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Limit shaping formed sections on grading machines

A mathematical model of a smooth transition zone extent is developed, enabling to determine the limit hemming angles, establish technology continuity and perform calculations of the interstand distance when designing product-line oriented roll-forming equipment.

Keywords: roll-forming, forming roll, formed section, hemming angle, smooth transition zone, interstand distance.

Introduction

In the last decade, formed sections have been widely used in machine-building and automotive industries (load-bearing elements of units, enclosing structures and decorative elements), aviation industry (aircraft stringers), construction of civil and military engineering objects (load-bearing elements of engineering structures, elements of roofing, air ducts, interior and exterior finish). At the same time, there arises a demand for frequent update of the section product line and assimilation of new kinds of sections.

The developers of roll-forming technologies from Ulyanovsk have designed a method of intensive deformation, used in the manufacture of formed sections [1]. As compared with the traditional roll-forming procedure [2], the method has a number of advantages (lower costs of the process equipment and associated implements, low power consumption, smaller shop floor areas required), which make it possible to assimilate small-lot production and, due to high mobility, bring production centres closer to the consumers, thus reducing product transportation costs.

An essential limitation in implementing technologies based on the intensive deformation method is the absence of a reliable mathematical model of the shaping process, one that would allow to predict the limit shaping of the formed sections in rolls. It is elimination of this gap in the roll-forming technology which is actually the objective of this publication.

Problem statement

For modelling of the shaping process taking place during roll-forming, the key factor is blank behaviour in the interstand space of roll-forming machine, in particular, dimensions of the smooth transition zone (STZ) of a hemmed flange. The existing known STZ models offered by Gun – Polukhin (for angle section) [3] and Bhattacharyya – Collins (for channel section of non-strain-hardening material) [4] relate to the traditional roll-forming and do not account for the factors influencing the limit angles of section elements' hemming: strain hardening of the blank, bend radius, and sagging of section bottom part.

In the modelling we shall use the variational method [5], and as the optimisation functionality we shall take the expression for plastic strain action on the hemmed flange, corner region, and bottom of the section. Let us make the following assumptions, physically consistent for the deformation process:

for the hemmed flange: 1) blank material is incompressible, hardening according to the linear law; 2) flange width does not change, and flange middle surface is circumscribed by the ruled surface; 3) shearing strains in the flange plane are insignificant; 4) corner region dimensions are small as compared with the flange width;

for the bend region: 1) a plane strain pattern is assumed (deformation in the forming direction $e_u = 0$); 2) middle surface curve radius remains constant at all deformation stages; 3) strain action in the compressed region is equal to that in the tensile region; 4) surface elements retain their areas during bending: $\alpha \rho dr = \alpha \rho_c d\rho_c$ (α – hemming angle; $d\rho$ and $d\rho_c$ – curve radii increments of the arbitrary and middle layers, respectively);

for the section bottom: 1) the bulging action is small as compared with the plastic compression action; 2) the length of the plastic region in the section bottom part is equal to the extent of hemmed flange's STZ; 3) lateral boundaries of the section bottom plastic region can be considered straight lines.

It is presumed that solving Euler equation [5] with proper boundary conditions will enable to obtain a functional dependence of flange hemming angle in an arbitrary point of the interstand space on the process determining factors. Satisfying the additional condition at the plastic region boundary will allow to obtain the calculated dependence for determining STZ length.

Problem solving

Hemmed flange. Fig. 1 shows the coordinates and parameters of shaping at the k-th transition with hemming angle $\theta_k = \theta(x_2) = \theta_k - \theta_{k-1}$, where x_2 – Cartesian coordinate. The equation for the flange middle surface circumscribed by the ruled surface appears as follows:

$$x_1 - C/2 = v\cos\theta(u);\ x_1 = u;\ x_3 = v\sin\theta(u),$$

where x_1, x_3 – Cartesian coordinates;

C – width of section bottom part;

v, u – curvilinear coordinates;

$\theta(u)$ – hemming angle.

