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Secondary radar azimuth accuracy with increased surveillance speed

The study introduces the dependence of the secondary radar aircraft azimuth accuracy on the space surveillance speed and the signal-to-noise ratio for various algorithms of the primary processing of radar data under the influence of normal Gaussian and Rice processes. The results obtained are practically applicable when upgrading software which implements the processing of radar data in existing radars.

Keywords: radar data processing, maximum likelihood method, target azimuth coordinate estimation

One of the key resources consumed during operation of a radar system (RS) with electronic or electronic-mechanical antenna beam sweeping is the resource of time. Its allocation determines the basic RS characteristics. To reduce time expenditures required for detecting aircraft (AC) and estimating their spatial coordinates, a two-stage detection and estimation procedure is applied in radars. At the first stage (surveillance) the AC search is performed at a high surveillance speed, with lower certainty of AC detection and coarse estimates of their coordinates. The second stage (estimation) is run at a slower surveillance speed, but only in the directions of LA probable location obtained as a result of surveillance stage execution. Slower surveillance speed at the second stage enables to achieve higher detection certainty and reduce coordinate estimation errors. Resulting from such time allocation, as compared with the single-stage detection and estimation procedure, the time expenditures are lower.

This paper focuses on the dependence of secondary radar azimuth estimation accuracy by the single-channel method on the surveillance speed and the signal-to-noise ratio for various algorithms of the primary processing of radar data (RD). It should be noted that the implication behind the term ‘secondary’ is that it is a radar receiving signals radiated by AC (i. e., a radar with active response).

Considering practical orientation of the study, we shall make some simplifications in derivation of the algorithms. Proceeding from this, the following actions will be done.

1. Building a model of RD primary processing by secondary radar under the influence of additive Gaussian noise on the wanted signal modulated by the radar radiation pattern (RP) envelope.

2. Developing a processing algorithm based on the maximum likelihood criterion.

3. Carrying out statistical modelling of the proposed algorithm (based on the maximum likelihood criterion) and conventional algorithm under the Gaussian noise influence. Under conventional algorithm we shall assume the algorithm based on the linear filtration with filter pulse-response characteristic corresponding to radar RP envelope.

4. Performing statistical modelling of the proposed algorithm under Rice distribution of the input process (signal-noise mixture) and comparing the obtained data with normal distribution results.

Let us consider a case when the radar with radiation time period t_α and azimuthal increment of radiations $\Delta\alpha$ is surveying a sector with width Θ. The number of radiations k in sector Θ will be defined by the expression

$$k = \frac{\Theta}{\Delta\alpha}.$$

The radar surveillance speed $\Omega\alpha$ of a sector with width Θ will be defined by the expression

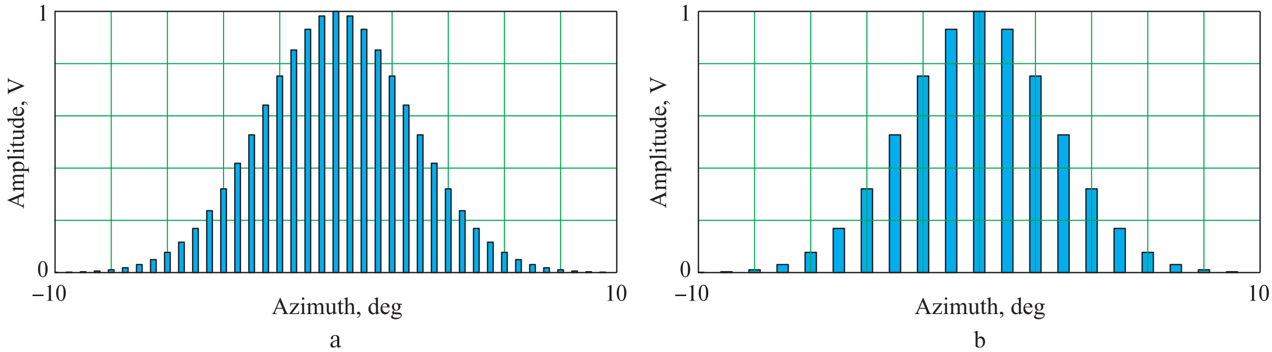

Fig. 1. Echo signal bursts (without noise influence):
a – $\Delta\alpha = 0.5°$; b – increment $\Delta\alpha = 1°$

$$\Omega\alpha = \frac{1}{kt_\alpha} = \frac{\Delta\alpha}{\Theta t_\alpha}.$$

Time period t_α and sector width Θ will be assumed constant, and increment $\Delta\alpha$ – variable. These expressions and the condition of fixed parameters are introduced so as to examine the dependence of estimation accuracy on the radiation increment, which is equivalent to dependence on the surveillance speed. For example, with radiation increment increased two-fold, the surveillance speed increases two-fold as well.

