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On development of a technique for estimating structural dynamics and strength of modular phased array

The purpose of the study was to develop a method of computational and experimental analysis to reduce the dimension of the problem, which makes it possible to simplify and accelerate the strength calculations. When using the method, one can take into account the stiffening effect of the carrier object, where the product will be installed, determine the transmission coefficients of vibration acceleration from the base of the structure to the individual units of the equipment to assess their strength and stability under the influence of mechanical factors. Moreover, the method allows for the strain-stress state analysis using the dynamic environment coefficients. Currently, the developed method is used in the design of several promising projects using modular phased arrays, both sea- and land-based.

Keywords: modular phased arrays, mathematical simulation, dynamics and strength, phased array deformations.

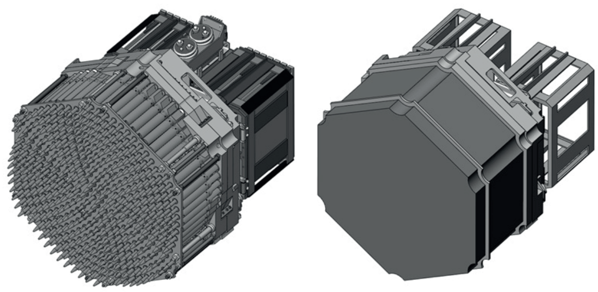
Introduction

Contemporary phased antenna arrays (PAA) are complex, large-sized and multi-connection structures with multiply repeating structural elements. Accurate computation of elastic systems, especially systems with distributed mass, is very difficult in itself, and computation of such structures like the PAA is currently infeasible, basically due to limited performance of computing systems. Public Joint Stock Company “Research and Production Corporation “Almaz” named by Academician A. A. Raspletin” has designed and is currently implementing two promising projects using sea- and land-based modular PAAs. The computation method has been tested at the product design stage.

Method of modular PAA structural dynamics and strength analysis

To enable application of computer-aided mechanical impact simulation systems, the computational model shall be simplified. It is reasonable to replace PAA modules with approximate models. This allows to reduce the computation model’s quantity of degrees of freedom and to study deformation of the antenna frame with account for PAA modules. For developing a simplified model, its loading pattern shall be taken into consideration.

As an example, we will analyse two modules with different loading patterns. Fig. 1 shows a ship-based radar system (RS) PAA module and its approximate model. It is obvious that an array of phase shifters is included in the module structure’s loading pattern and contains multiple duplicate elements, such as phase shifters and holes in the body and in the disc. To reduce the dimension of the problem, a transition was made from the array of phase shifters equally spaced on the surface between the body and the disc to a 3D continuum having certain anisotropic properties.



a b

Fig. 1. Ship-based RS PAA module (a) and its approximate model (b)

Fig. 2 shows a land-based RS PAA module and its approximate model. The structure design is based on eight transceiver modules aggregated as a single unit using a baseplate and two attachment plates to fasten the PAA to the antenna frame. This example shows the structure’s power circuit without an array of phase shifters.

The computation model development is followed by computation of the system’s natural

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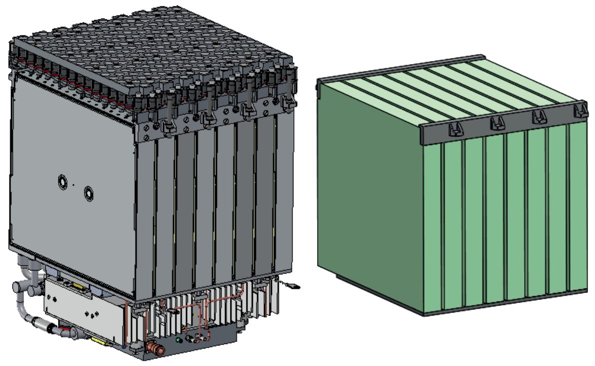


Fig. 2. Land-based RS PAA module (a) and its approximate model (b)

frequencies and shape using any finite element software package. Further, the response to external impact shall be determined for each node of the finite-element model [1]:

$$[\mathbf{K}]\{\mathbf{u}\}+[\mathbf{C}]\frac{\partial}{\partial t}\{\mathbf{u}\}+ +[\mathbf{M}]\frac{\partial^2}{\partial t^2}\{\mathbf{u}\}+\{\mathbf{F}(t)\}=0, \tag{1}$$

where $[\mathbf{K}]$ – system's stiffness matrix with regard to inter-element elastic linkage;

$\{\mathbf{u}\}$ – displacement column-vector;

$[\mathbf{C}]$ – system elements' damping matrix with regard to energy dissipation;

$[\mathbf{M}]$ – system mass matrix;

t – time;

$\{\mathbf{F}(t)\}$ – external forces matrix.