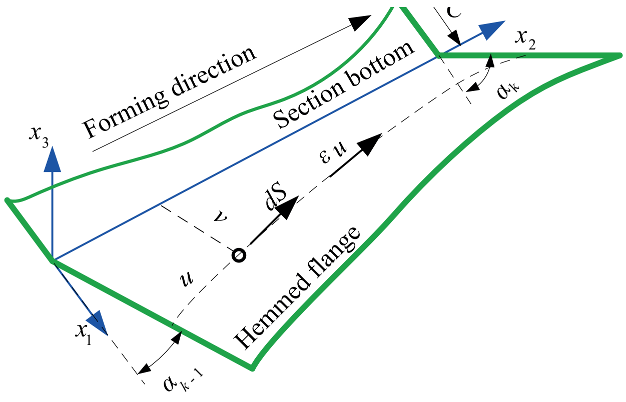

Fig. 1. Diagram of flange hemming and curvilinear coordinates

The length of the linear element of arc dS is determined by the expression:

$$dS = \sqrt{E\cdot(du)^2 + 2F\cdot du\cdot dv + G\cdot(dv)^2},$$

where E, F, G – coefficients of the first quadratic form [5].

The longitudinal and lateral logarithmic strains e_u and e_v, with account for the formulas for dS, x_1, x_2, x_3, are defined by the relationships [3]:

$$e_u = \ln\left(\frac{dS\,|_{dv=0}}{du}\right) = \ln\left(\sqrt{E}\right) = \frac{1}{2}\cdot\ln E; \qquad (1)$$

$$e_v = \ln\left(\frac{dS\,|_{du=0}}{dv}\right) = \ln\left(\sqrt{G}\right) = 0.$$

Applying the incompressibility condition and strain intensity definition, with formula (1) taken into consideration, the value of strain intensity e_i can be found as follows:

$$e_i = \frac{2}{\sqrt{3}}\cdot\ln\left(1 + v^2\cdot\left(\frac{\partial\theta(u)}{\partial u}\right)^2\right). \qquad (2)$$

Specific strain action $A_{\text{п}}^{\text{уд}}$ on the flange, taking into consideration linear hardening [6], can be obtained by integration of specific action increment:

$$A_{\text{п}}^{\text{уд}} = \int \sigma_i\cdot d\varepsilon_i = \sigma_{\text{т}\,0}\cdot\varepsilon_i + \frac{\lambda\cdot\varepsilon_i^2}{2} \approx$$

$$\approx M + N\cdot\left[v^2\cdot\left(\frac{\partial\theta(u)}{\partial u}\right)^2\right], \qquad (3)$$

where σ_i – intensity of strains;

ε_i – intensity of deformations;

λ – linear hardening modulus;

M and N – magnitudes characterising mechanical properties of the blank and defined by relationships:

$$M = \frac{2\sigma_{\text{т}\,0}}{\sqrt{3}} + \frac{2}{3}\lambda;\ N = \frac{2\sigma_{\text{т}\,0}}{\sqrt{3}} + \frac{4}{3}\lambda, \qquad (4)$$

where $\sigma_{\text{т}0}$ – blank material yield strength.

Corner region. To compute specific action associated with shaping of the corner region, let us specify the coordinates of an arbitrary point in it (Fig. 2):

$$x_1 - \frac{C}{2} = \rho\sin\left(\frac{\theta(u)}{2} + \gamma\right) - \rho_c\cdot\sin\left(\frac{\theta(u)}{2}\right);$$

$$x_2 = u; \qquad (5)$$

$$x_3 = \rho_c - \rho\cdot\cos\left(\frac{\theta(u)}{2}\right),$$

where ρ – current radius;

ρ_c – radius of corner region middle surface;

γ – current angle reckoned from the angle bisector.

