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Optimum regular local spline interpolation of signals

The study deals with an optimum approach to regular local signal interpolation by means of generalised splines. For the special case of local regular polynomial spline interpolation we derive quasi-optimal interpolation bases and provide corresponding recommendations dealing with selecting interpolation order and order of smoothness.

Keywords: signal interpolation, interpolating function, quantization interval, spline interpolation, spline basis.

Introduction

The study is devoted to the problem of regular signal interpolation with the spectrum limited by frequency ω_m in case of its correct quantization. This problem has an accurate solution represented by the Kotelnikov interpolation series. However, the interpolation based on the Kotelnikov series is global. This factor makes it difficult to implement interpolation in practice and results in searching for approximate solutions with preference given to local methods of interpolation.

In case of local interpolation, the interpolating function is formed as a set of fragments describing it at each quantization interval. When searching approximate solutions to the interpolation problem, it is necessary to take into account that a signal with the limited spectrum is a smooth function, thus giving preference to smooth blending of interpolating function fragments, i.e. the use of splines.

Classic mathematical approaches to interpolation problems are described in detail in [1].

Spline interpolation is one of promising modern trends in the interpolation theory. Mathematical properties of splines are studied in detail in [2, 3]. Practical aspects of spline interpretation are analysed in [4]. However, most of the specified studies of splines are devoted to global interpolation. Local spline interpolation regarding aircraft characteristics is analysed in [5], and regarding signals – in [6–12].

The common feature typical of local spline interpolation methods is the ambiguity associated with a random selection of methods of approximate calculation of restored signal's derivatives at nodes where interpolating fragments meet. This results in a variety of appropriate methods of local spline interpolation along with lack of any general recommendations on their use in practice. This study is an attempt to eliminate the existing ambiguity that implies the formulation of, and a solution to, the problem of optimization. This approach allowed to elaborate recommendations for choosing local spline interpolation of a signal.

In case of regular interpolation, the sampling grid is uniform and infinite, while an interpolating function is formed at each particular quantization interval in accordance with the same rules. Let us designate coordinates of interpolation nodes as $t_n = nT$, $s_n = s(nT)$, where $n = 0, \pm1, \pm2, \ldots$; T – signal quantization period. We will write the expression for the interpolating function as follows

$$\psi(t) = \sum_{n=-\infty}^{+\infty} s_n \varphi_0(t - t_n) = \sum_{n=-\infty}^{+\infty} s(nT)\varphi_0(t - nT), \quad (1)$$

where generating function $\varphi_0(t)$ satisfies the condition

$$\varphi_0(kT) = \begin{cases} 1, & k = 0, \\ 0, & k = \pm1, \pm2, \pm3, \ldots . \end{cases} \quad (2)$$

Since the quantization period can be reduced with no limit, interpolation process convergence is enabled if the following condition is satisfied [13]

$$\Phi_0\left(\frac{2\pi k}{T}\right) = \begin{cases} T, & k = 0, \\ 0, & k = \pm1, \pm2, \pm3, \ldots, \end{cases} \quad (3)$$

where $\Phi_0(\omega)$ – spectral density $\varphi_0(t)$.

The following normalizations are based on conditions (2) and (3)

$$\int_{-\infty}^{+\infty} \varphi_0(t)dt = T,$$

$$\int_{-\infty}^{+\infty} \Phi_0(\omega)d\omega = 2\pi.$$

Further, we will analyse isotropic interpolation, i.e. interpolation that implies "equality" of signal values preceding and following any selected instant of time. Along with that, the generating function and its spectral density are even-symmetrical and real-valued functions.

Local fundamental interpolation generalised spline bases

In case of spline interpolation, fragments of interpolating function $\psi_n(t)$, describing it at each particular quantization interval $t \in [t_n; t_{n+1}]$, are joined at interpolation nodes in such a manner so that the ambiguity of its first derivatives r (value r characterises the degree of smoothness) is provided together with consistency condition (2):

$$\psi_n(t_n) = s_n, \quad \psi_n(t_{n+1}) = s_{n+1},$$

$$\psi_n'(t_n) = \sum_{m=-M}^{M} d_m^{(1)} s_{n+m},$$

$$\psi_n'(t_{n+1}) = \sum_{m=-M}^{M} d_m^{(1)} s_{n+1+m}, \qquad (4)$$

$$\ldots,$$

$$\psi_n^{(r)}(t_n) = \sum_{m=-M}^{M} d_m^{(r)} s_{n+m},$$

$$\psi_n^{(r)}(t_{n+1}) = \sum_{m=-M}^{M} d_m^{(r)} s_{n+1+m},$$

where $\{d_m^{(i)}\}_{m=-M}^{M}$ – coefficients of linear algorithm of estimation of the i-th derivative of a signal;

M – derivative estimation algorithm order.

