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# Procedure of synthesising astatic digital tracking systems analytically from continuous prototypes

The article deals with the problem of synthesising astatic digital tracking systems analytically, taking into account the requirements for astaticism quality and order; solving this problem leads to finding the desired transfer function of a discrete system based on a continuous prototype (a continuous transfer function). The advantage of the method specified above is that it makes it possible to eliminate the procedure bias caused by quantising desired transfer functions of continuous systems when synthesising astatic discrete systems. This bias is inherent in other synthesis methods that assume that reducing the sampling period will also lower the constant error. We used the MATLAB environment to develop our software implementing this method for automated construction of a desired transfer function of a discrete system.

*Keywords:* digital tracking system, continuous prototype, discrete prototype, desired transfer function, astaticism, quality indices, MATLAB.

## Introduction

Nowadays, with available cutting-edge computer technologies, all tracking systems (TS) are supposed to be digital systems. The application of digital methods of data processing in TS allows to implement complex control algorithms and ensure their fast modernization.

Often, a tracking system shall meet predetermined technical requirements at the design stage (control time, overshoot, astaticism order, system coefficients implementation accuracy), so, in accordance with the design specifications, a TS needs to be synthesized with the preset astaticism order based on control inputs and certain quality indices [1, 2].

The problem is set to develop the procedure for synthesizing astatic tracking systems in the MATLAB environment in accordance with preset quality indices. The synthesis procedure is to be developed in the *MATLAB* environment, while simulation – in the *MATLAB/Simulink* extension applied for dynamic system modelling. The input parameters of the procedure are the preset quality indices and astaticism order, the output parameters are the desired discrete transfer function (TF). Besides, the process of determination of the desired discrete TF is subject to mathematical modelling in the *MATLAB/Simulink* extension.

## Mathematical support

The tracking system diagram presented for mathematical modelling is shown in Fig. 1. Signal $g$ is sent to the system input, signal $y$ – to the system output. Here also, $W_{yg}(p/z)$ – the transfer function of a closed-type TS (where argument $p$ is used for continuous TSs, and $z$ – for discrete TF), offset (mismatch) $\varepsilon = g - y$.

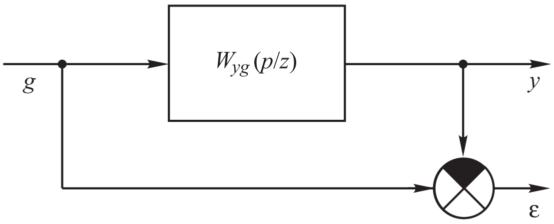

**Fig. 1.** Diagram of tracking system for modelling

The desired discrete TS TF is determined based on standard TFs of continuous TSs. According to the research paper [3], the synthesis problem includes three stages.

***The first stage.*** It is necessary to obtain a continuous prototype – TF $W_{yg}(p)$ of a continuous closed-type TS that meets the requirements to control quality, astaticism order and physical realizability conditions: control time $t_p^*$, overshoot $\sigma_g^*$ and astaticism order $v_g^*$, as well as physical realizability condition $m \le n$ for continuous systems (where m, n – polynomial orders of the TF numerator and denominator, respectively).

We will represent a continuous prototype of the TS astatic in terms of the control input [1]:

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$$W(p,\,\omega_0)=\frac{H_0(p)}{H(p)}=$$
$$=\frac{\Delta_{\nu_{g-1}}\omega_0^{n-\nu_g+1}p^{\nu_g-1}+\ldots+\Delta_1\omega_0^{n-1}p+\Delta_0\omega_0^n}{\Delta_n p^n+\Delta_{n-1}\omega_0 p^{n-1}+\ldots+\Delta_1\omega_0^{n-1}p+\Delta_0\omega_0^n},\qquad(1)$$

where $H_0(p), H(p)$ – polynomials of the continuous TF numerator and denominator, respectively;

$\Delta_i$ – standard coefficients (to be taken from the table of standard coefficients [1, 2] with account for requirements to the order of the system and astaticism, as well as overshoot);

$\nu_g$ – system astaticism order depending on control input.

The above formula also contains time-scale factor $\omega_0$, needed to take into account the preset control time and calculated by formula

$$\omega_0=t_{pm}/t_p^*,\qquad(2)$$

where $t_{pm}$ – table value of control time at $\omega_0=1$ [1].

If $\omega_0=1$, expression (2) describes a standard TF. At $\omega_0<1$, the transient process in a system takes more time, and at $\omega_0>1$ – it takes less time.