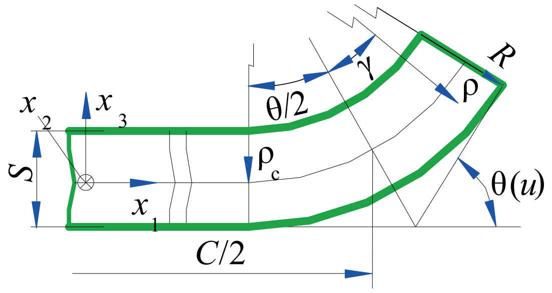

Fig. 2. Coordinates and parameters of section corner region

The coefficients of the first quadratic form, with consideration of equations (5), are expressed by the relationships:

$$E=1;\ F=\rho\sin(\theta+2\gamma);\ G=\rho^2. \qquad (6)$$

The arc length for this case is determined from the formula:

$$dS=\sqrt{E\cdot(d\rho)^2+2\cdot F\cdot d\rho\cdot d\gamma+G(d\gamma)^2}. \qquad (7)$$

Expressing circumferential and radial strains, with formulas (6) and (7) considered, and accounting for the incompressibility condition and strain intensity definition, we can find the specific action of corner region forming:

$$A_{\text{уг}}^{\text{уд}}=\frac{2}{\sqrt{3}}\sigma_{\text{т}\,0}\left(\frac{\rho}{\rho_{\text{д}}}-1\right)+\frac{2\lambda}{3}\left(\frac{\rho}{\rho_{\text{д}}}-1\right)^2, \qquad (8)$$

where $\sigma_{\text{д}}$ – radius of the neutral layer of strains.

Section bottom. Specific strain action on the section bottom in the elastic-plastic boundary condition can be expressed by the relationship:

$$A_{\text{д}}^{\text{уд}}=(\sigma_{\text{т}\,0}+\lambda\varepsilon_{\text{пр}})\varepsilon_{\text{пр}}, \qquad (9)$$

where $\varepsilon_{\text{пр}}$ – reference magnitude of limit elastic strain [6].

A further problem consists in determining the total plastic strain action on the flange, corner region, and bottom part of the section, using relationships (3), (8), and (9), respectively.

The total flange shaping action will be found by integrating specific action (3) for the plastically deformed region of the flange:

$$A_{\text{п}}^{\text{полн}}=\int_0^L\int_0^b\int_0^s A_{\text{п}}^{\text{уд}}ds\cdot dv\cdot du=$$

$$=M\cdot b\cdot s\cdot L+\frac{N\cdot s\cdot b^3}{3}\cdot\int_0^L\left(\frac{d\theta(u)}{du}\right)^2du, \qquad (10)$$

where L – smooth transition zone extent;

s – blank thickness;

b – flange width;

M and N are defined by relationships (4).

The total action of corner region plastic strain is computed by integrating specific action (8) for the tensile region, with subsequent doubling of the result:

$$A_{\text{уг}}^{\text{полн}}=2\eta\int_0^L\int_{\rho_{\text{д}}}^{r+s}A_{\text{уг}}^{\text{уд}}\cdot\rho\cdot\theta(u)\cdot d\rho\cdot du=$$

$$=2\eta\cdot s^2\cdot Q\cdot\int_0^L\theta(u)du, \qquad (11)$$

where r – bend radius;

η – calibration coefficient of corner region stiffness effect;

Q – constant depending on blank mechanical properties and relative bend radius ($\bar{r}=r/s$) with accuracy up to 1 %:

$$Q\approx\frac{\bar{r}}{3}\sigma_{\text{т}\,0}+\frac{2+3\cdot\bar{r}}{10}\cdot\lambda. \qquad (12)$$

The total action of section bottom strain is determined by integrating specific action (9), taking into account boundary approximation for the bulging region:

$$A_{\text{д}}^{\text{полн}}=\int_0^L Z\cdot\theta(u)\cdot du, \qquad (13)$$

where coefficient Z is defined by the relationship:

$$Z=(\sigma_{\text{т}\,0}+\lambda\varepsilon_{\text{пр}})\varepsilon_{\text{пр}}s\,C/2\theta_k. \qquad (14)$$