Fig. 1 shows an example of the interpolation fragment.

Interpolating function fragment $\psi_n(t)$ is determined by values of M preceding and M following samples s_{n-M}, ..., s_{n-1}, s_n, s_{n+1}, s_{n+2}, ..., s_{n+M+1} relative to interval $t \in [t_n; t_{n+1}]$ ($M \geq 0$ – interpolation order) and is supposed to be a generalised polynomial

$$\psi_n(t) = \sum_{k=0}^{N-1} C_{n,k} w_k(t - t_n). \qquad (5)$$

Here, $\{w_k(t)\}_{k=0}^{N-1}$ – linearly independent system of functions (primary basis);

$C_{n,k}$ – coefficients arcwise connected with signal samples due to expression (1):

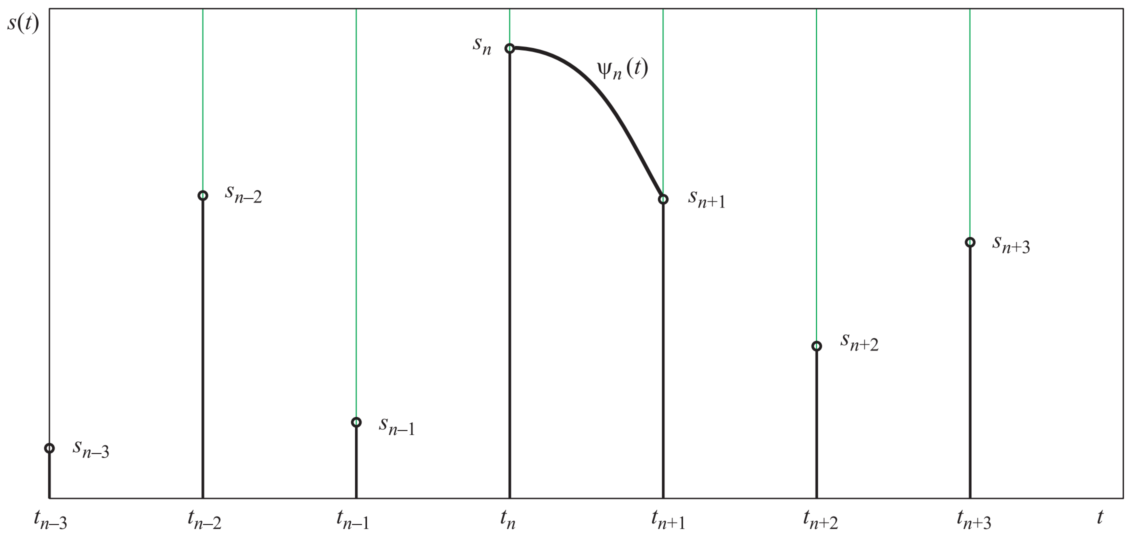

Fig. 1. Example of interpolating fragment

$$C_{n,k} = \sum_{m=-M}^{M+1} a_{m,k} s_{n+m}, \tag{6}$$

where $a_{m,k}$ – constants to be determined by means of the interpolation method and similar for each particular quantization interval. The generalised polynomial order is determined by the number of equations in the system (4) and correlates with the degree of smoothness

$$N = 2r + 2. \tag{7}$$

With account for recovery of a unit sample, we get the following expression for the generating function from formulae (5) and (6)

$$\varphi_0(t) = \\ = \begin{cases} \sum_{k=0}^{N-1} a_{(M+1),k} w_k(t - t_{(-(M+1))}),\ t \in [t_{(-(M+1))};\ t_{(-M)}], \\ \ldots, \\ \sum_{k=0}^{N-1} a_{0,k} w_k(t - t_0),\ \ t \in [t_0;\ t_1], \\ \ldots, \\ \sum_{k=0}^{N-1} a_{(-M),k} w_k(t - t_M),\ \ t \in [t_M;\ t_{M+1}], \end{cases} \tag{8}$$

