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Panchenko B. A., Ponomarev O. P., Denisov D. V.

## Quick calculation of Luneberg lens scattering signature

The paper focuses on solving the problem of diffraction of a linear polarization electromagnetic wave on multilayer bodies of a spherical shape, i.e. a metal sphere with a dielectric cover and Luneberg lens. For this purpose tensor Green function method was used. Within the research we described the electrodynamic system verifying the expressions obtained for the calculation of electromagnetic fields. Furthermore, we compared the obtained results of calculations with the results of simulation of a similar problem in the software package *ANSYS HFSS*. Finally, we estimated the computational burden and computing time for solving the problem in two ways.

**Keywords:** tensor Green functions, Luneberg lens, *ANSYS HFSS*.

### Introduction

Diffraction of a linear polarization electromagnetic wave on multilayer objects is an interesting problem regarding calculation of the field dissipation on spherical bodies such as a metal sphere with a protective dielectric cover and a Luneberg lens antenna. Metal spheres with a dielectric cover raise interest, on the one hand, due to frequent use of primitive diffraction objects for calibration in anechoic chambers and in equipment for measuring radar cross-section (RSC) of real devices and objects; on the other hand, due to common use of covers for objects exposed to electromagnetic effect. Such covers are used for protection against external effects and for radio camouflage applications by reducing the radar scattering factor within a required angle range.

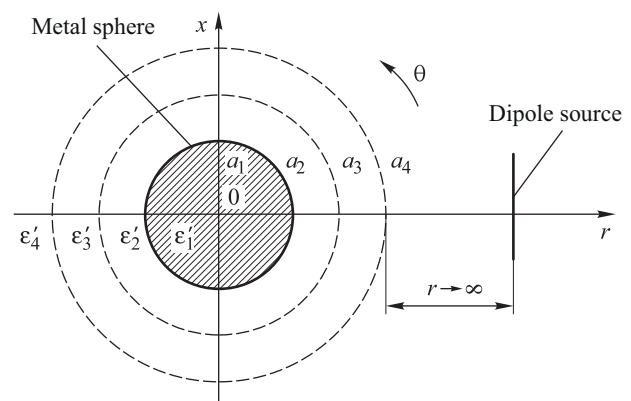
Operating in the diffraction mode, the Luneberg lens forms a directional scattering pattern on the shadow side; this property of the lens can be of practical use in various applications. For example, using a remote feed element mounted in the far region on the illuminated side of the lens, it is possible to generate an auxiliary beam to the main feed element mounted on the spherical surface, thus enabling a multi-beam operation mode. Also, using a remote feed element, it is possible to suppress interference, the angular position of which relative to the lens is known and does not coincide with the direction of the operating directional pattern. Therefore, the Luneberg lens

operating in the diffraction mode allows to expand functional capabilities of a Luneberg lens-based antenna system [1].

### Solving the Luneberg lens diffraction problem

Fig. 1 shows an infinitely distant electromagnetic field source at infinity ( $r \rightarrow \infty$ ), which generates the field of linear vertical polarization  $E_0$ , propagating in direction  $\mathbf{p}$  towards the diffraction object. The incident field is scattered on a spherical object which has the number of layers  $L$ . Boundaries of layers are coordinate surfaces in a spherical coordinate system. Depending on electrophysical parameters of each layer, regarding the problem under consideration, a spherical object can be represented as the Luneberg lens with a random number of layers or as a metal sphere with a multilayer cover.