Total strain action and STZ length. Variation of the total action, which is a sum of actions (10), (11), and (13), is expressed by the relationship:

$$\delta A^{\text{полн}}=\delta\int_0^L\left[Y\cdot\left(\frac{d\theta(u)}{du}\right)^2+W\cdot\theta(u)\right]\cdot du=$$

$$=\delta\int_0^L\xi(\dot{\theta},\theta,u)\cdot du, \qquad (15)$$

where function ξ is an integrand (in square brackets), in which coefficients Y and W are determined by the values of constants (4), (12), and (14):

$$Y=Nsb^3/3;\ W=2\eta s^2Q+Z. \qquad (16)$$

Solving the variation problem for functionality (15) with a moving boundary comes down to integrating Euler equation [5]:

$$\frac{\partial \xi}{\partial \theta}-\frac{d}{du}\left[\frac{\partial \xi}{\partial \dot{\theta}}\right]=0,$$

from which we have:

$$\theta(u)=\left(\frac{W}{4Y}\cdot u+C_1\right)\cdot u+C_2, \qquad (17)$$

where C_1, C_2 – constants determined from the boundary conditions:

$$\theta(u)\,|_{u=0}=0; \quad \frac{d\theta}{du}\Big|_{u=0}=0.$$

In the final form solution (17) appears as follows:

$$\theta(u)=Wu^2/(4\,Y). \qquad (18)$$

To determine STZ extent, it is necessary to use the additional condition for the current process transition $\theta(u)\,|_{u=L}=\theta_k$, after which it follows from (18):

$$L=\sqrt{\frac{4\cdot Y\cdot\theta_k}{W}}. \qquad (19)$$

If we assume that section bottom is absolutely stiff – $C = 0$ ($Z = 0$), and the material is non-hardening – $\lambda = 0$, then the following dependence is obtained from (19):

$$L=\sqrt{\frac{4\cdot b^3\theta_k}{\sqrt{3}\cdot s\cdot \bar{r}\cdot\eta}},$$

which, in the absence of hardening, must coincide with the known models of Gun – Polukhin and Bhattacharyya – Collins [3, 4], wherefrom the value of calibration coefficient determining corner region stiffness $\eta=\sqrt{3}/2\,\bar{r}$ is determined.

After simple transformations, the sought STZ length of the k-th transition can be finally obtained from formula (19) in the form of dependence:

$$L_k=\left[4\cdot b^3\cdot\theta_k\cdot\left(\frac{2\sigma_{\text{т}\,0}}{\sqrt{3}}+\frac{4}{3}\cdot\lambda\right)\Big/\;3\sqrt{3}\times\right.$$

$$\left.\times\left(\frac{\bar{r}\sigma_{\text{т}\,0}}{3}+\frac{2+3\bar{r}}{10}\lambda\right)\frac{s}{\bar{r}}+\frac{3\bar{C}s\varepsilon_{\text{пр}}}{2\theta_k}\left(\sigma_{\text{т}\,0}+\lambda\varepsilon_{\text{пр}}\right)\right]^{\frac{1}{2}}, \qquad (20)$$

where $\bar{r}$, $\bar{C}$ – relative radius and relative width, respectively, of the section bottom.

Discussion of results

An analysis of formula (20) with the use of *MathCAD*-2000*Pro* package reveals a weak dependence of STZ length on relative bend radius $\bar{r}$ (under its variation from 3 to 1, the STZ extent reduces by 8 %) and hardening modulus λ (under its variation from 100 to 400 MPa, the STZ extent reduces by 6 %). In both cases the corner region stiffness increases, which prevents the flange from bending. It will be noted that in the previous models the influence of the bend radius on the STZ length was not accounted for.