We use the two-parameter Rayleigh model as a damping model:

$$[\mathbf{C}]=\alpha[\mathbf{M}]+\beta[\mathbf{K}], \tag{2}$$

where α, β – constants.

Frequently investigated systems can be reduced to an equivalent single-mass system. For this purpose, we shall select frequencies corresponding to the maximum contributions the vibration mode makes to kinetic and potential energy of elastic structural deformations in the direction of a particular coordinate axis. Frequency selection is available if correlation between frequencies is subtle. In order to check the system for compliance with the above condition, the Mandelstam's correlation criterion is applied [2]. For the obtained *basic* frequencies, reaction forces formed in structure supports will be maximum; therefore, inertial loads will reach their maximum values in the selected direction. The *basic* frequency vibration mode is often similar to the distorted form of the system exposed to static load. This allows to consider the structure behaviour under overloads via dynamic response factors and to run a statistical analysis.

Adequate evaluations of factors α and β of the Rayleigh model (2) for complex structures can be obtained by means of experiments only. For this purpose, we experimentally studied damped vibrations of standard-type shipboard equipment, including those based on PAA models. Experimental data were processed using the method developed at the Central Aerohydrodynamic Institute named after N. E. Zhukovsky. According to the method, we can determine natural frequencies and relevant damping logarithmic decrements. A numeric sequence of the recorded damping transient process from the experiment obeys the equation

$$\mathbf{X}_k=\sum_{i=1}^{n}\mathbf{A}_i e^{-\delta_i f_i k\Delta t}\cos(2\pi f_i k\Delta t+\varphi_i), \tag{3}$$

where $\mathbf{X}_k$ – vector, the elements of which are values of signals generated by measuring sensors at time $k\Delta t$;

k – sampling number equal to 0, 1, 2, …;

n – structure's quantity of degrees of freedom;

i – vibration mode number;

$\mathbf{A}_i$ – initial vibration amplitude vector;

δ_i – vibration damping logarithmic decrement;

f_i – vibration frequency (Hz);

Δt – time interval between samples; it is considered constant during recording;

φ_i – initial vibration phase.

The real structure is replaced with a viscoelastic model with the equivalent absorption capacity. This model is the result of harmonic linearisation of dissipative characteristics. In this case, the damping coefficient is

$$c_i = \delta_i / 2\pi, \qquad (4)$$

where δ_i – vibration damping logarithmic decrement.

Damping decrement is corrected by means of linear approximation of the experimental dependence of the vibration damping decrement on $k\Delta t$.

To determine the Rayleigh model parameters based on experimental data, the Lavrentiev – Tikhonov regularisation method was applied for solving "ill-posed" problems, including degenerate and ill-conditioned systems of equations [3, 4]:

$$\left([\mathbf{A}]^T[\mathbf{A}] + \lambda[\mathbf{E}]\right)(\mathbf{X}) = [\mathbf{A}]^T(\mathbf{B}), \qquad (5)$$

where

$$(\mathbf{X}) = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}; \quad \mathbf{A} = \begin{bmatrix} 1 & \omega_1^2 \\ 1 & \omega_2^2 \\ \cdot & \cdot \\ 1 & \omega_n^2 \end{bmatrix}; \quad (\mathbf{B}) = \begin{bmatrix} 2\omega_1 c_1 \\ 2\omega_2 c_2 \\ \cdot \\ 2\omega_n c_n \end{bmatrix}.$$

Here, ω – basic natural frequency of the system's equivalent single-mass model;

$\lambda = 0.5\sqrt{Nt}$ – regularization parameter;

N – number of variables equal to 2;

t – computation accuracy equal to 10^{-8};

$[\mathbf{E}]$ – unity matrix.