where the coefficients are determined from the solution to the matrix equation

$$\boldsymbol{A} = \boldsymbol{W}^{-1}\boldsymbol{D}, \tag{9}$$

where

$$\boldsymbol{A} = \begin{bmatrix} a_{-M,0} & a_{-M+1,0} & \ldots & a_{M+1,0} \\ a_{-M,1} & a_{-M+1,1} & \ldots & a_{M+1,1} \\ \ldots & \ldots & \ldots & \ldots \\ a_{-M,N-1} & a_{-M+1,N-1} & \ldots & a_{M+1,N-1} \end{bmatrix},$$

$$\boldsymbol{W} = \begin{bmatrix} w_{0,0} & w_{1,0} & \ldots & w_{N-1,0} \\ w_{0,1} & w_{1,1} & \ldots & w_{N-1,1} \\ w'_{0,0} & w'_{1,0} & \ldots & w'_{N-1,0} \\ w'_{0,1} & w'_{1,1} & \ldots & w'_{N-1,1} \\ \ldots & \ldots & \ldots & \ldots \\ w^{(r)}_{0,0} & w^{(r)}_{1,0} & \ldots & w^{(r)}_{N-1,0} \\ w^{(r)}_{0,1} & w^{(r)}_{1,1} & \ldots & w^{(r)}_{N-1,1} \end{bmatrix},$$

$$\boldsymbol{D} = \\ = \begin{bmatrix} 0 & \ldots & 0 & 1 & 0 & 0 & \ldots & 0 \\ 0 & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 \\ d^{(1)}_{-M} & \ldots & d^{(1)}_{-1} & d^{(1)}_{0} & d^{(1)}_{1} & d^{(1)}_{2} & \ldots & 0 \\ 0 & \ldots & d^{(1)}_{-2} & d^{(1)}_{-1} & d^{(1)}_{0} & d^{(1)}_{1} & \ldots & d^{(1)}_{M} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ d^{(r)}_{-M} & \ldots & d^{(r)}_{-1} & d^{(r)}_{0} & d^{(r)}_{1} & d^{(r)}_{2} & \ldots & 0 \\ 0 & \ldots & d^{(r)}_{-2} & d^{(r)}_{-1} & d^{(r)}_{0} & d^{(r)}_{1} & \ldots & d^{(r)}_{M} \end{bmatrix}, \tag{10}$$

where $w^{(i)}_{k,m} = w^{(i)}_k(mT)$;

$a_{m,k} = [\boldsymbol{A}]_{k,m+M}$.

Therefore, generating function (8) can be determined using equation (9). Then, with account for (1), we can write the following expression for each interpolating fragment

$$\psi_n(t) = \sum_{m=-M}^{M+1} s_{n+m}\varphi_0(t-(n+m)T). \tag{11}$$

According to expression (11), computational costs for calculating the interpolating function value at a certain time are equal to $2M+1$ multiplications, provided the generating function value corresponding to this instant of time is stored in computer memory. The latter is typically the case, since the instants of time when the interpolating function is calculated, are known beforehand. Along with that, computational costs are determined based only on the interpolation order and do not depend on the degree of smoothness or selection of the primary basis. However, if the specified condition is not satisfied, N multiplication operations for calculating (8) as well as multiplications n_w needed for calculating primary basis functions are added per each multiplication $2M+1$. In accordance with equation (7), the greater the degree of smoothness, the greater the computational costs, which are equal to $(2M+1)((2r+2)+n_w)$.

Analysis of convergence at local regular spline interpolation of signals

According to [13], the convergence of the regular interpolation method takes place if the method allows to recover the unity. With account for the unity recovery, we will get convergence conditions at local spline interpolation from system of equations (4):

$$\sum_{m=-M}^{M} d_m^{(i)} = 0,\ i = 1, ..., r. \tag{12}$$

According to the signal differentiation theorem for the Fourier transform, we will make an additional statement that the sequence of coefficients of the odd-order derivative estimation algorithm shall be odd-symmetrical, and in case of an even-order derivative, it shall be even-symmetrical.