Fig. 3 shows STZ length dependence on the hemmed flange width and the hemming angle, and Fig. 4 – STZ dependence on the blank thickness and hemming angle for the parameter values within the range allowed by the technology (indexing of transitions is conventionally omitted). The unspecified parameters for calculations were taken in their mean values: $\theta = 15°$; $\sigma_{\text{т}0} = 200$ MPa; $\lambda = 200$ MPa; $s = 1$ mm; $b = 50$ mm; $C = 50$ mm; $r = 2$ mm; $\varepsilon_{\text{пр}} = 0.002$. The analysis of Figs. 3 and 4 shows that the STZ extent substantially depends on the hemming angles, hemmed flange width, and blank thickness, whereas other factors have considerably lower influence.

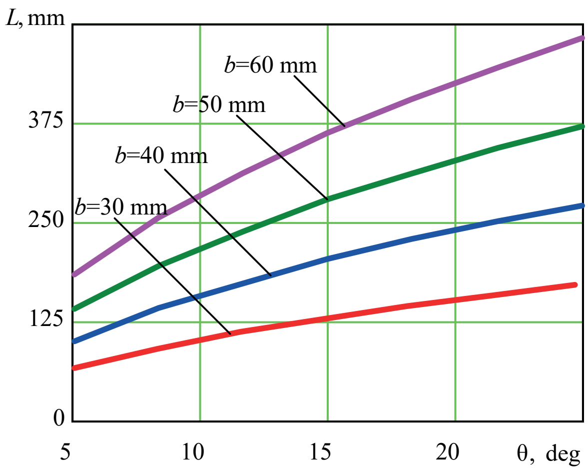

Fig. 3. STZ length vs. flange width and hemming angle

The STZ length is an important parameter allowing to set the limit strain boundaries, establish technology continuity and calculate the interstand distance for newly designed roll-forming machines.

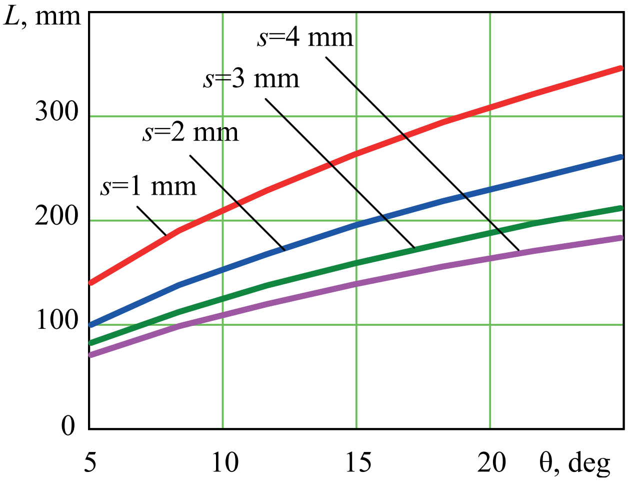

Fig. 4. STZ length vs. blank thickness and hemming angle

Production applications

Shaping of sections with edge stiffening elements (ESE). The designed model (20) can be applied for section with ESE after its reducing to channel-type section with straight flanges of effective thickness (Fig. 5). The effective thickness is determined by the formula [7]:

$$s_{пр}=3J_p/b^3, \qquad (21)$$

where J_p – polar moment of inertia of hemmed flange carrying an ESE.

Subsequent procedures for revealing the ultimate potential for shaping are performed similarly to those applied to U-section with straight flanges [10].

Blank reforming. In case of excessive hemming angles of wide-flanged sections, the STZ length may exceed the interstand distance of a roll-forming machine. This leads to blank reforming and hence, to undue energy consumption, lowering of plastic properties of the blank, which hinders its further processing; worsening

