UDC 629.7.058.68

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Allan variance in dynamically tuned inertial-unit gyro error analysis

The purpose of the study was to consider an algorithm for obtaining the measurement information from a dynamically tuned gyroscope in the mode of an angular velocity sensor and output signal noise component estimate, the algorithm being based on the Allan variance method. The results obtained were evaluated.

Keywords: navigation system, Allan variance, dynamically tuned gyroscope, calculation of noise components

Introduction

The source of data in a navigation system is a gyro inertial unit (GIU), comprising gyroscopes and accelerometers. The components and characteristic features of the initial data sensors (IDS) are determined by the requirements of a particular system. Navigation system accuracy will depend on the accuracy of the sensing elements (SE) proper and that of the processing channel. The IDS used for measuring angular motion parameters in serially produced strapdown inertial navigation systems (SINS) and advanced systems manufactured at JSC Arzamas Instrument-Building Plant named after P. I. Plandin are dynamically tuned gyroscopes (DTG) in the mode of angular velocity sensors (AVS).

To improve SINS navigation accuracy, development of new approaches and methods is required for perfection of those technical characteristics that provide control of the output parameters capable of resolving the output signal into components for subsequent analysis of each one of them.

This paper offers a new algorithm and software for processing DTG output signal in the AVS mode, developed by M. A. Sharova under supervision of S. S. Diadin at JSC Arzamas Instrument-Building Plant named after P. I. Plandin. Numerical values of errors present in the data are determined and analysed.

Review of the output data evaluation methods

At present, widely used is the classical method of gyroscope output signal evaluation, according to which the total error of the output signal additive component, which can be found in each output response measurement, is the sum of random (Δ^0) and systematic (Δ_c) components:

$$\Delta = \Delta_c + \Delta^0. \qquad (1)$$

An output response measuring model is shown in Fig. 1, with the following designations introduced:

ω – true value of angular velocity;

$\tilde{\omega}$ – angular velocity with account of systematic and random components;

Ω – angular increment with account of systematic and random components.

However, electronic equipment and errors in the gyroscope output signal produce additional error, commensurate with the error of instrument itself; such error cannot be detected by the classical method, which is the latter’s drawback.

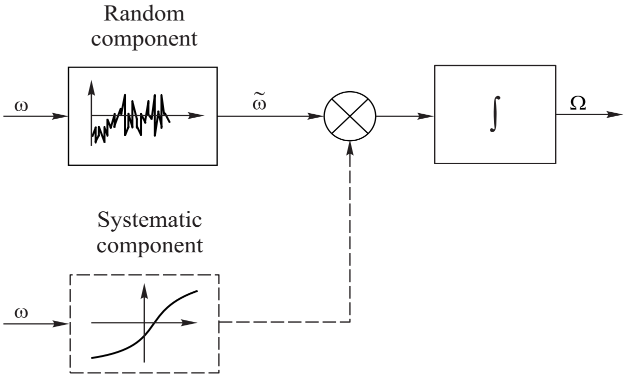


Fig. 1. Model for measuring angular velocity with account of systematic and random components

A magnitude varying randomly under the influence of multiple factors which affect the instrument considering measurement of a single parameter is called a random error.

A magnitude which is under the influence of several factors and whose value remains constant over time is a systematic error.

The classical method of DTG drift errors analysis, applied in angular velocity measurement, consists in determination of the systematic and random components by each of the sensitivity axes, since the DTG has two measurement channels, implemented according to the following algorithm.

1. Assess gyroscope drift by 8 positions referring to the design documentation.

2. Make compensation for the vertical and horizontal components of Earth's rotation.

3. Calculate systematic and random components from the obtained values on successive time intervals by means of the averaging method.

The many known methods of stochastic noise analysis in the inertial sensors' output data include a method of spectral power density (SPD) in the frequency domain, a method using correlation function, as well as dispersion methods. However, the most known and used one is the Allan variance method.

Allan variance method

Lying in the base of the Allan variance method is the method of cluster analysis, which implies splitting of the whole dataset into groups of certain length.