$$d_m^{(2i+1)} = -d_{(-m)}^{(2i+1)},\ d_0^{(2i+1)} = 0,$$
$$i = 0, 1, 2, ...,\ \ m = -M, ..., M;$$
$$d_m^{(2i)} = d_{(-m)}^{(2i)},\ i = 1, 2, ...,\ \ m = -M, ..., M. \tag{13}$$

Analysis of distortions at local regular interpolation of signals

With account for Fourier transform (1), regarding linearity properties and time delay for the spectral density of the interpolating functions, we will write the following expression

$$\Psi(\omega) = \Phi_0(\omega)\sum_{n=-\infty}^{+\infty} s(nT)e^{-j\omega nT}.$$

Since the discrete signal spectrum is

$$S_д(\omega) = \sum_{n=-\infty}^{+\infty} s(nT)e^{-j\omega nT} = \frac{1}{T}\sum_{k=-\infty}^{+\infty} S(\omega - \omega_д),$$

where $S(\omega)$ – spectral density of signal $s(t)$;

$\omega_д = \dfrac{2\pi}{T}$ – sampling rate.

Let us write the latest expression in the following form

$$\Psi(\omega) = S_д(\omega)\Phi_0(\omega) = \Phi_0(\omega)\frac{1}{T}\sum_{k=-\infty}^{+\infty} S(\omega - \omega_д).$$

The spectral density of the interpolating function is the result of separation of initial signal spectrum $S(\omega)$ from discrete signal spectrum $S_д(\omega)$ by multiplying generating function $\Phi_0(\omega)$ (Fig. 2) by the spectral density. Such separation is supposed to be accurate if $\Phi_0(\omega)$ is a rectangular function that is not an option in case of local interpolation, since generating function $\varphi_0(t)$ is time limited, so its spectral density $\Phi_0(\omega)$ is an analytical function, which is not the null equation and cannot become zero whichever the finite interval might be. In this respect, if $\omega_д > 2\omega_m$, interpolating function $\psi(t)$ can be viewed in a formal way as the result of distortions of initial signal $s(t)$. In case of such distortions, the amplitude of each frequency signal harmonic ω changes irrespective of other harmonics when multiplied by $\Phi_0(\omega)$, with the signal superimposed by aliasing of frequency oscillations $k\omega_д + \omega$, $k = 0, 1, 2, \dots$, with amplitudes proportional to $\Phi_0(k\omega_д + \omega)$. There is no combination interaction between signal harmonics. That is why distortions can be minimised individually for each harmonic.

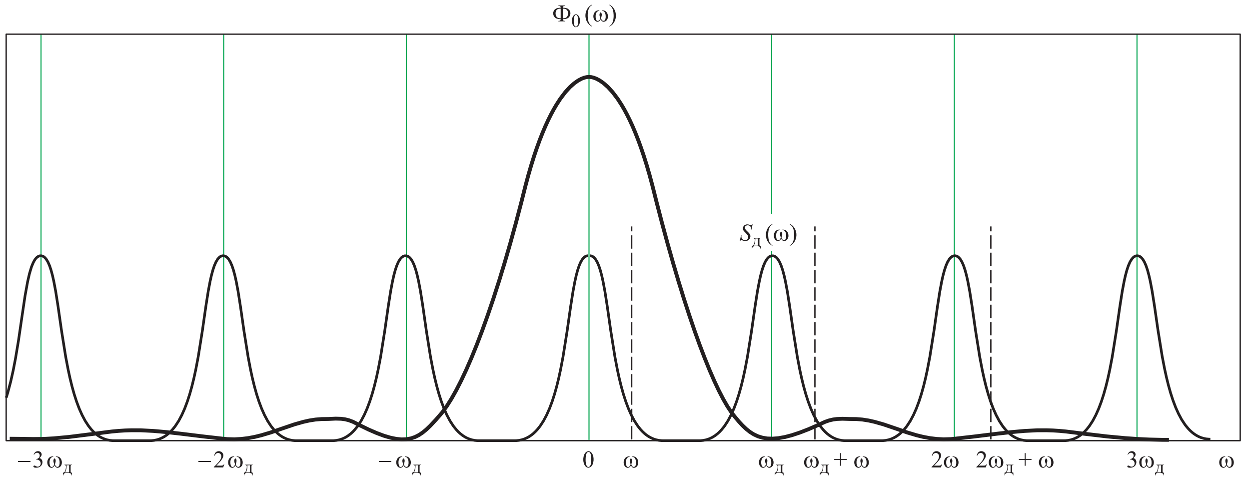

Fig. 2. Graphs of discrete signal spectrum $S_д(\omega)$ and spectral density of generating function $\Phi_0(\omega)$