We applied two methods to enable electromagnetic excitation by a remote source: using the electrodynamic system of tensor Green



**Fig. 1.** Geometry of the wave diffraction problem on a multilayer sphere

function (TGF) [2–4] and the finite element method in the *ANSYS HFSS* software environment. In the first case, the formula for calculating the electric field intensity of a diffracted wave was written in a general form (formula derivation is described in [2, 4]):

$$\mathbf{E}(\theta) = E_0 \frac{\exp(-ik_0 r)}{(k_0 r)(k_0 a)} \times \sum_{n=1}^{\infty} \left( \frac{2n+1}{n(n+1)} (-1)^n \times \begin{pmatrix} (\mathbf{a}_\theta \cos \varphi \tau_n(\theta) - \mathbf{a}_\varphi \sin \varphi \pi_n(\theta)) M_n - (\mathbf{a}_\theta \cos \varphi \pi_n(\theta) - \mathbf{a}_\varphi \sin \varphi \tau_n(\theta)) N_n \end{pmatrix} \right) \quad (1)$$

where  $k_0 = \frac{2\pi}{\lambda}$  – wave number;

$$\tau_n(\theta) = \frac{\partial P_n^l(\cos \theta)}{\partial \theta}, \quad \pi_n(\theta) = \frac{P_n^l(\cos \theta)}{\sin \theta};$$

$$M_n = \frac{i\tilde{\mathbf{Z}}_n(a)j_n(k_0 a) - j'_n(k_0 a)}{i\tilde{\mathbf{Z}}_n(a)h_n(k_0 a) - h'_n(k_0 a)},$$

$$N_n = \frac{i\tilde{\mathbf{Y}}_n(a)j_n(k_0 a) - j'_n(k_0 a)}{i\tilde{\mathbf{Y}}_n(a)h_n(k_0 a) - h'_n(k_0 a)} - \text{characteristic}$$

elements of function;

$P_n^l(\cos \theta)$  – Legendre function;

$h_n(x)$ ,  $j_n(x)$  – spherical Riccati – Bessel functions [5];

$\tilde{\mathbf{Z}}_n$ ,  $\tilde{\mathbf{Y}}_n(a)$  – oriented directional impedances and admittances to be determined via sequential recalculation from the external boundary of a multilayer sphere to the centre through partial layer-regions (calculation procedure is described in [2]).

To check the derived formula, we analysed the classical problem of diffraction of the linear polarization wave on a metal ball, which had been solved by many scholars. Diagrams of the secondary (scattered) field on a metal ball are shown in Fig. 2.

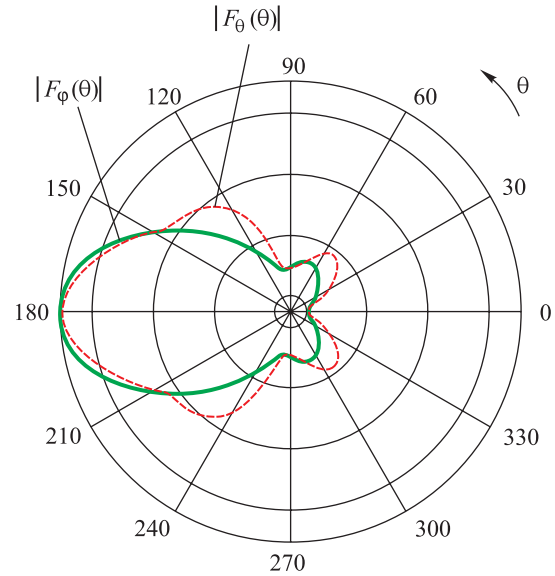


Fig. 2. Scattered field patterns on metal sphere  $k_0 a = 3$

When calculating diagrams as per the procedure proposed herein, we obtained the results close to solutions to the problem of diffraction on a metal sphere, which were published in [6]. This may justify the obtained formula representation (1).

### Luneberg lens diffraction using the *ANSYS HFSS* software

Fig. 3 shows the electrodynamic model of a linear polarization electromagnetic wave on the Luneberg lens, using the *ANSYS HFSS* software package. A linear vertical polarization electromagnetic wave falls onto a multilayer sphere in the direction of Z-axis. The incident wave's electric field vector is oriented along X-axis. Parameters of sphere layers correspond to the Luneberg law [1]: four layers with normalized radii  $a$  equal to 0.53, 0.75, 0.93 and 1 cm and with dielectric permittivity  $\epsilon'_L$  with values of 1.86, 1.57, 1.28 and 1.