The echo signal bursts without the influence of noise, with $\Delta\alpha$ equal to 0.5 and 1°, are given in Fig. 1. The RP width as per half-power level equals to 4.5°.

Now we shall introduce a mathematical model of RD primary processing (Fig. 2).

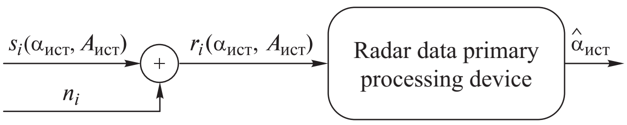

Fig. 2. Mathematical model of RD primary processing

In the model shown in Fig. 2 the following terms are used:

$s_i(\alpha_{ист}, A_{ист})$ – sampling of wanted signal modulated by radar RP;

i – sampling number;

$\alpha_{ист}$ – azimuth true value;

$A_{ист}$ – amplitude true value;

n_i – noise sampling;

$r_i(\alpha_{ист}, A_{ист})$ – sampling of wanted signal and noise mixture;

$\hat{\alpha}_{ист}$ – azimuth estimate.

The expression for model shown in Fig. 2 can be written as follows:

$$r_i(\alpha_{ист}, A_{ист}) = s_i(\alpha_{ист}, A_{ист}) + n_i. \quad (1)$$

Under Gaussian noise, the likelihood function (LF) p (r|α, A) can be represented by the expression [1]:

$$p(\mathbf{r} \mid \alpha, A) = \\ = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi}\sigma_n} \exp \frac{-\left[r_i(\alpha_{ист}, A_{ист}) - s_i(\alpha, A)\right]^2}{2\sigma_n^2}. \quad (2)$$

where $\mathbf{r} = \{r_1, r_2, \ldots, r_M\}$ – vector of signal-noise mixture samplings;

σ_n – noise root-mean-square deviation (RMSD);

M – number of echo signals in a burst;

α – azimuth assumed value;

A – amplitude assumed value.

The maximum-likelihood criterion implies searching for such a pair of α and A values at which the LF will be the highest.

It is known that the LF logarithm has the maximum value at the same α and A values as the LF itself. It makes it possible to proceed and consider the LF logarithm, which is convenient in practice, since the product by M in expression (2) changes to the sum by M in the following formula:

$$\ln p(\mathbf{r} \mid \alpha, A) = \ln \frac{1}{(\sqrt{2\pi}\sigma_n)^M} + \\ + \frac{1}{2\sigma_n^2} \sum_{i=1}^{M} -\left[r_i(\alpha_{ист}, A_{ист}) - s_i(\alpha, A)\right]^2. \quad (3)$$

Function $\ln p(\mathbf{r}|\alpha, A)$ can be represented as a 3D figure in the Cartesian coordinates X, Y, Z. Let the values of α and A be conventionally assigned to axes X and Y, and the function value – to the vertical axis Z.

When considering expression (3), it is worth mentioning that term $\ln \frac{1}{(\sqrt{2\pi}\sigma_n)^M}$ makes function $\ln p(\mathbf{r}|\alpha, A)$ shift up or down relative to axis Z, and multiplier $\frac{1}{2\sigma_n^2}$ influences function expansion or contraction along the same axis. At the same time, neither of the terms of the expression affects the position of the function maximum in the horizontal coordinates.

Term $r_i^2(\alpha_{\text{ист}}, A_{\text{ист}})$, obtained after removal of parentheses in the right-hand part of expression (3), is a constant for the considered implementation of $\mathbf{r}$ and does not depend on the values of α and A. Hence, this term influences function shifting up and down relative to axis Z but does not influence the position of the function maximum in the horizontal coordinates.