We divide the entire volume of data subject to analysis, as shown in Fig. 2, into a certain number of groups of equal length, with averaging interval

$$T = nt_0,$$

where n – number of samples in the measurement interval ($n = 1, 2, ... < N/2$);

t_0 – meter sampling increment.

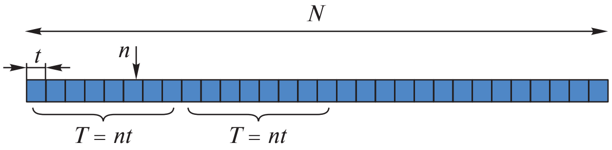


Fig. 2. Data structure diagram used in Allan variance calculation

The number of groups is determined by formula

$$K = \frac{N}{n}.$$

Here, N – the number of measurements.

Let us calculate the mean data value for each separate group:

$$\bar{\Omega}_k(T) = \frac{1}{T}\int_{t_k}^{t_k+T} \Omega(t)dt. \qquad (2)$$

Here, $\Omega_k(t)$ – mean output velocity value in a group, starting from the k-th point and comprising n data points;

t_k – meter sampling increment in the k-th point.

The next mean value of the group's output velocity can be expressed as follows:

$$\bar{\Omega}_{next}(T) = \frac{1}{T}\int_{t_{k+1}}^{t_{k+1}+T} \Omega(t)dt, \qquad (3)$$

where $t_{k+1} = t_k + T$.

Having calculated the difference between the mean values of each pair of the adjacent groups, we have:

$$\xi_{k+1,k} = \bar{\Omega}_{next}(T) - \bar{\Omega}_k(T). \qquad (4)$$

Set ξ, determined by equation (4) for each group with averaging interval T, forms an aggregate of random magnitudes. In this case, it is ξ dispersion value for all groups of the same size that is of interest.

Having calculated mean square of these differences, we divide it by a certain coefficient and obtain:

$$\sigma^2(T) = \frac{1}{2}\left\langle\left[\bar{\Omega}_{next}(T) - \bar{\Omega}_k(T)\right]^2\right\rangle = \frac{1}{2(N-2n)}\sum_{k=1}^{N-2n}\left[\bar{\Omega}_{next}(T) - \bar{\Omega}_k(T)\right]^2. \qquad (5)$$

Brackets $\langle\ \rangle$ in equation (5) determine the averaging operation for a set of groups.

A logarithmic plot of Allan variance $\sigma(T)$ dependence on averaging time T is a direct indication of the types of random processes present in the gyroscope output data, since the value of Allan variance $\sigma^2(T)$ is a measurable magnitude.

The Allan variance method is a technique for classifying and assessing the level of noise in the output signal associated with statistical properties of intrinsic random processes affecting gyroscope operation. As applies to DTG, its inherent errors when operating in the AVS mode, which can be analysed by means of the Allan variance method, are determined.

Noise components on the base of Allan variance method

The most part of errors as per the Allan variance method contained in a single angular velocity measurement can be represented as a model

$$I_\omega = \omega + R_\omega + K_\omega + B_\omega + N_\omega + Q_\omega. \qquad (6)$$

Here, I_ω – measurements (deg/h);

R_ω – angular velocity trend;
K_ω – angular velocity random walk;
B_ω – zero instability;
N_ω – angular random walk;
Q_ω – quantisation noise.

Given in Fig. 3 is Allan deviation curve for a signal changing under the influence of various stochastic processes which correspond to the gyroscope errors.

Angular velocity trend R_ω is a systematic error representing determinate deviation of an output signal received from the considered sensor over large periods of time. Gyroscope angular random walk N_ω is an additive white noise component, manifesting itself as noise or slow variation of output signals over time [1–16]. Angular velocity random walk K_ω is a random drift of physical magnitudes, such as angular velocity, really measured by the sensor considered. Output signal quantisation noise Q_ω are the errors introduced into an analogue signal when encoding it to the digital form [12, 13]. Zero instability B_ω is an electronic noise observed in virtually any electronic devices; its sources can be inhomogeneities in the conducting medium [16].