The frequency of an incident electromagnetic wave is 10 GHz, the lens radius is 9 cm,  $k_0 a \approx 6\pi$ . Fig. 4 shows results of comparison of scattering patterns calculated with the help of the *ANSYS HFSS* and formula (1) in *MATLAB*.

When analysing the diagram shown in Fig. 4, we may notice that in the angle range

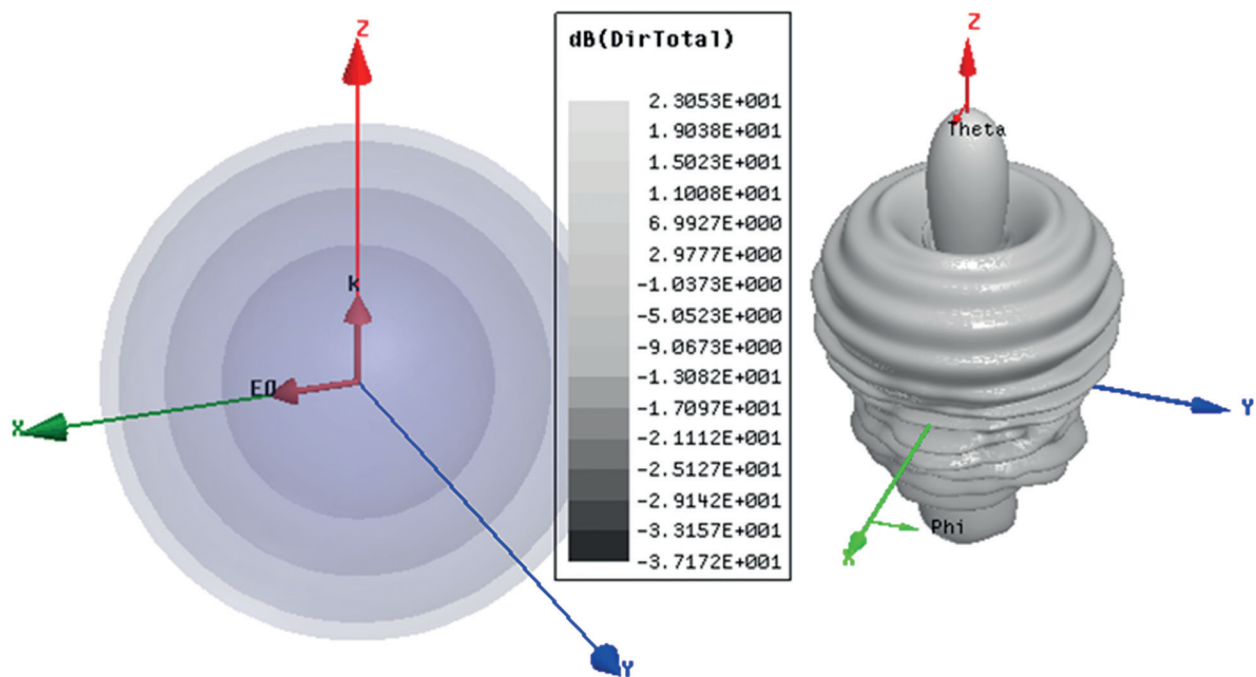


Fig. 3. Model of Luneberg lens in *ANSYS HFSS* software

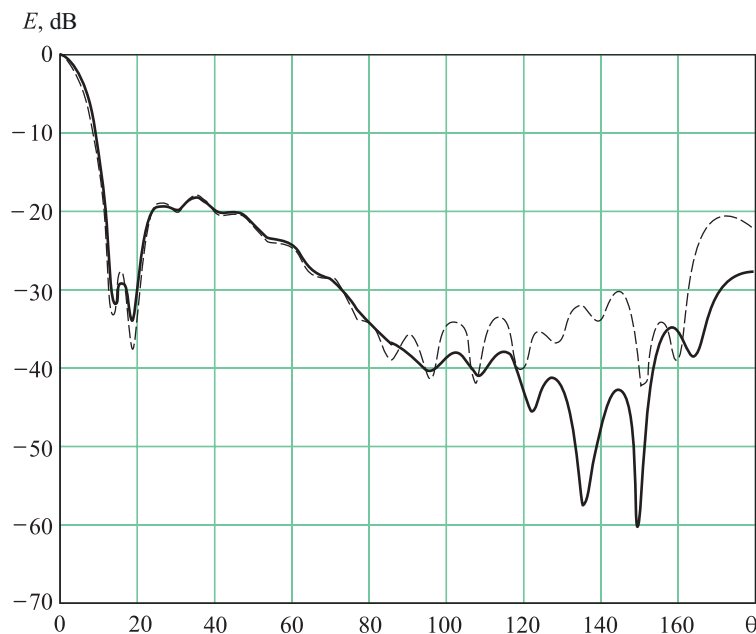


Fig. 4. Scattering patterns of Luneberg lens with electrical radius

$$k_0 a = 6\pi;$$

— — *MATLAB*; --- — *ANSYS HFSS*

of 0 to 90 degrees the results of scattering patterns calculations obtained by different methods are in a good agreement and demonstrate quite accurate repeatability, thus allowing for the conclusion that the calculation formula (1) is written correctly. The difference in radiation in the far region (in the range of 90 to 180 degrees)

for both methods can be caused by different settings of the primary source of radiation in the far region due to mathematical features of the applied methods. When using the *ANSYS HFSS*, the source of radiation is set as an incident electromagnetic wave, while the TGF method implies an infinitely distant point source (in this case,

the Luneberg lens is located in the far region relative to the source).