The PAA design shall account for the influence of the rigidity of the carrier on which the article will be installed. However, sometimes it is impossible due to the lack of initial data on structural parameters of the carrier, which is often designed in parallel to the antenna. In particular, the ship-based antenna design shall prevent mechanical resonance in the ship's operational vibration frequency range and prevent residual plastic strain caused by intense overloads after shock impacts. In ship-based RS design, special attention shall be paid to the interface between the antenna to be developed and the carrier ship's topside, providing their sufficient rigidity along with limited weight. The load-carrying structure of the antennas in question shall be developed in compliance with the requirements for rigidity, strength and resistance when exposed to vibration and shock impact, but with no account for deformations of the ship topside. To take into account ship topside stiffness $G_{\text{н}}$, the mathematical model uses the elastic foundation beam theory. For antenna design, the elastic foundation is the topside.

As an example, let us specify natural frequencies and vibration modes of the antenna, determined with account for different levels of topside stiffness $G_{\text{н}}$. Fig. 3 shows the lower basic mode of natural structural vibrations corresponding to the maximum mass contribution in the direction perpendicular to the antenna aperture at $G_{\text{н}}$, that is equal to 10^8 and 10^5 kg/cm.

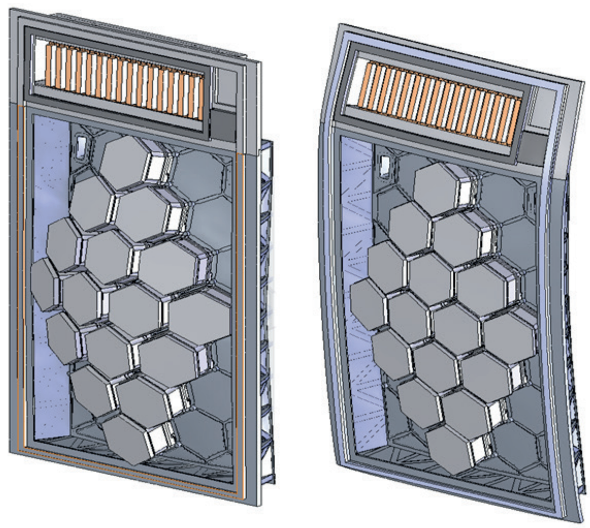
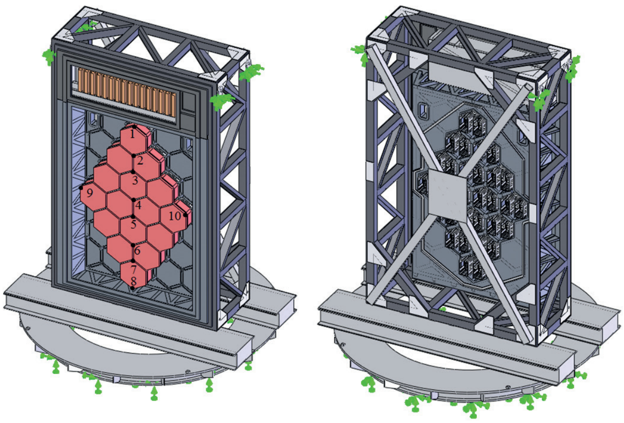


Fig. 3. Lower basic mode of natural vibrations of the topside – antenna system at $G_{\text{н}} = 10^8$ (a) and $G_{\text{н}} = 10^5$ kg/cm (b)

Frequencies are equal to 51.5 and 29.1 Hz, respectively. It is shown that if $G_{\text{н}}$ is taken into account, this leads to a considerable change of the lower basic vibration frequency of the topside – antenna system and mechanical overloads in the antenna structure.

The consistency of the developed model and simulation method is proved by experimental verification of the antenna's natural frequencies and vibration modes. In the course of the experiment we installed ten acceleration sensors, including eight sensors arranged along the long side of the antenna aperture (Fig. 4).



a b

Fig. 4. Installation of sensors on the antenna in the rigging unit:
a – on the antenna aperture side;
b – rear side of the antenna

The experimental basic natural frequency of the antenna installed in the rigging unit was 43.7 Hz (Fig. 5).

According to simulation results, the frequency of $f = 47.1$ Hz corresponds to the first basic vibration mode of the antenna. The difference between the calculated and experimental frequencies is about 7 %, between vibration modes – about 15 % (Fig. 6). The above frequency calculation errors are mainly caused by the fact that it is impossible to take into account the characteristics of the test bench and tooling.