Taking this into account, we shall simplify expression (3) for more convenient practical application. Function $L(\alpha, A)$ is introduced, while those terms of the expression that do not affect the values of arguments α and A at which the LF logarithm, and hence the LF itself, have the maximum value, are excluded from the formula:

$$L(\alpha, A) = \sum_{i=1}^{M}\left[r_i(\alpha_{\text{ист}}, A_{\text{ист}}) s_i(\alpha, A) - \frac{1}{2} s_i^2(\alpha, A)\right]. \quad (4)$$

Let us write down the expression for estimation of parameters $\hat{\alpha}_{\text{ист}}$ and $\hat{A}_{\text{ист}}$ by the maximum of function $L(\alpha, A)$, which is essentially an estimate by the maximum-likelihood criterion:

$$\hat{\alpha}_{\text{ист}}, \hat{A}_{\text{ист}} = \arg\max_{\alpha, A} L(\alpha, A), \quad (5)$$

where $\hat{A}_{\text{ист}}$ – amplitude estimate.

In the proposed algorithm the maximum of function $L(\alpha, A)$ is searched for by the brute force method by α and A according to expression (5). Estimation of amplitude $\hat{A}_{\text{ист}}$ within the framework of this task is an auxiliary one, and estimation of azimuth $\hat{\alpha}_{\text{ист}}$ – the sought one.

For azimuthal coordinate estimation, linear filtration of signal-noise mixture with filter pulse-response characteristic, corresponding to the RP envelope, is customarily applied. A cross-correlation function is produced, whose argument of the maximum is the sought estimate.

Let us evaluate efficiency of the proposed algorithm in comparison with the conventional one. As the evaluation criterion, we shall use RMSD of azimuth estimation error $\sigma(\delta\alpha)$:

$$\sigma(\delta\alpha) = \sqrt{\frac{1}{N-1}\sum_{j=1}^{N}\left[\delta\alpha(j) - \frac{1}{N}\sum_{j=1}^{N}\delta\alpha(j)\right]^2}, \quad (6)$$

where N – number of samplings;

$\delta\alpha(j) = \hat{\alpha}_{\text{ист}}(j) - \alpha_{\text{ист}}(j)$ – azimuth estimation error in the j-th sampling.

It should be mentioned that selection of noise component of the estimation error as the evaluation criterion is deliberate.

The results of statistical modelling of the conventional and the proposed algorithms under the influence of additive Gaussian noise are shown in Fig. 3.

Given the wide use in radars of the quadrature method, which implies reception of signal followed by summary calculation of the modulus of a vector formed by the quadratures, we shall perform computations with account of this fact. As is known, distribution of the input process in this case becomes Rician.

It is worth reminding that the proposed algorithm was developed for a normal (Gaussian) input process. However, further below we shall purposefully undertake modelling of the proposed algorithm under Rice distribution condition. As was said in the beginning of this paper, the author had made certain assumptions conditioned by orientation towards practical application.

May it be pointed out that strictly mathematically correct would be the development of an algorithm with Rice distribution. This, however, requires radio design engineers of certain competence level, who are well familiar with the literature on the subject, e. g. [1, 2].

From the practical viewpoint, application of the approach described in this paper is