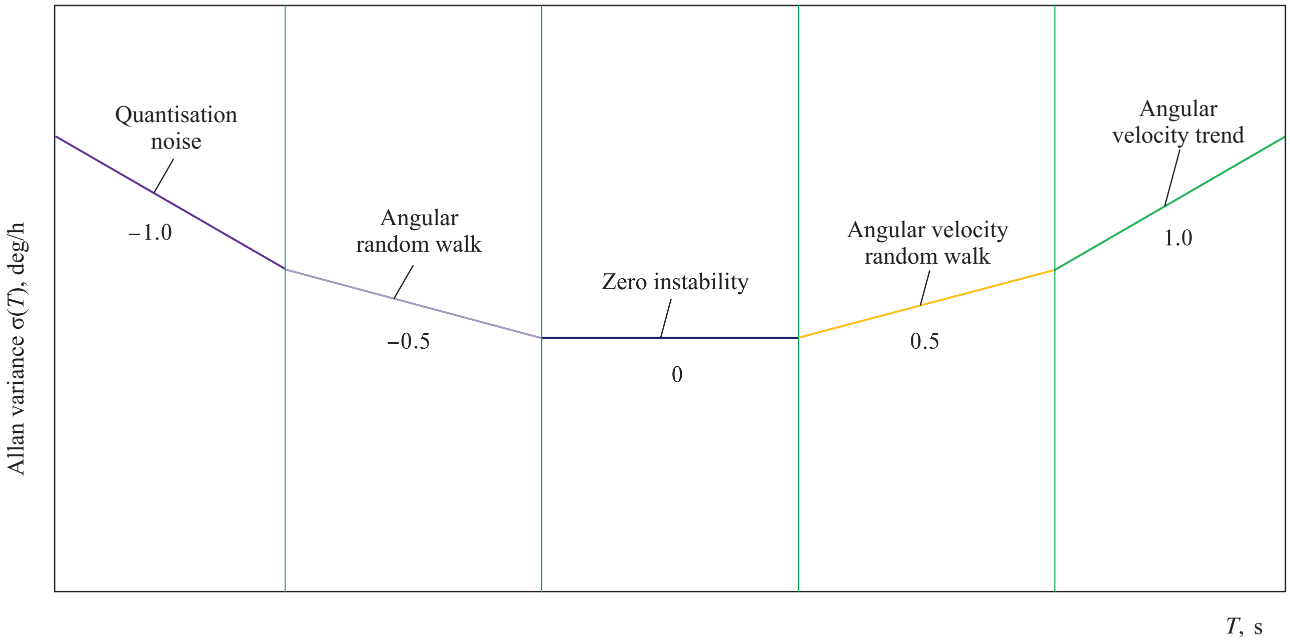


Fig. 3. Correspondence of Allan variance segments to typical processes

Development and implementation of algorithms for output response analysis

Resulting from experimental setup (Fig. 4) operation, a DTG output signal in the AVS mode was obtained.

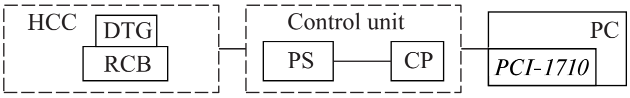


Fig. 4. Block diagram of experimental setup for conducting tests:
HCC – heat and cold chamber; DTG – dynamically tuned gyroscope; RCB – remotely controlled bracket;
PS – power source; CP – control panel

The experimental setup, whose block diagram is given in Fig. 4, comprises the following elements:

- control unit, comprising a power source and a control panel;
- heat and cold chamber, placed into which was DTG fixed in a remotely controlled bracket;
- PC with installed *Advantech PCI-1710* card.

For correct operation of the *Advantech PCI-1710* card, it is necessary to install on the user's computer the software supplied in a set with the card. This software enables to set up the equipment and save settings in *Windows* registry, as well as use the settings in calling up *API* functions for working with the driver. The use of this driver makes accessible a wide range of functions in various programming environments for enhancing hardware performance, such as the event handler function, which enables to find out the status of data transmission or reception.