If the structure is exposed to sinusoidal vibration, the dynamic response factor for a system with lumped parameters is calculated by formula

$$K_{дин} = \sum_i \sqrt{\frac{1+\alpha_i^2/Q_i^2}{(1-\alpha_i^2)+\alpha_i^2/Q_i^2}}. \qquad (6)$$

Here, $\alpha_i = f_0/f_i$;

f_0, f_i – baseplate vibration frequency and system natural frequency, respectively;

$Q_i = \sqrt{(\pi/\delta_i)^2 + 1/4}$ – Q-factor of the i-th vibrating circuit of the system;

δ_i – natural vibration logarithmic damping decrement of the i-th vibrating circuit of the system.

Even for the most primitive dynamic systems there are no analytical expressions of peak reaction values under shock impact, that is why

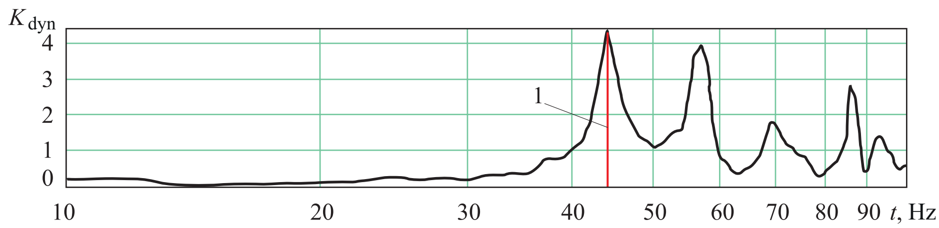


Fig. 5. Antenna dynamic response factor K_{dyn} at the location of one of the sensors:
1 – lower basic vibration frequency

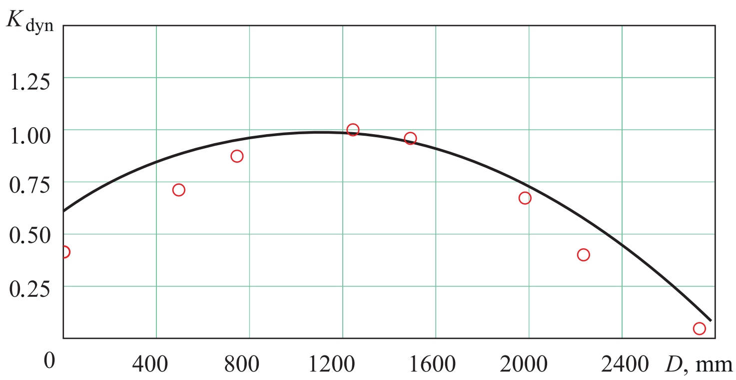


Fig. 6. Comparison of experimental (○)
and calculated (—) first basic vibration modes of the antenna:
D – aperture length

we consider the kinematic excitation equation for a single-mass structural model:

$$\omega^2 q + 2\omega c \frac{d}{dt} q + \frac{d^2}{dt^2} q = -I(t). \quad (7)$$

Here, $q = 0$, $\dot{q} = 0$;

$I(t)$ – sinusoidal impact pulse in the form of

$$I(t) = \begin{cases} A \sin\left(\frac{2\pi}{\tau} t\right), & t \le \tau; \\ 0, & t > \tau, \end{cases} \quad (8)$$

where A – impact pulse amplitude;

τ – impact pulse duration.

Absolute acceleration on antenna structure

$$a(t) = \frac{d^2}{dt^2} q + I(t). \quad (9)$$

With respect to the obtained accelerations that occur under vibrations and shock impact, the stress-strain state of the antenna and the deflection shape can be determined. Fig. 7 shows a scaled representation of the antenna deflection shape under shock impact.

As an example, let us also analyse the PAA to be designed as part of a land-based system.