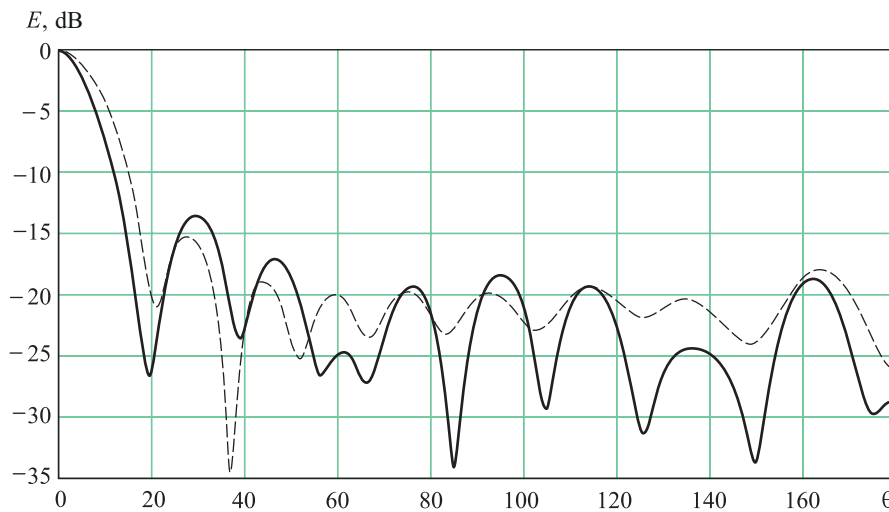
Besides, this problem is easy to adapt for solving the problem of diffraction on a metal ball by changing the number of layers and their electrophysical parameters. Fig. 5 shows scattering patterns calculated by two methods: the TGF method and using the *ANSYS HFSS* software.

When the *ANSYS HFSS* software is used, the difference in results is associated with calculation accuracy and a number of separations into finite areas. The table below shows the time spent for solving the problems related to computation of spherical objects of different electrical size such as the Luneberg lens and a metal sphere. For computations, we used the following PC configuration: *Intel Core i5 3210M CPU 2.5 GHz*, RAM 8 GB, licensed *ANSYS HFSS* software package for electrodynamic analysis.

When analysing the results given in the table, we may notice that the computation time for inhomogeneous spherical structures using the TGF method is at least 2 times shorter that the computation time when we apply the finite element method in the *ANSYS HFSS* software environment. Such a significant advantage in saving of computation time allows to integrate the proposed calculation method into various automated systems for a real-time analysis.