Distortions of an individual harmonic component of the signal will be characterised by the distortion coefficient to be determined as the ratio of the current value of total unwanted harmonics, with occur during interpolation, to the current value of the harmonic to be restored. Along with that, if we analyse only the interpolation that provides convergence, i.e. with account for condition (3), $\Phi_0(0) = T$, we write the expression for the distortion coefficient:

$$K_и(\omega) = \frac{1}{T}\times \times\sqrt{(T-\Phi_0(\omega))^2+\Phi_0^2(\omega_д+\omega)+\Phi_0^2(2\omega_д+\omega)+\Phi_0^2(3\omega_д+\omega)+...} \quad (14)$$

In practical calculations, the finite number of unwanted harmonics shall be taken into account

$$K_и^N(\omega) = \frac{1}{T}\times \times\sqrt{(T-\Phi_0(\omega))^2+\Phi_0^2(\omega_д+\omega)+\Phi_0^2(2\omega_д+\omega)+...+\Phi_0^2(N\omega_д+\omega)} \quad (15)$$

The analysis of dependence $K_и(\omega)$ can be taken as a basis for determining the required value of sampling rate and/or selection of the interpolation method at a given level of distortions. Selection of the admissible distortion level $K_{и,доп}$ depends on the specifics of the problem to solve and defines limit frequency $\omega_{гр}$ for the interpolation method to satisfy the condition $\leq \omega_{гр}) \leq K_{и,доп}$. To restore a signal with maximum spectrum frequency ω_m with a given distortion level, it is necessary to satisfy the condition $\omega_m \leq \omega_{гр}$, while each harmonic of frequency signal ω is restored with $K_и(\omega) \leq K_{и,доп}$.

Formulation of the problem of optimum interpolation spline basis synthesis

The task is to find a set of elements of matrix $\boldsymbol{D}$ (10) that satisfies convergence conditions (12) and ensures maximum limit frequency $\omega_{гр}$ of the distortion coefficient (14) at a given distortion level $K_{и,доп}$, primary basis $\{w_k(t)\}_{k=0}^{N-1}$, interpolation order M and degree of smoothness r:

$$\omega_{гр}\left(\{d_m^{(1)}\}_{m=-M}^{M}, ..., \{d_m^{(r)}\}_{m=-M}^{M}\right) \to \max. \quad (16)$$

With such problem formulation, the best suitable method is the method of spline interpolation that allows, all other things being equal, to restore a signal at a given admissible distortion level with a minimum sampling rate margin.

Methods of solving the problem of synthesis of optimum interpolation spline basis

The problem of synthesis of optimum interpolation spline basis (16) is a multi-parameter single criterion problem of optimization. It was difficult to obtain the closed form of the target function. So, we had to find approximate (quasi-optimal) solutions using numerical methods, including random search methods and genetic algorithm [14, 15] and to develop high-volume software.

For calculations, we took into account the tendency of the generating function's spectral density to decrease, and expression (15) was used instead of expression (14) for $K_и^{10}(\omega)$. For calculations per expression (15), we solved the problem of the generating function spectral analysis at local polynomial spline interpolation in a general form [16].

Also, we should note that adding of interrelations (13) allows to reduce a number of problem parameters.

Certain quasi-optimal local polynomial spline bases

In case of polynomial spline interpolation, the primary basis is formed by power functions $w_k(t) = t^k$, $k = 0, ..., N-1$, where N is determined (7). Generating functions of optimum interpolation spline bases at $K_{и,доп} = 0.001$ for interpolation of order $M = 1, 2, 3$ and degrees of smoothness $r = 1, 2$ are given in the table. The Table also contains values of limit frequencies expressed in normalised units. Distortion coefficient graphs are shown in Fig. 3. The line numbers shown in Fig. 3 correspond to the numbers given in the rightmost column of the table.