Fig. 5 illustrates interaction between hardware and software.

Operation with the analogue-to-digital converter (ADC) card starts with its initialisation. First, it is necessary to determine the constants, command words, card control parameters, declare structures, and allocate memory for data arrays. Then check the card for errors and create stream for reading the incoming data. This done, the card is ready for operation and waiting for the input data in the standby mode.

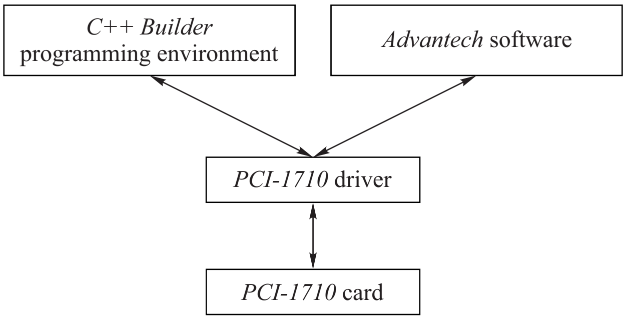


Fig. 5. Hardware and software interaction

The procedure of ADC module preparing for operation can be represented in the form of a block diagram (Fig. 6).

Operation of the setup (see Fig. 4) consists in simulation of gyro inertial unit operation, whose measurement time we take equal to 10 min.

Implementing in action the setup given in Fig. 4, we collect experimental data, write them

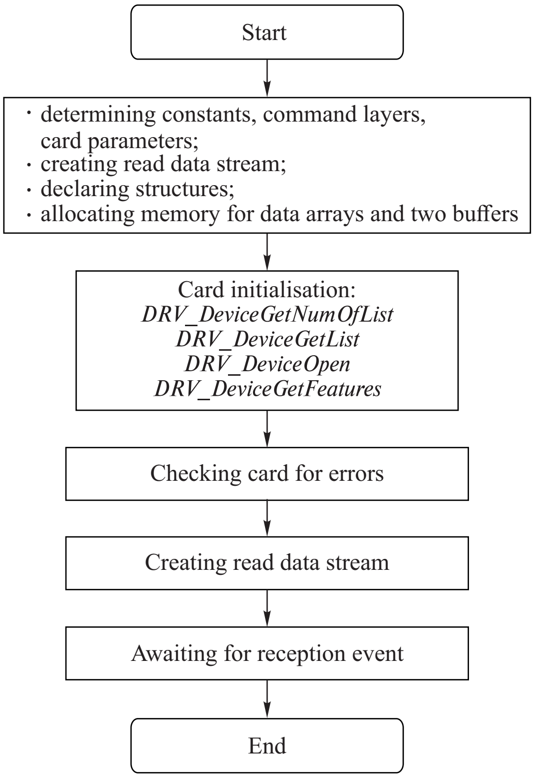


Fig. 6. Block diagram of ADC preparation for work

Experimental data fragment

Measurement channel, deg/h	
X	*Y*
91.8	–139.00
Ditto	–116.13
68.38	–139.93
…	…
114.28	–103.76
57.14	–127.55

down, save, and export to files *.*txt*. A fragment of the experimental data is given in the table.

To have an initial notion of the data, it is necessary to convert them [2]. To do that, the mean value, dispersion, and RMS deviations are calculated [10]. The next step is graphical representation of the data. This is done using an additional module created in the *C++ Builder 6* programming environment [3]. Shown in Fig. 7 is a graph of gyroscope output signal by axes *X* and *Y* under normal climatic conditions (22 °C).