***The second stage.*** To obtain a discrete prototype using a continuous prototype, the latter is subject to the Z-transform [1, 2]:

$$W(z)=\frac{z-1}{z}Z_T\left\{\frac{W(p,\omega_0)}{p}\right\},\qquad(3)$$

where $Z_T\{\cdot\}$ – designation of the $Z_T$-transform of the function given in curly brackets;

$T$ – sampling period.

Mathematical transformation in accordance with formula (3) is performed with the help of the integrated function *MATLAB c2d* with the selected sampling period [4].

Thus, the obtained discrete prototype in closed state $W(z)$ by the polynomial control input $g(kT)=g(t)$ (where $k$ – cycle number) is described by the expression represented in a general form:

$$W_{yg}(z)=$$
$$=\frac{\eta_0+\eta_1 z+\eta_2 z^2+\ldots+\eta_{m-1}z^{m-1}+\eta_m z^m}{\delta_0+\delta_1 z+\delta_2 z^2+\ldots+\delta_{n-1}z^{n-1}+\delta_n z^n}.\qquad(4)$$

Here, $\eta_i$, $\delta_i$ – coefficients of the discrete TF numerator and denominator, respectively.

***The third stage.*** It is necessary to obtain the desired discrete TF. The accuracy of discrete systems (DS) is determined by the value of the error, with which a system generates the polynomial control input. To determine errors of discrete systems caused by polynomial control inputs, the error rate estimation method is applied. This method served as the basis for formulating and establishing the theorem on DS astaticism conditions [5].

*Theorem 1. On DS astaticism conditions.* A discrete system with the TF of type (4) has the $\nu_g$-th order astaticism with respect to polynomial control input $g(kT)$ of degree $r_g=\nu_g$, if the TF coefficients satisfy the conditions:

$$\sum_{i=\nu-1}^{m}\binom{i}{\nu-1}\eta_i=\sum_{i=\nu-1}^{n}\binom{i}{\nu-1}\delta_i,$$
$$\nu=\overline{1,\nu_g};\qquad(5)$$
$$\sum_{i=\nu_g}^{m}\binom{i}{\nu_g}\eta_i\neq\sum_{i=\nu_g}^{n}\binom{i}{\nu_g}\delta_i,$$

where $\binom{i}{k}$ – binomial coefficients calculated by formula $\binom{i}{k}=i!/k!(i-k)!$

If condition (5) for the preset astaticism order is not satisfied with respect to discrete prototype (4), new values of coefficients $\delta_i^*$ and $\eta_i^*$, shall be determined with the help of a number of methods [6, 7]. Then condition (5) shall be rechecked with account for new coefficients. If the discrete TF obtained by changing the coefficients fully complies with these conditions, it is considered the desired discrete TF for the TS being synthesized.

The stepwise algorithm described above was implemented in the *MATLAB* environment in the form of software procedures and functions. The block diagram illustrating the algorithm operation is shown in Fig. 2.