Conclusion

We proposed a unified generalised approach to mathematical description of methods of local regular spline interpolation of signals using generalised polynomials. We analysed the convergence

Generating functions of optimum interpolation spline bases

r	M	$\varphi_0(t)$	$(\omega T)_{гр}$	No.
1	1	$\varphi_0(t)=\begin{cases}1-2.4586\tau^2+1.4586\tau^3, & \tau=\lvert t/T\rvert,\ \lvert t\rvert\le T,\\ -0.5414\tau+1.0829\tau^2-0.5414\tau^3, & \tau=\lvert t/T\rvert-1,\ T<\lvert t\rvert\le 2T\end{cases}$	0.75	1
	2	$\varphi_0(t)=\begin{cases}1-2.2343\tau^2+1.2343\tau^3, & \tau=\lvert t/T\rvert,\ \lvert t\rvert\le T,\\ -0.7657\tau+1.3826\tau^2-0.6169\tau^3, & \tau=\lvert t/T\rvert-1,\ T<\lvert t\rvert\le 2T,\\ 0.1489\tau-0.2978\tau^2+0.1489\tau^3, & \tau=\lvert t/T\rvert-2,\ 2T<\lvert t\rvert\le 3T\end{cases}$	1.02	2
	3	$\varphi_0(t)=\begin{cases}1-2.1173\tau^2+1.1173\tau^3, & \tau=\lvert t/T\rvert,\ \lvert t\rvert\le T,\\ -0.8827\tau+1.4979\tau^2-0.6152\tau^3, & \tau=\lvert t/T\rvert-1,\ T<\lvert t\rvert\le 2T,\\ 0.2675\tau-0.4840\tau^2+0.2165\tau^3, & \tau=\lvert t/T\rvert-2,\ 2T<\lvert t\rvert\le 3T,\\ -0.0511\tau+0.1021\tau^2-0.0511\tau^3, & \tau=\lvert t/T\rvert-3,\ 3T<\lvert t\rvert\le 4T\end{cases}$	1.02	3
2	1	$\varphi_0(t)=\begin{cases}1-1.0182\tau^2-4.3122\tau^3+7.2100\tau^4-2.8796\tau^5, & \tau=\lvert t/T\rvert,\ \lvert t\rvert\le T,\\ -0.5310\tau+0.5091\tau^2+1.6588\tau^3-2.7208\tau^4+1.0839\tau^5, & \tau=\lvert t/T\rvert-1,\\ & T<\lvert t\rvert\le 2T\end{cases}$	0.70	4
	2	$\varphi_0(t)=\begin{cases}1-1.1779\tau^2-2.8917\tau^3+5.0617\tau^4-1.9922\tau^5, & \tau=\lvert t/T\rvert,\ \lvert t\rvert\le T,\\ -0.7448\tau+0.5957\tau^2+2.1489\tau^3-3.2373\tau^4+1.2375\tau^5, & \tau=\lvert t/T\rvert-1,\\ & T<\lvert t\rvert\le 2T,\\ 0.1315\tau-0.0068\tau^2-0.7685\tau^3+1.0314\tau^4-0.3876\tau^5, & \tau=\lvert t/T\rvert-2,\\ & 2T<\lvert t\rvert\le 3T\end{cases}$	1.36	5
	3	$\varphi_0(t)=\begin{cases}1-1.36\tau^2-1.7925\tau^3+3.5101\tau^4-1.3576\tau^5, & \tau=\lvert t/T\rvert,\ \lvert t\rvert\le T,\\ -0.8451\tau+0.7473\tau^2+1.7872\tau^3-2.6821\tau^4+0.9927\tau^5, & \tau=\lvert t/T\rvert-1,\\ & T<\lvert t\rvert\le 2T,\\ 0.2461\tau-0.0570\tau^2-1.1009\tau^3+1.4422\tau^4-0.5304\tau^5, & \tau=\lvert t/T\rvert-2,\\ & 2T<\lvert t\rvert\le 3T,\\ -0.0537\tau-0.0103\tau^2+0.3533\tau^3-0.4607\tau^4+0.1715\tau^5, & \tau=\lvert t/T\rvert-3,\\ & 3T<\lvert t\rvert\le 4T\end{cases}$	1.75	6