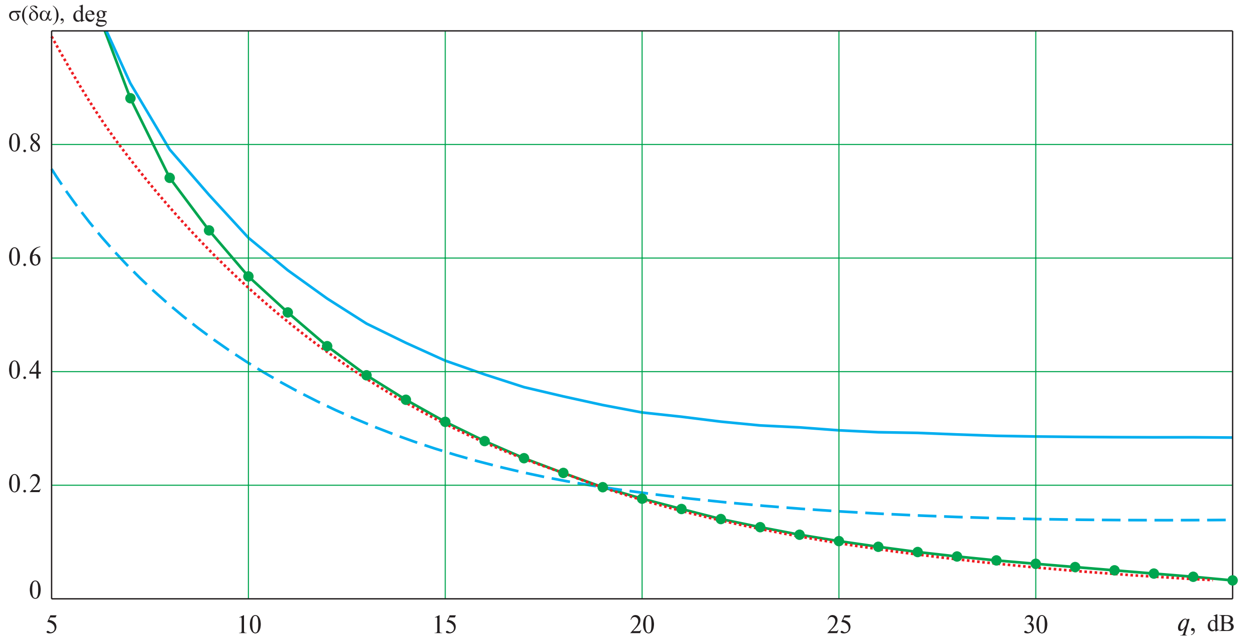

Fig. 3. Dependence of azimuth estimation error RMSD σ(δα) on signal-to-noise ratio q under Gaussian noise influence:
– Δα = 0.5° (customary algorithm); – Δα = 1° (customary algorithm); – Δα = 1° (proposed algorithm); – Δα = 1° (Kramer–Rao boundary)

considerably more simple and, at the same time, feasible. Thus, it is known that under great signal-to-noise ratios Rician distribution is drawing closer to Gaussian one. Given this, according to [2], with signal-to-noise ratio being equal to 16.9 dB or higher, substitution of Rice distribution for Gauss, when carrying out even theoretical calculations, does not entail substantial errors.

Fig. 4 illustrates the results of statistical modelling of the proposed algorithm for normal and Rician distributions.