To calculate Allan variance $\sigma^2(T)$, we shall select data averaging interval *T* divisible by the meter sampling increment:

$$\sigma^2(T) = \frac{1}{2\times T^2(N-2n)}\sum_{k=1}^{N-2n}\left(\theta_{k+2n} - 2\theta_{k+n} + \theta_k\right)^2. \quad (7)$$

Here, θ_k – angle accumulated as a result of integration of the output velocity values, calculated by the formula

$$\theta_k = \int_0^{kt_0} \Omega(t)dt.$$

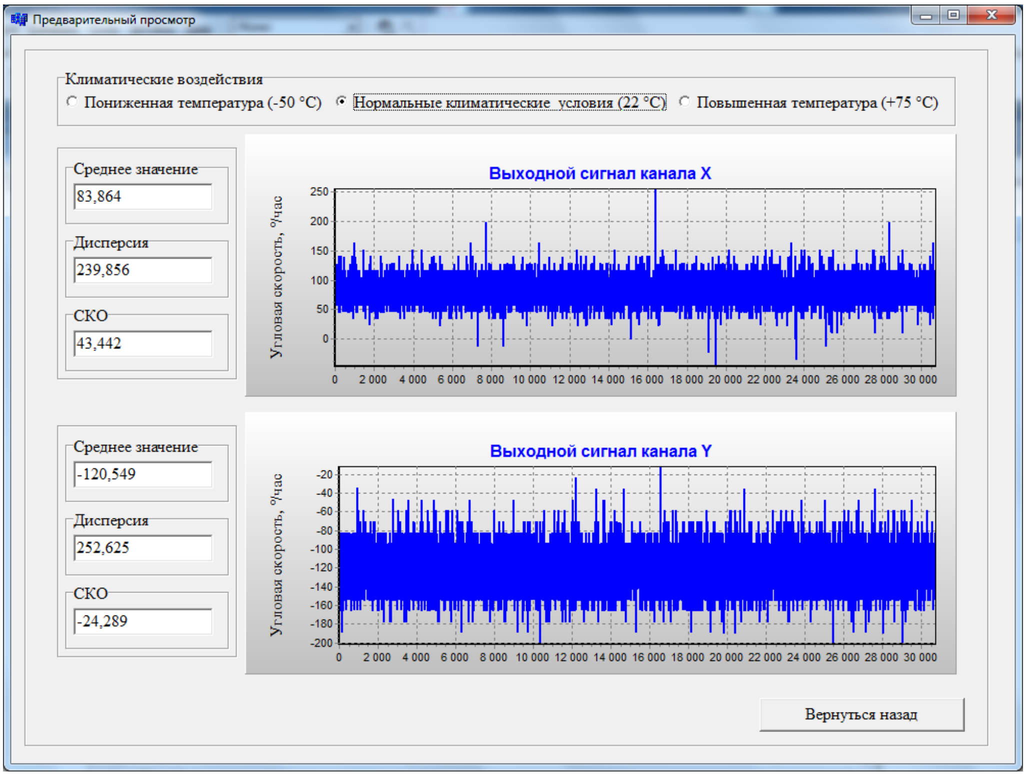
A block diagram of an algorithm implementing the Allan variance method is given in Fig. 8.

Allan variance $\sigma^2(T)$ based on the known noise components [6, 7] can be represented by means of polynomial approximant $p^2(T)$ as:

$$\sigma^2(T) \approx p^2(T) = \\ = \frac{3Q^2}{T^2} + \frac{N^2}{T} + \frac{2B^2}{\pi}\ln 2 + \frac{K^2T}{3} + \frac{R^2T^2}{2}. \quad (8)$$

Here, *R*, *K*, *B*, *N*, *Q* – coefficients of polynomial $p(T_k)$.

