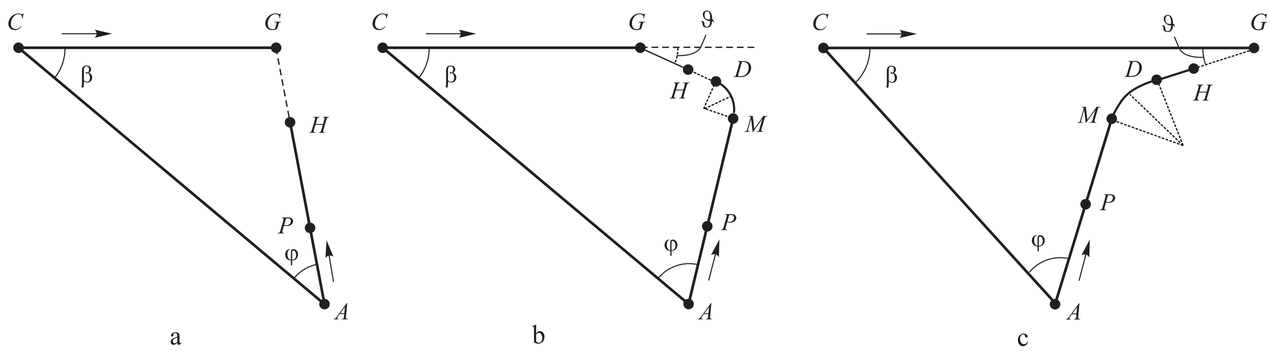

Fig. 4. Aircraft route patterns in case of direct interception (a), in case of manoeuvre and head-on engagement (b) in case of manoeuvre and tail-chase engagement (c): *A* – aircraft flight origin; *C* – initial target location; *P* – balanced flight leg origin; *CG* – target penetration range; *MD* – flight leg with aircraft manoeuvre; *H* – aircraft position at missile launch; *G* – position of target when hit by air-to-air missile

certain equations that determine interception route elements in case of manoeuvring become transcendental equations.

To overcome the ensuing complications, the penetration range in case of head-on engagement $\left(r_{\text{пр}}^{\text{ППС}}\right)$ has been obtained in the form of the function of parameter φ:

$$r_{\text{пр}}^{\text{ППС}}(\varphi) = v_4(t_1+t_2+t_3)+ \\ +v_4\frac{Sq_1(\varphi)+t_1q_2(\varphi)+t_2q_3(\varphi)+t_3q_4(\varphi)-R_0q_5(\varphi)}{\left(v_Bv_P\sin(\beta+\varphi-\theta)+v_Bv_4\sin(\beta+\varphi)-v_4v_P\sin\theta\right)\sin\theta}, \quad (5)$$

where $q_1(\varphi)=\left(v_P\sin(\beta+\varphi-\theta)\sin^2+\left(v_B\sin(\beta+\varphi)-\right.\right.$
$\left.\left.-v_p\sin\theta\right)\sin\varphi\right)\sin\theta\left(\sin(\beta+\varphi)\right)^{-1};$

$$q_2(\varphi)=q_3(\varphi)-v_1v_P\sin\theta\sin(\beta+\varphi-\theta);$$

$$q_3(\varphi)=v_4\left(v_p\sin\theta-v_B\sin(\beta+\varphi)\right)\sin\theta;$$

$$q_4(\varphi)=q_3(\varphi)-v_kv_B\sin(\beta+\varphi-\theta)\sin\theta;$$

$$q_5(\varphi)=2(v_B+v_P)\sin\frac{\beta+\varphi-\theta}{2};$$

$v_i, t_i, R_0, \beta, \theta$ – preset initial data.