### Conclusion

The obtained results may be used for determining diffraction and radar-location characteristics of objects. The proposed mathematical apparatus offers high-speed performance for calculating the Luneberg lens scattering signature and can be efficiently applied for primary analysis of Luneberg lens-based antenna systems. The results



**Fig. 5.** Scattering on a metal sphere of size  $k_0 a = 4\pi$ :  
— — TGF; --- — *ANSYS HFSS*

Results of computations in *ANSYS HFSS* and *MATLAB* software

Sizes of a spherical body	Luneberg lens computation time in <i>HFSS</i> , s		Metal sphere computation time in <i>HFSS</i> , s		Luneberg lens computation time in <i>MATLAB</i> , s	Metal sphere computation time in <i>MATLAB</i> , s
Solution method	<i>Direct Solver</i>	<i>Iterative Solver</i>	<i>Direct Solver</i>	<i>Iterative Solver</i>	Tensor Green functions	Tensor Green functions
$k_0 a = \pi$	38	22	13	13	0.2	0.24
$k_0 a = 2\pi$	129	26	14	14	0.6	0.27
$k_0 a = 4\pi$	182	83	21	22	1.5	0.29
$k_0 a = 6\pi$	1024	227	30	33	2.2	0.4
$k_0 a = 8\pi$	4320	839	50	54	3.6	0.59



of the developed method may be used in design of scanning antenna radar systems for target detection and tracking, including multi-beam solutions.

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**Panchenko Boris Alekseevich** – Doctor of Engineering Sciences, Professor, Ural Federal University named after the first President of Russia B. N. Yeltsin. Author of over 300 published works, 7 monographs among them. Science research interests: technical electrodynamics, antenna theory and technology.

**Ponomarev Oleg Pavlovich** – Doctor of Engineering Sciences, Professor, Ural Federal University named after the first President of Russia B. N. Yeltsin. Deputy Director General for Scientific and Technical Development, Chief Designer, Joint Stock Company “Ural Industrial Enterprise “Vector”. Author of over 100 published works. Science research interests: diffraction theory, computational simulation of antennas.

**Denisov Dmitry Vadimovich** – Candidate of Engineering Sciences, Associate Professor, Siberian State University of Telecommunications and Informatics, Leading Design Engineer, Joint Stock Company “Ural Industrial Enterprise “Vector”. Author of over 20 published works. Science research interests: telecommunications, antenna theory and technology.

## Быстрый расчет характеристик рассеяния линзы Лüneберга

Методом тензорных функций Грина решена задача дифракции электромагнитной волны линейной поляризации на многослойных телах сферической формы – металлической сфере с диэлектрическим укрытием и линзе Лüneберга. Приведено описание используемого электродинамического аппарата с проверкой полученных выражений для расчета электромагнитных полей. Проведено сравнение полученных результатов расчетов с результатами моделирования аналогичной задачи в программном пакете *ANSYS HFSS*. Представлена оценка затрат вычислительных ресурсов и машинного времени при решении задачи двумя способами.

**Ключевые слова:** тензорные функции Грина, линза Лüneберга, *ANSYS HFSS*.

**Панченко Борис Алексеевич** – доктор технических наук, профессор Уральского федерального университета имени первого Президента России Б. Н. Ельцина, автор более 300 печатных работ, в том числе 7 монографий. Область научных интересов: техническая электродинамика, теория и техника антенн.

**Пономарев Олег Павлович** – доктор технических наук, профессор Уральского федерального университета имени первого Президента России Б. Н. Ельцина. Заместитель генерального директора по научно-техническому развитию, главный конструктор Акционерного общества «Уральское производственное предприятие «Вектор». Автор более 100 печатных работ.

Область научных интересов: теория дифракции, численное моделирование антенн.

**Денисов Дмитрий Вадимович** – кандидат технических наук, доцент «Сибирского государственного университета телекоммуникаций и информатики», ведущий инженер-конструктор Акционерного общества «Уральское производственное предприятие «Вектор». Автор более 20 печатных работ. Область научных интересов: телекоммуникации, теория и техника антенн.