Each one of the obtained coefficients *R*, *K*, *B*, *N*, *Q* of the polynomial (8) is a separate noise



Климатические воздействия
○ Пониженная температура (-50 °C) ● Нормальные климатические условия (22 °C) ○ Повышенная температура (+75 °C)

Среднее значение 83,864
Дисперсия 239,856
СКО 43,442

Среднее значение -120,549
Дисперсия 252,625
СКО -24,289

Fig. 7. Graph of gyroscope output signal by axes *X* and *Y* under normal climatic conditions (22 °C)

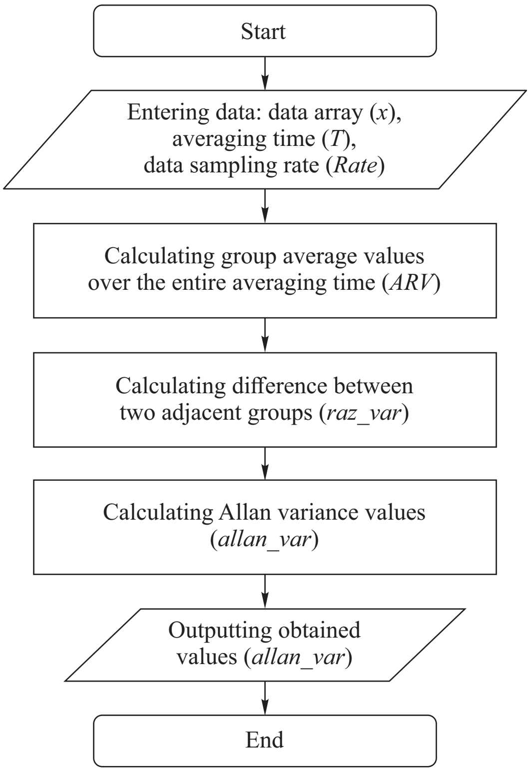


Fig. 8. Block diagram of algorithm based on the Allan variance method

component present in the gyroscope output signal [4, 5, 8]. An efficient method for obtaining coefficient values, on condition that Allan variance is a polynomial function of 5 unknown coefficients, is the method of least squares.

The essence of this method consists in finding coefficients R, K, B, N, Q that would conform to the condition of the minimum square sum of Allan variance $\sigma^2(T)$ deviations from the approximating surface:

$$\Phi=\sum_{k=0}^{K}[\sigma(T_k)-p(T_k)]^2 \xrightarrow{\{R,K,B,N,Q\}} \min, \qquad (9)$$

where $\sigma(T_k)$ – Allan variance;

$p(T_k)$ – polynomial approximant.

For expression (9) to be true, all partial derivatives of functional Φ by all $\{R,K,B,N,Q\}$ must be equal to zero:

$$\frac{\partial\Phi\{R,K,B,N,Q\}}{\partial\{R,K,B,N,Q\}}=0. \qquad (10)$$

Now we have a system of equations for finding 5 indeterminates $\{R,K,B,N,Q\}$:

$$\frac{\partial\Phi\{R,K,B,N,Q\}}{\partial\{R,K,B,N,Q\}}=$$
$$=2\sum_{k=0}^{K}\left[\left(\sigma(T_k)-p(T_k)\right)\frac{\partial\sigma(T_k)}{\partial\{R,K,B,N,Q\}}\right]=0, \qquad (11)$$

which can be more conveniently written in the matrix form as

$$[A]*[C]=[B],$$

$$[A]=\begin{bmatrix} T_1^{-2} & T_1^{-1} & 1 & T_1 & T_1^2 \\ T_2^{-2} & T_2^{-1} & 1 & T_2 & T_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ T_i^{-2} & T_i^{-1} & 1 & T_i & T_i^2 \end{bmatrix},\ [C]=\begin{bmatrix} 3Q^2 \\ N^2 \\ \frac{2B^2}{\pi}\ln 2 \\ K^2 \\ R^2 \end{bmatrix},$$

$$[B]=\begin{bmatrix} \sigma_1^2 \\ \sigma_2^2 \\ \vdots \\ \sigma_i^2 \end{bmatrix}.$$

Here, $[A]$ – time matrix of data averaging;

$[C]$ – vector designating the sought coefficients;

$[B]$ – vector reflecting the obtained Allan variance value.

The coefficient matrix, in accordance with the least squares algorithm can be resolved as follows:

$$[C]=-(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T B, \qquad (12)$$

where $\mathbf{A}^T$ – transposed matrix $\mathbf{A}$.