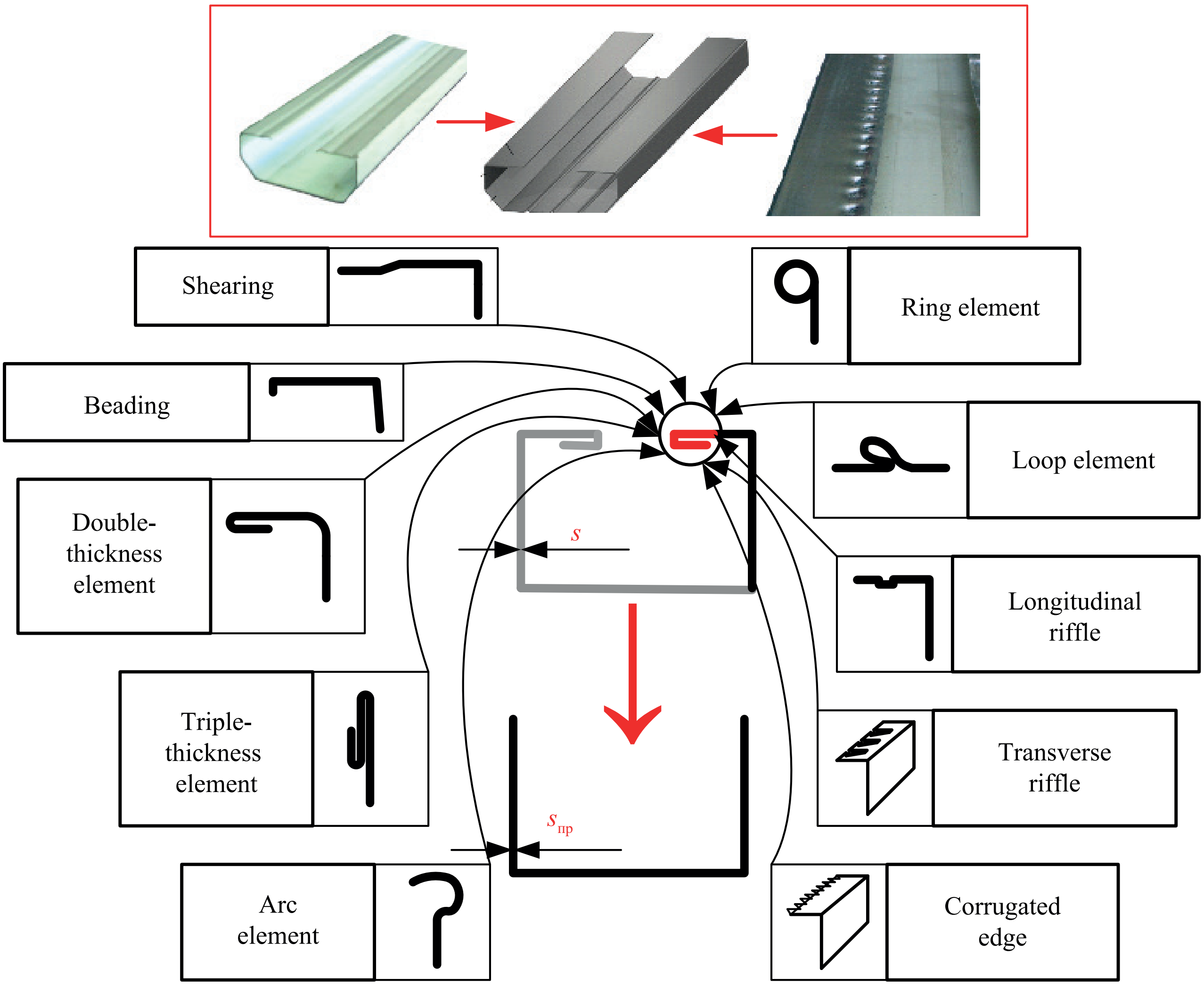

Fig. 5. Reducing section with different types of edge stiffening elements to a channel section form with effective flange thickness

of section surface quality, risk of section defects appearing in the form of flange fracture or edge buckling [7]. When developing a technology, hemming angles for each transition should be checked for compliance with the criterion:

$$\theta_k \leq \text{root}(L(\theta) - L_M, \theta), \qquad (22)$$

where $L(\theta)$ – function defined by formula (20);

L_M – roll-forming machine interstand distance;

root($L(\theta) - L_M$, θ) – limit hemming angles.