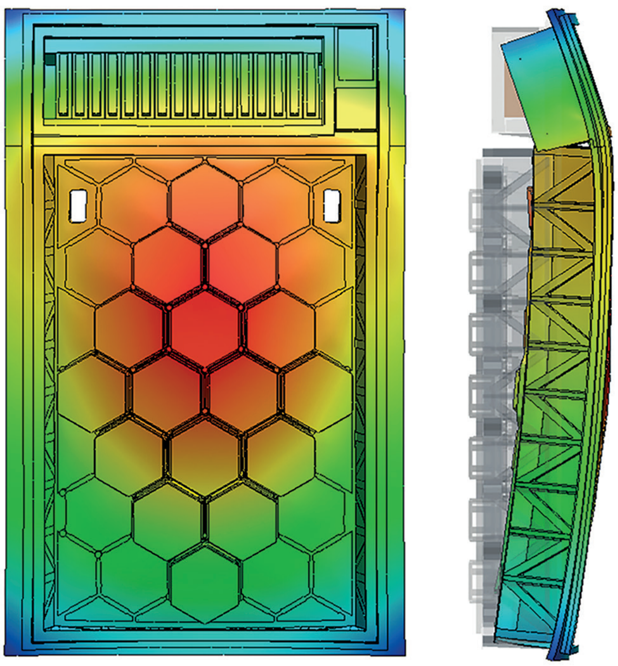


Fig. 7. Deflection shape and displacement distribution in the antenna

According to technical requirements, the structure shall be robust and rigid with account for the given severe limitations of weight-and-dimensional parameters. To provide repairability of the antenna, the load-carrying frame of the antenna device (AD) is designed as a split-type structure, parts of which are screwed together with threaded connections. The AD is installed on the antenna container (AC) (Fig. 8), which is mounted on vehicle chassis. The product is not meant to be operated as a mobile unit, but it may be exposed to overloads (multiple mechanical shocks) during transportation. The AD is lifted up to its operational position using three hydraulic cylinders for deployment. Lifting operation is associated with the risk of asynchronous behaviour of hydraulic cylinders, resulting in deformation and overstress in the AD load-carrying frame. This also affects fasteners of PAA modules installed in the frame.

There were several design versions, depending on the applicable material of its components, as well as on the number and location of supports.

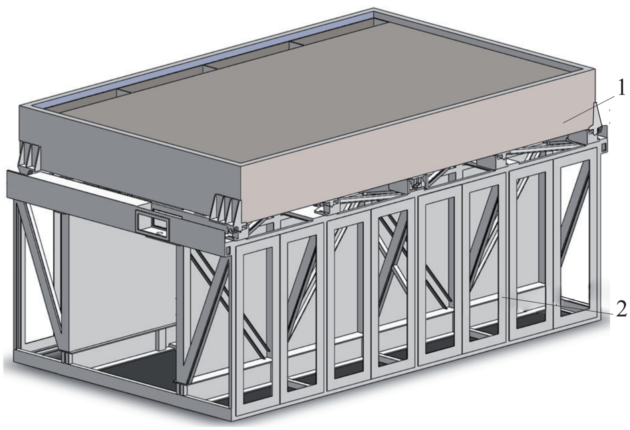


Fig. 8. 3D structural model:
1 – antenna device; 2 – antenna container

Based on the analysis of all design versions (under loads in accordance with the technical requirement specification), a titanium alloy was selected for making the AD load-carrying frame. The frame is attached to the AC with five hinge joints and five support-retainers.

At the AD design stage, the boundary conditions were established to fully restrain the AD at the AC attachment points. Fig. 9 shows a vibration mode corresponding to the natural frequency with the rigid fixation. Fig. 10 shows a vibration mode with account for the AC elasticity.

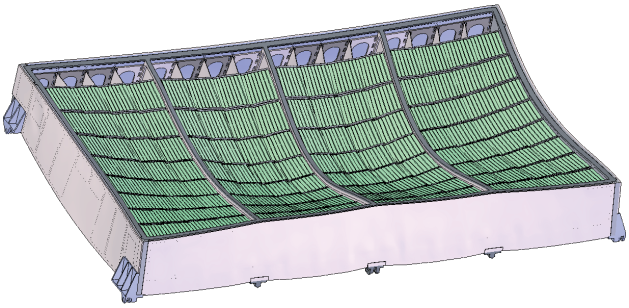


Fig. 9. AD structural vibration mode with rigid fixation and natural frequency of 64.6 Hz

The minimum natural frequency of the antenna device with account for the AC elasticity is considerably lower than the frequency of the rigidly fixed AD and is equal to 32 Hz.