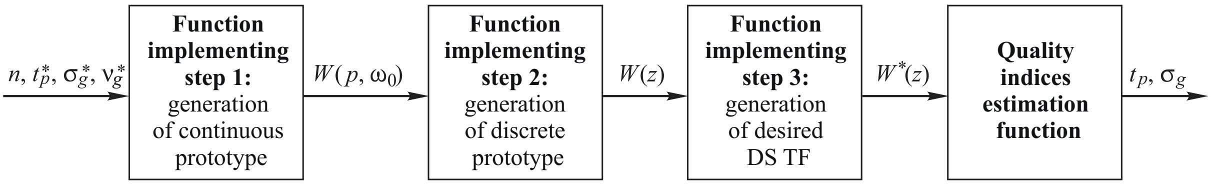
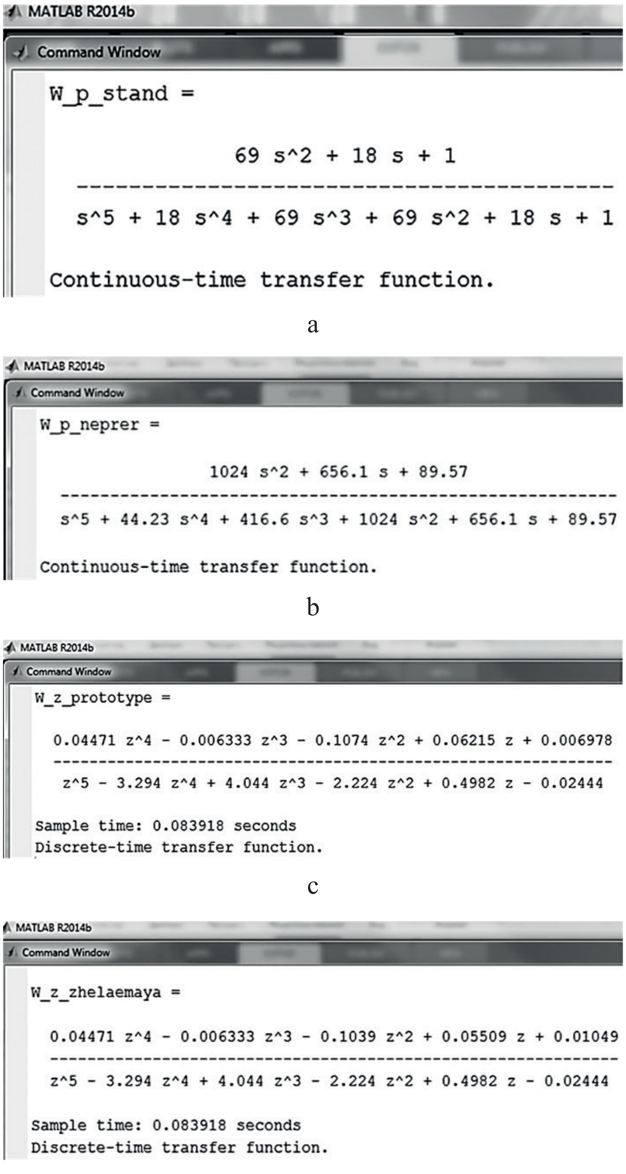


**Fig. 2.** Procedure of analytical synthesis with the component of analysis of TS quality indices

Fig. 3 shows an example of computation of the desired discrete TF using a continuous prototype. The following data is used in the example:

| | |
|---|---|
| System order, $n$ | 5 |
| Preset astaticism order, $\nu_g^*$ | 3 |
| Control time, $t_p^*$, s | 3.7 |
| Overshoot, $\sigma_g^*$, % | ≤ 25 |
| System type | discrete |



```
W_p_stand =

            69 s^2 + 18 s + 1
  -------------------------------------------
  s^5 + 18 s^4 + 69 s^3 + 69 s^2 + 18 s + 1

Continuous-time transfer function.
```

a

```
W_p_nepre r =

                 1024 s^2 + 656.1 s + 89.57
  ---------------------------------------------------------
  s^5 + 44.23 s^4 + 416.6 s^3 + 1024 s^2 + 656.1 s + 89.57

Continuous-time transfer function.
```

b

```
W_z_prototype =

   0.04471 z^4 - 0.006333 z^3 - 0.1074 z^2 + 0.06215 z + 0.006978
   ----------------------------------------------------------------
     z^5 - 3.294 z^4 + 4.044 z^3 - 2.224 z^2 + 0.4982 z - 0.02444

Sample time: 0.083918 seconds
Discrete-time transfer function.
```

c

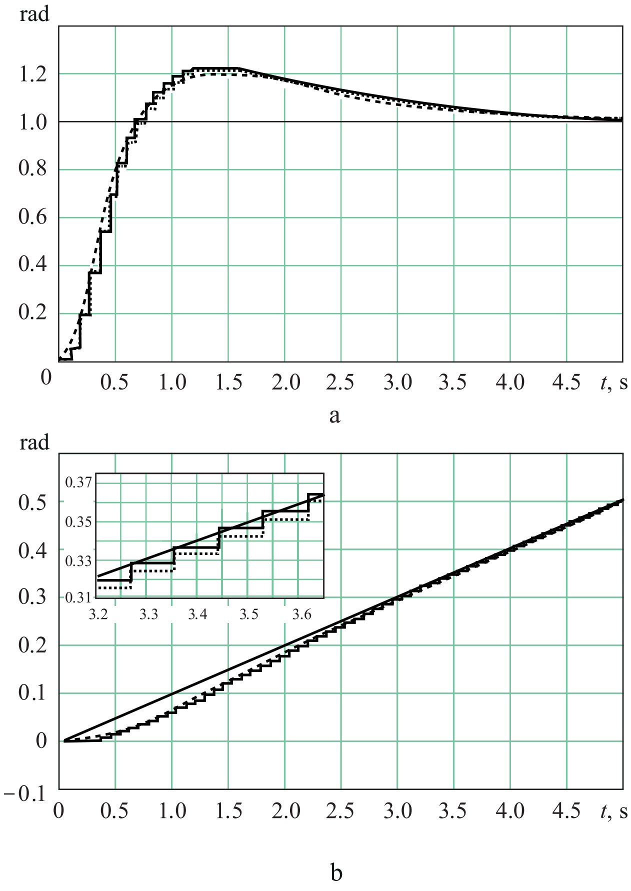
```
W_z_zhelaemaya =

   0.04471 z^4 - 0.006333 z^3 - 0.1039 z^2 + 0.05509 z + 0.01049
   ---------------------------------------------------------------
   z^5 - 3.294 z^4 + 4.044 z^3 - 2.224 z^2 + 0.4982 z - 0.02444

Sample time: 0.083918 seconds
Discrete-time transfer function.
```