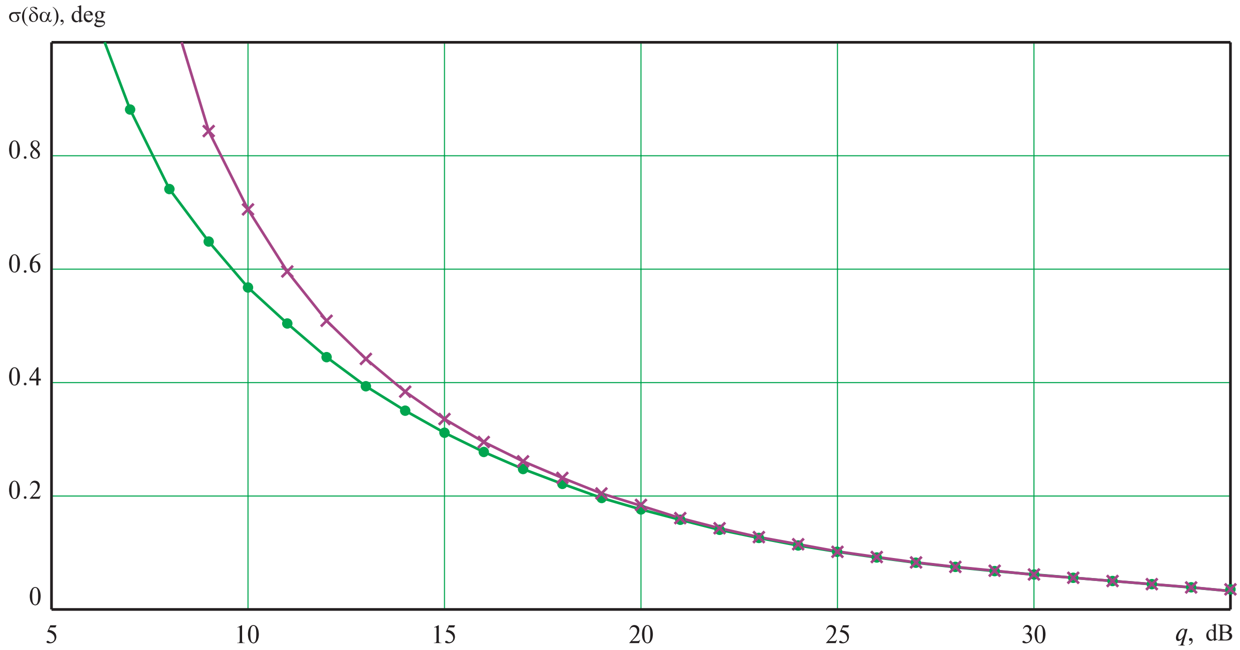

Fig. 4. Dependence of azimuth estimation error RMSD σ(δα) on signal-to-noise ratio q for the proposed processing algorithm at Δα = 1°:
– normal process; – Rician distribution

Let us consider implementation of the proposed algorithm on the base of programmable logic integrated circuit (PLIC). For processing of a burst containing up to 25 echo signals, it is desirable to have 25 hardware multiplier blocks – *DSP* blocks (if they are unavailable, the multiplying operation can be substituted for expansion and summation). Values $\sum_{i=1}^{M}\frac{1}{2}s_i^2(\alpha, A)$ from expression (4) are known a priori and can be written in the memory block in advance. Computing and finding the maximum of function $L(\alpha, A)$ under availability of 25 *DSP* blocks running in parallel, with 80 possible values of α and 16 possible values of A, on clock frequency of 100 MHz takes about 15 μs. At the same time, radiation period t_α is, as a rule, considerably greater and may reach hundreds of microseconds or units of milliseconds. A small number of possible values of A, which allows to reduce the computing time, is achieved due to normalisation of the input implementation.

May it be pointed out that in the example presented for implementation the selected constants are conditioned by the considered model: RP width about 4.5°, with radiation increment about 1° and the resultant estimation accuracy as given in Figs. 3 and 4. In so doing, the considered model relates to an existing radar.

Conclusion

Under conventional processing algorithm, the root-mean-square deviation (RMSD) of azimuth estimation error σ(δα) increases along with the increase of radiation increment Δα and, as one would expect, with the growth of the signal-to-noise ratio it tends to the value $\Delta\alpha/\sqrt{12}$.

With signal-to-noise ratio being equal to 19 dB and higher, the use of the proposed algorithm makes it possible to achieve the same azimuth estimation accuracy as under the conventional algorithm, or higher than that, by increasing two-fold the azimuthal increment of radiation and, respectively, increasing two-fold the airspace surveillance speed.

With signal-to-noise ratio over 12 dB, the estimation made according to the proposed algorithm is close to the potential accuracy [1] (Kramer – Rao boundary). With the SNR decrease, starting from 12 dB the estimation performed according to the proposed algorithm is more and more at variance with the potential accuracy. This variance is explained by the presence of so-called 'abnormal' errors [3].

As a result of the calculations, it was established that at SNR equal to 20 dB and radiation increment Δα equal to the RA width as per half-power level, the RMSD of the error of azimuth estimation made according to the proposed algorithm amounts to 1/10 of Δα, which in practice is a fairly good result.

With signal-to-noise ratio higher than or equal to 15 dB, the use of the proposed algorithm under Rician distribution of the input process makes it possible to obtain azimuth estimation error RMSD which is by no more than 10 % different from the azimuth estimation error RMSD obtained under normal distribution (and with potential accuracy). It can be claimed that the algorithm has a certain invariance to the distribution law, estimate unbiasedness and efficiency at high SNR.

With the proposed algorithm implemented on the PLIC base, the computing time for finding the maximum of function $L(\alpha, A)$ amounts to tens of microseconds, which is considerably less than radiation period t_α, which makes, as a rule, hundreds of microseconds or units of milliseconds.

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Точность измерения азимута вторичным радиолокатором при увеличенной скорости обзора

Представлена зависимость точности измерения азимутальной координаты летательного аппарата вторичным радиолокатором от скорости обзора пространства и отношения сигнал – шум при различных алгоритмах первичной обработки радиолокационной информации в условиях воздействия нормального гауссовского и райсовского процессов. Дополнительным моментом является доступность практического применения полученных результатов при модернизации программного обеспечения, реализующего обработку радиолокационной информации в существующих радиолокаторах.

Ключевые слова: обработка радиолокационной информации, метод максимального правдоподобия, оценка азимутальной координаты цели.

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