The right-hand part of formula (22) is presented in a form convenient for determining limit angle $\theta_{\text{пр}}$ (Fig. 6) in the *MathCAD*-2000*Pro* environment for automation of calculations, but it can as well be obtained in explicit form from formula (20) at $L_k = L_M$. If the limit angles are exceeded in as many as one of the transitions, it will inevitably necessitate revision of the section shaping pattern and correction of the hemming angles.

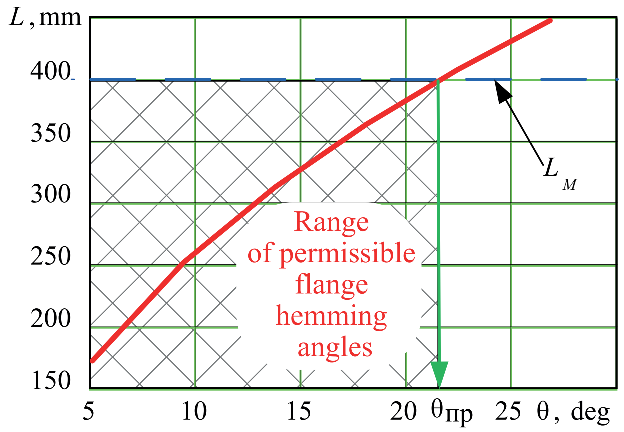

Fig. 6. Determining limit hemming angles

Selection of the hemming angles is also influenced by the section bottom width, which matters when solving the issue of technology continuity.

Technology continuity. It is shown in [7] that section bottom width affects the forming mode, in particular, the STZ length and longitudinal deformations of the hemmed flanges. For sections with wide bottom, its forming conditions become “smoother” as compared with a narrow-bottom section, with all other conditions being equal, which allows to reduce the number of transitions. This is quite important for the technology continuity.

For example, there is a proven and assimilated technology for producing a six-transition type section with ESE, having bottom width of 50 mm. An issue arises of applying this technology to forming a section with the same parameters except for the bottom width (which quite frequently occurs in practice).

Model (20), in which the STZ length is given as a function of relative bottom width (Fig. 7), responds to this issue. An average hemming angle over one transition, for a type profile with bottom width of 50 mm and total hemming angle of 90°, is 15°, corresponding to which is STZ length equal to 258 mm. To retain in the interstand space the same configuration of the hemmed flange of a section with bottom width of 150 mm, the hemming angle over one transition must be 17.3°, and the number of transitions (as shown by calculation using the model of the number of transitions [7]) – 5.2 (actually, 5 transitions). However, if the bottom width of a manufactured section is 0 mm (stiff bottom), then an average hemming angle has to be equal to 13.5°, which will take 6.7 transitions (actually, 7 transitions). In Fig. 7 the procedure of determining said average hemming angles over one transition is shown with arrows. Using this method of technology designing, it is no longer required to perform calculations of strain stability, assess the risks of blank reforming, neither run a full cycle of experimental tryout, which reduces the costs of technology development by 9–15 % [8, 10].

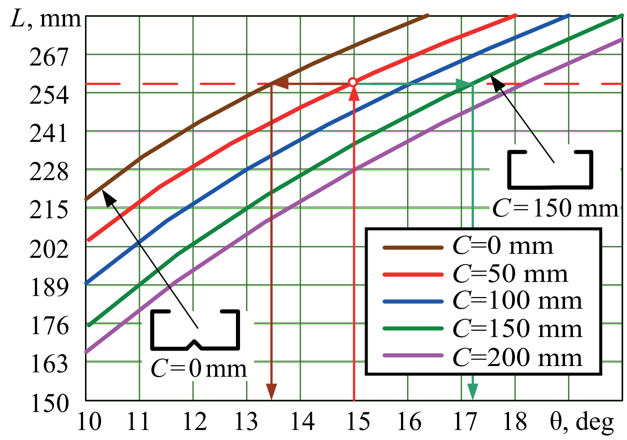

Fig. 7. Revealing technology continuity for fabrication of sections with different bottom thickness on the basis of STZ extent model