d

**Fig. 3.** Results of procedure:
a – standard continuous TF; b – continuous prototype;
c – discrete prototype; d – desired DS TF

For a comparative analysis of quality indices resulted from the TF synthesis, Fig. 4 shows responses of a continuous prototype, a discrete prototype and the desired DS TF at a unit step input and at a ramp input. In case of a unit step input, the system response will be a transient parameter.

The system’s quality indices are estimated by the transient response (Fig. 4, a). The table above represents the results of quality indices



b

**Fig. 4.** Simulation results:
a – transient responses of continuous prototype (----), discrete prototype (·········), desired DS TF (——) at unit step input; b – response of continuous prototype (----), discrete prototype (·········), desired DS TF (——) at ramp input

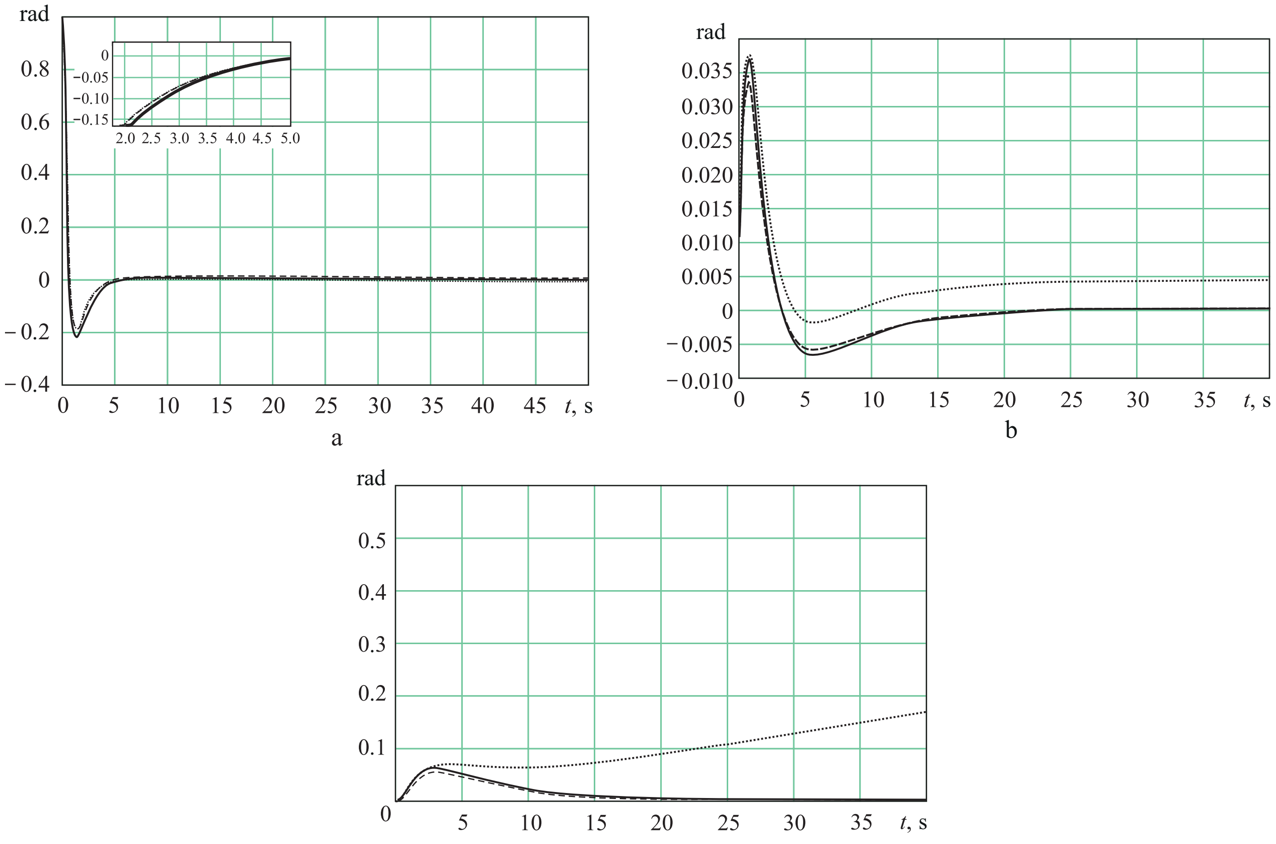
Results of estimation of tracking systems quality indices