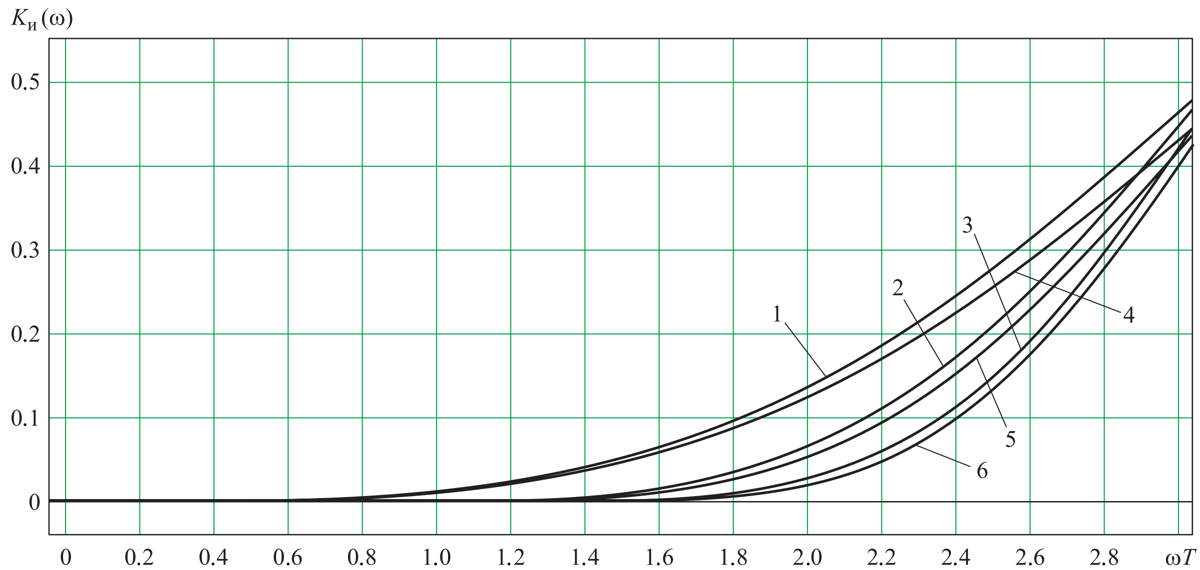

Fig. 3. Table values of distortion coefficient for interpolation methods:
1 – $M=1$, $r=1$; 2 – $M=2$, $r=1$; 3 – $M=3$, $r=1$;
4 – $M=1$, $r=2$; 5 – $M=2$, $r=2$; 6 – $M=3$, $r=2$

at local regular spline interpolation of signals. We analysed distortions at regular interpolation and proved their non-linear behaviour. We introduced the distortion coefficient that characterises the quality of signal restoration with the limited spectrum, taking into account distortions of signal harmonic components.

Using numerical and analytical methods, we managed to find quasi-optimal solutions to the set problem at hand regarding the synthesis of the optimum interpolation spline basis in the form of generating functions of fundamental interpolation polynomial spline bases at interpolation of order $M = 1, 2, 3$ and degrees of smoothness $r = 1, 2$. We worked out the following recommendations for method application in practice: admissible distortions are provided if the maximum frequency in the signal spectrum does not exceed the limit frequency of the distortion coefficient.

The interpolation methods under consideration have been developed to the level of application in practice and can be selected with regard to the specifics of the problem to solve, with account for relation between the scope of computations and the limit frequency of the distortion coefficient. Thus, in order to increase the limit frequency, it is preferable to increase the degree of smoothness and the local interpolation order; however, this requires more computations and higher computation accuracy.

Beyond the scope of this paper is the search for optimum local polynomial spline bases at higher orders of local interpolation and degrees of smoothness.

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Оптимальная регулярная локальная сплайновая интерполяция сигналов

Рассмотрен оптимальный подход к регулярной локальной интерполяции сигналов обобщенными сплайнами. В частном случае локальной регулярной полиномиальной сплайн-интерполяции получены квазиоптимальные интерполяционные базисы, приведены соответствующие рекомендации по выбору порядка интерполяции и степени гладкости.

Ключевые слова: интерполяция сигнала, интерполирующая функция, интервал дискретизации, сплайн-интерполяция, сплайн-базис.

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Область научных интересов: теория сигналов, цифровая обработка сигналов, интерполяция, экстраполяция и аппроксимация сигналов, оптимальная обработка сигналов.