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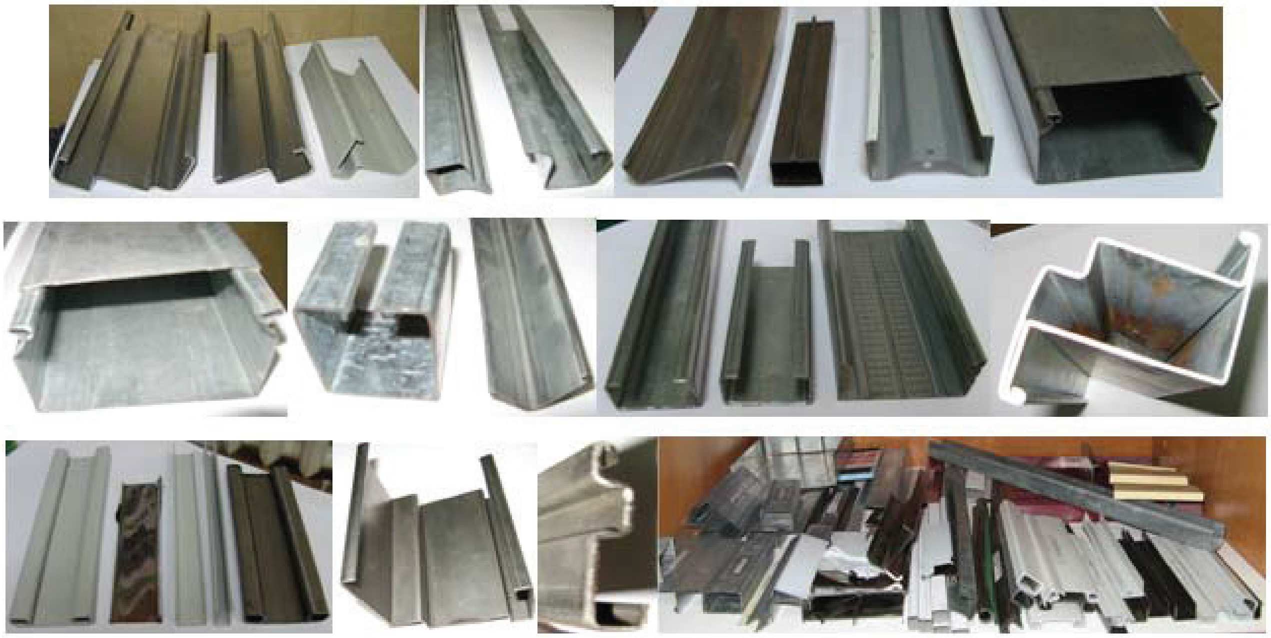

Fig. 8. Sections with ESE obtained at developer enterprises of Ulyanovsk city

When forming sections with ESEs, due to the “superstructure”, a load-bearing flange has greater stiffness than a straight one, therefore it is necessary to take into account element stiffness based on the “local stiffnesses” method [7], using formula (21). The total STZ length should be determined as a sum of STZ lengths of the load-bearing flange and peripheral element. This is especially important in fabrication of multi-element formed sections produced as per semi-closed patterns [8]. Some of the sections with edge stiffening elements shown in Fig. 8 are fabricated with the use of the technology continuity principle.

Calculation of interstand distance for a product-line oriented roll-forming machine. The same model (20) can be used when determining a required number of stands for a product-line oriented roll-forming machine [9]. At the outset, for each of the sections to be manufactured on a given machine, a required number of transitions is calculated using the model of the number of transitions [10], and for sections with the largest and smallest number of transitions an average hemming angle for a single transition is determined. Then, proceeding from the obtained hemming angles, the STZ length is calculated by formula (20). The largest of the two calculated values is taken as the sought interstand distance.

Conclusion

Application of the developed model of the smooth transition zone length makes it possible to solve the issues of limit strain, technology continuity, and calculation of the interstand distance of roll-forming machines.

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