As for manoeuvring and tail-chase engagement, the similar procedure for calculating the penetration range has given the following estimate:

$$r_{\text{пр}}^{\text{ЗПС}}(\varphi) = v_4(t_1+t_2+t_3)+ \\ +v_4\frac{t_1k_1(\varphi)+t_2k_2(\varphi)+t_3k_1(\varphi)+Sk_3(\varphi)+R_0k_4(\varphi)}{\left(v_Bv_P\sin\alpha+v_Bv_4\sin(\beta+\varphi)-v_4v_P\sin\theta\right)\sin\theta}, \quad (6)$$

where $k_1(\varphi)=v_p\left(v_4\sin\beta-v_1\sin\varphi\right)\sin\theta+$
$+\left(v_1v_p\sin(\beta+\theta)-v_4v_B\sin\beta\right)\sin(\beta+\varphi);$

$$k_2(\varphi)=v_4\left(v_p\sin\theta-v_B\sin(\beta+\varphi)\right)\sin\beta;$$

$$k_3(\varphi)=\left(v_B\sin\varphi-v_p\sin(\beta+\theta)\right)\sin\beta;$$

$$k_4(\varphi)=\left(v_p+v_B\right)\left(\sin(\beta+\theta)\sin(\beta+\varphi)-\right. \\ \left.-\sin\theta\sin\varphi\right)\text{ctg}\frac{\beta+\theta-\varphi}{2}.$$

Further steps to determine elements of the best suitable interception route in case of manoeuvring shall include the following procedures: calculation of numerical values $r_{\text{пр}}(\varphi)$ for $\varphi\in\left[\varphi_{\min},\varphi_{\max}\right]$; calculation of values $\hat{\varphi}=\underset{\varphi\in\left[\varphi_{\min},\varphi_{\max}\right]}{\operatorname{argmin}} r_{\text{пр}}(\varphi)$; calculation of route elements at $\varphi=\hat{\varphi}$.

Results of calculation of relationship $r_{\text{пр}}^{\text{ЗПР}}(\varphi)$ are shown in Fig. 5.

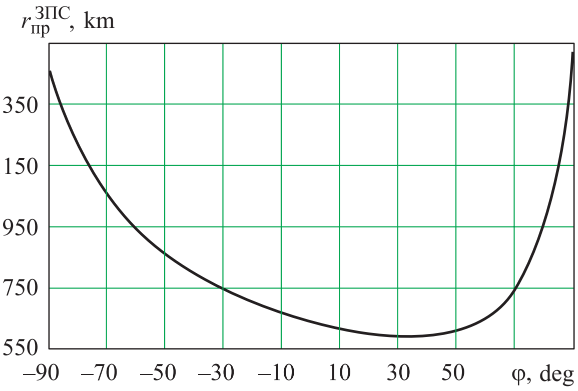

Fig. 5. Angular dependence of penetration range for manoeuvre and tail-chase engagement

The behaviour of the represented curve indicates that if φ varies within ±20° of the value at which the minimum target penetration range is achieved, an increase in the range itself does not exceed 10 % of the minimum value. Therefore, when determining the extreme value of the penetration range, it is reasonable to use sample spacing $\Delta\varphi \in [1°, 10°]$.

The expressions (4)–(6) allow to obtain numerical estimates of the aircraft flight route when using three aircraft-to-air-target approach algorithms. Fig. 6 shows a block diagram of the route elements calculation algorithm that allows to minimize the target penetration range.

This algorithm ensures optimization of the aircraft flight route by the air target penetration range, which is not a strict limitation. The scope of application of the algorithm is determined by the possibility of formalization of the optimization parameter represented as a function of route elements, the analytical dependence of which on initial data can be determined within the scope of the approach proposed above. In particular, the parameters such as aircraft flight range, air-to-air missile flight range, etc. are used for flight route optimization.