| System type | Quality indices | | | | Astaticism order | | Result |
|---|---|---|---|---|---|---|---|
| | $t_p$, s | | $\sigma_g$, % | | $\nu_g$, | | |
| | Required value* | Simulation results | Required value* | Simulation results | Required value* | Simulation results | |
| Continuous prototype | 3.5 | 3.46 | $\leq 20$ | 19.8 | 3 | 3 | Fails to comply with the system type |
| Discrete prototype | Ditto | Ditto | Ditto | Ditto | Ditto | 1 | Fails to comply with the astaticism order |
| Desired DS TF | 3.7 | 3.62 | $\leq 25$ | 22 | Ditto | 3 | Complies with requirements |
| * As initial data for calculations, the value of the relevant parameter is taken | | | | | | | |

estimation for both the desired discrete TF and its continuous prototype.

We should mention that during synthesis, the quality indices of a continuous prototype shall be selected with a certain margin at the first stage. The quality indices of the desired discrete TF comply with required ones, but demonstrate minor degradation in comparison with a continuous prototype. This is sort of payback for the preset astaticism order achieved by the system.

In case of a unit step input (Fig. 5, a), the error between the control input and the output signal for all three systems tends to zero in the steady-state condition. In case of a ramp input



a

b

c

**Fig. 5.** System errors at unit step input (a), ramp input (b) and square-law ramp input (c):
—— – desired discrete TF; ---- – continuous prototype; ········ – discrete prototype

(Fig. 5, b) and a square-law ramp input (Fig. 5, c), it is evident that only the error of the continuous prototype and desired DS TF tends to zero, while the discrete prototype mismatch has the constant error value in case of a ramp input and a discrepant error value in case of a square-law input. This proves the discrete prototype does not possess the astaticism, the order of which is higher than the first order.

## Conclusion

While investigating the problem of TS synthesis, we used the method of calculation of the desired TF of a discrete TS based on its continuous prototype resulting from standard TFs. The *MATLAB* environment offers a developed software application for automatic generation of the desired TF of discrete TSs, implementing the developed stepwise algorithm for solving the problem of analytical synthesis. The paper gives an example of the numerical simulation demonstrating the operation of the implemented software procedure for determining desired TFs of astatic digital TSs.

The approach demonstrated in the course of simulation has illustrated the advantage over the previous approach for simulating digital tracking systems. The value of dynamic error that inevitably occurs during calculation of a discrete prototype has been reduced by shortening the time-dependent sampling period. In particular, revised discrete prototype coefficients allowed to achieve the preset astaticism order being executed at the third stage of the algorithm. Moreover, using the developed software, we have managed to estimate and analyse the system’s quality indices with the desired discrete TF.

The developed software allows to automate the process of analysis and synthesis of digital TSs and has practical relevance for design and mathematical simulation of various high-precision servo, guidance and tracking systems.

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# Процедура аналитического синтеза астатических цифровых следящих систем по непрерывным прототипам

Рассмотрена задача аналитического синтеза астатических цифровых следящих систем при заданных требованиях к качеству и порядку астатизма, в результате ее решения определена желаемая передаточная функция дискретной системы на основе ее непрерывного прототипа (непрерывной передаточной функции). Преимуществом указанного метода является то, что с помощью него удается устранить методическую ошибку при синтезе астатических дискретных систем, вызванную дискретизацией желаемых передаточных функций непрерывных систем. Данная ошибка присуща другим методам синтеза, в которых уменьшение систематической ошибки предполагается за счет уменьшения периода дискретизации по времени. Программа по автоматическому построению желаемой передаточной функции дискретной системы, реализующая указанный метод, была разработана в среде *MATLAB*.

*Ключевые слова:* цифровая следящая система, непрерывный прототип, дискретный прототип, желаемая передаточная функция, астатизм, показатели качества, *MATLAB*.

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