A block diagram of the algorithm implementing the least squares method for obtaining the coefficients is given in Fig. 9.

For the convenience of work in studying DTG output signal for the presence of noise components and for displaying respective information and graphs, a software package in the *C++ Builder 6* programming environment was developed. The runtime program includes the following functions:

- measuring and writing to *.*txt* files DTG output signals by means of analogue input/output card *PCI-1710*;

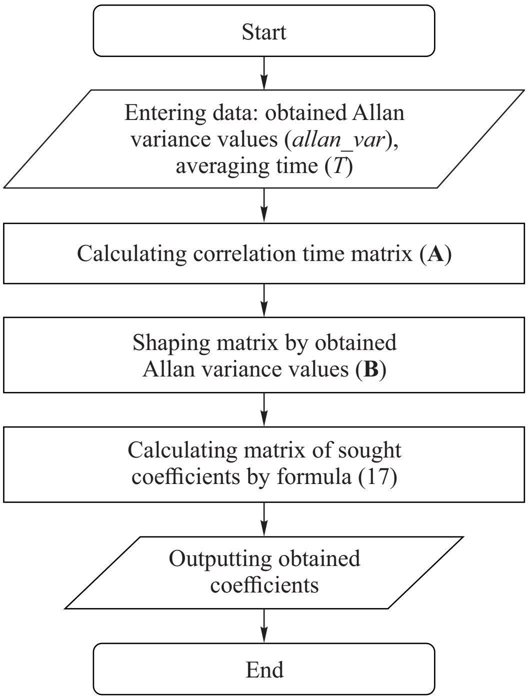
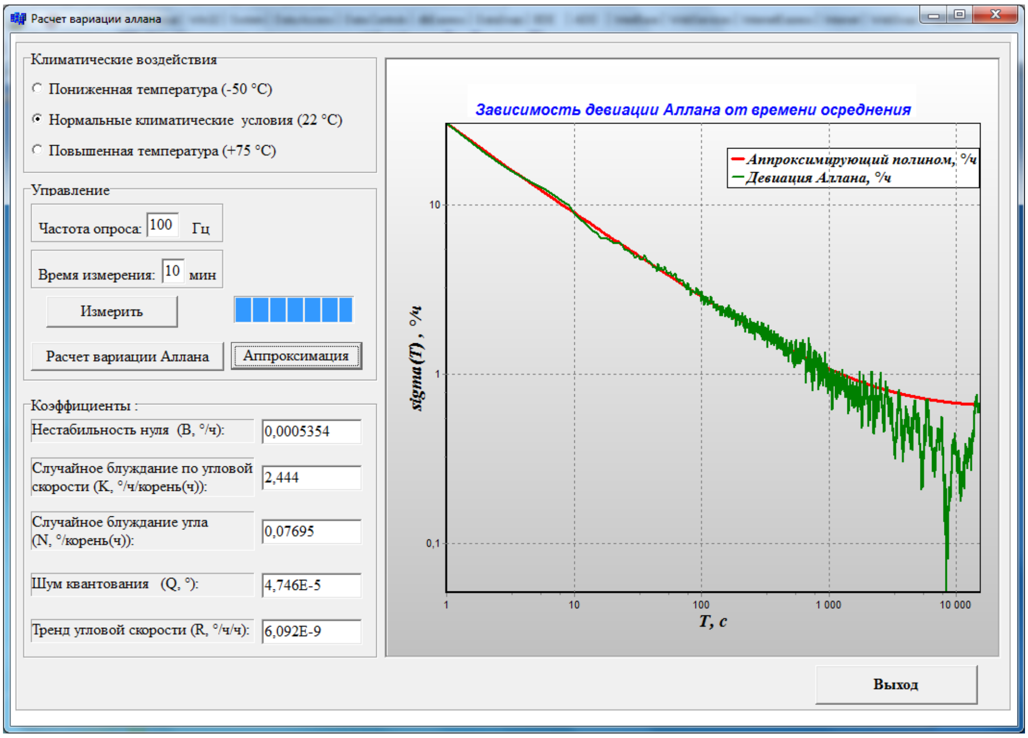


Fig. 9. Block diagram of algorithm of the least squares method

- calculating Allan variance values and displaying the obtained results in the graphical form;
- approximating the obtained Allan variance values to obtain numerical values of the coefficients corresponding to certain errors.