In general, the scope of correct application of the algorithm is determined by initial data values, at which the solutions (4)–(6) are

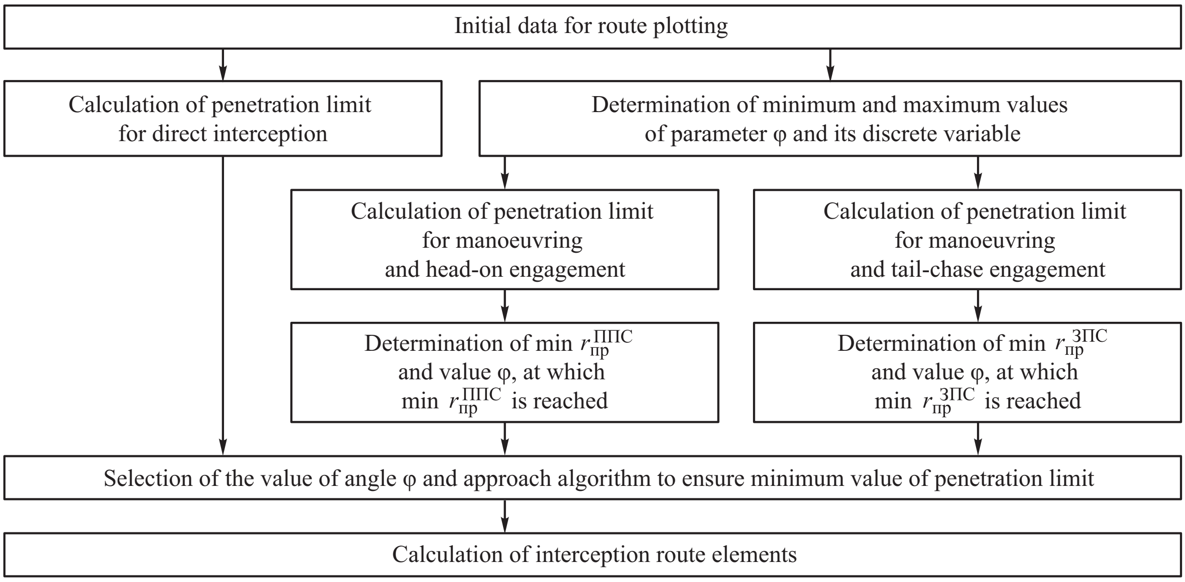

Fig. 6. Block diagram of the route elements calculation algorithm to minimize the air target penetration range

available.

Conclusion

The above-mentioned algorithms are intended to be used on various flight legs and can function independently of one another. This gives an opportunity for shared or stand-alone usage of the algorithms when calculating a general aircraft flight route.

The problem is solved in a deterministic formulation; therefore, the application of algorithms is available only if monitoring of accumulated errors is implemented (for example, in case of target position extrapolation).

The proposed algorithms for calculating route elements prove that a solution of practical importance can be found in case of incomplete formalization of the problem regarding aircraft flight route optimization. Implementation of algorithms improves computational efficiency and enables multiple recalculations of fight route characteristics in case of accumulation of errors, which may be caused, for example, by non-uniform and unequal target motion or by a change of tactical situation during interceptor flight. The algorithms can be used by air traffic control tower (command centre) personnel or in aircraft onboard systems for automation of the said calculations or for operations of air traffic controllers (combat control officers) and aircraft crew operators.

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Алгоритмы оптимизации маршрута полета летательного аппарата

Предложены алгоритмы оптимизации маршрута полета летательного аппарата при обходе стационарной опасной зоны, в качестве которой рассматривается зона поражения группировки зенитных ракетных комплексов, и при перехвате воздушной цели. Оптимизация маршрута обхода включает выполнение процедур поиска безопасного коридора и построения маршрута обхода с минимальной длиной. Оптимизация маршрута перехвата основана на определении аналитической зависимости дальности проникновения перехватываемой цели от усредненных значений высотно-скоростных характеристик каждого из участков маршрута и на нахождении условий минимальной дальности проникновения цели в охраняемое воздушное пространство.

Ключевые слова: оптимизация маршрута, безопасность полета, опасная зона, рубеж проникновения, перехват воздушных целей.

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