After program startup, the main window “Allan variance calculation” opens, serving for central control of the program module. In the right-hand part of the window is a field for displaying a graph of the values obtained from all calculations. By pressing button “Measure”, an additional module with the initial data graphs is called up (see Fig. 7). In the left-hand part of the window is a control field, where it is possible to enter climatic effects, meter sampling rate, and measurement time. Field “Coefficients”, located below, is intended for displaying the final results of all calculations. Besides, the main window features buttons by means of which sampling of the output signals received from DTG and writing them to file is performed, followed by displaying the received signals in a graph in the additional module, running calculation of Allan variance deviations and approximation procedure with subsequent building of a graph of the results obtained.

The result of module operation is finding respective coefficients and outputting an additional graph with consideration of the coefficients obtained (Fig. 10).



Климатические воздействия
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● Нормальные климатические условия (22 °C)
○ Повышенная температура (+75 °C)

Управление
Частота опроса: 100 Гц
Время измерения: 10 мин

Коэффициенты :
Нестабильность нуля (B, °/ч): 0,0005354
Случайное блуждание по угловой скорости (K, °/ч/корень(ч)): 2,444
Случайное блуждание угла (N, °/корень(ч)): 0,07695
Шум квантования (Q, °): 4,746E-5
Тренд угловой скорости (R, °/ч/ч): 6,092E-9

Fig. 10. Window with approximating curve graph

Conclusion

Resulting from application of a new algorithm for the analysis of the additive component of DTG output signal, an algorithm and software were developed, enabling to calculate 4 basic noise components: zero instability, angular velocity random walk, angular random walk, and angular velocity trend, as well as to determine quantisation noise present in the DTG output signal and occurring as a result of analogue signal conversion into digital one.

Practical application of a new algorithm based on the Allan variance method in serial production on a lot of DTG samples made it possible to single out and analyse random components of errors. The results obtained can be used at the stage of GIU calibration.

Notably, the obtained coefficient values remain stable and do not change from start to start, which testifies to reliability of the obtained results and adequacy of both the algorithm and the developed SW.

The developed SW is the first and indispensable step in preparing GIU, which use DTG as their part, for manufacturing application in SINS of advanced products. Resulting from trials of the developed SW in serial production of DTG in amount of two month's worth of the output, the random component was reduced from 30 to 50 % as compared with the classical method of assessing sensor output signals. Further trials of the method in serial production at the stage of GIU calibration are anticipated.

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Science research interests: gyroscopic devices and their use in strapdown inertial navigation systems.

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Science research interests: aircraft control systems, gyroscopic devices as part of a gyroinertial unit, gyrostabilizing platforms, free gyroscopes and free gyroscope systems.

Анализ погрешностей динамически настраиваемого гироскопа для гироинерциального блока методом вариации Аллана

Рассмотрен алгоритм получения измерительной информации с динамически настраиваемого гироскопа в режиме датчика угловых скоростей и оценки шумовых составляющих выходного сигнала, основанный на методе вариации Аллана. Проведена оценка полученных результатов.

Ключевые слова: навигационная система, вариация Аллана, динамически настраиваемый гироскоп, вычисление шумовых составляющих.

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Область научных интересов: гироскопические приборы и их применение в бесплатформенных инерциальных навигационных системах.

Дядин Сергей Серафимович – заместитель главного конструктора по динамически настраиваемым гироскопам и физическим платформам Акционерного общества «Арзамасский приборостроительный завод им. П. И. Пландина», г. Арзамас.
Область научных интересов: системы управления летательными аппаратами, гироскопические приборы в составе гироинерциального блока, гиростабилизирующие платформы, свободные гироскопы и системы на их основе.