The AC elasticity is taken into account as elastic foundation stiffness coefficients. Therefore, when exposed to shock impact, the AD is displaced at attachment points as well, unlike the design calculations, where the attachment points were rigidly fixed. In this case, the deflection value is the difference between displacements of the antenna edge and centre and will be reduced by approximately 30 % due to a decrease in

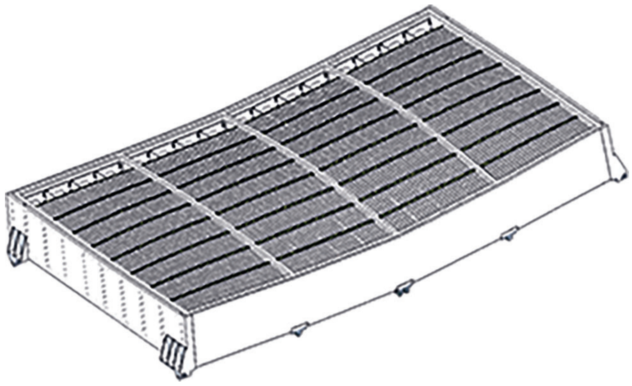


Fig. 10. AD structural vibration mode with account for the AC elasticity at natural frequency of 32 Hz

the natural frequency. The structure is exposed to asymmetric cyclic load.

In order to analyse the stress-strain state of PAA modules and devices included in the AC, the structure was split into individual constructs to reduce the dimension of the problems to solve (Fig. 1), since the qualitative analysis of a detailed model of the entire system may be unreasonable or infeasible due to high computation capacity required for problem solving.

To split the structure into constructs, empirical idealisation criteria are applied, developed similarly to the U.S. Nuclear Regulatory Commission's criteria [5]. Let us designate the ratios of masses and partial frequencies of a high level construct (M_1, f_1) and a low-lever construct (M_2, f_2) as follows

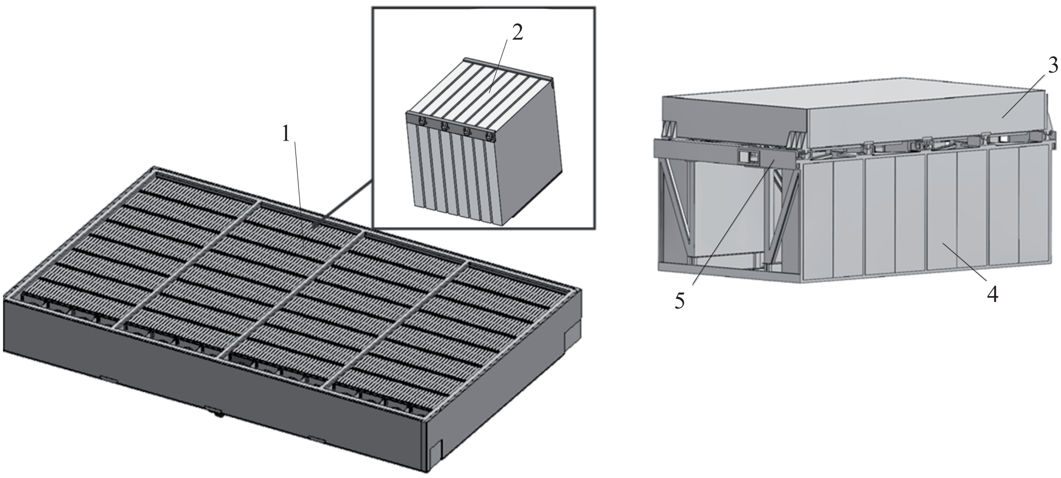


Fig. 11. Splitting into constructs the land-based system to be analysed:
1 – module model; 2 – approximate module model; 3 – antenna device; 4 – devices (16 pcs); 5 – antenna container

$$R_M = M_2 / M_1; \qquad (10)$$

$$R_f = f_2 / f_1. \qquad (11)$$

In this case:

• if $R_M < 0.01$, low-level construct models can be omitted for idealisation of a high-level construct model;

• if $0.01 \le R_M \le 0.1$, a high-level construct model shall include approximate low-level construct models;

• if $R_M > 0.1$ and $R_f \ge 2$, a low-level construct with the standard-type attachment to a high-level construct can be viewed as a weight-and-dimensional mockup.

The method applied to link individual constructs of different levels helps determine loads acting on PAA modules and system components (loads acting on different structural members may considerably vary) and conduct an in-depth analysis of their stress-strain state. To carry out a numerical analysis of pulse propagation through a structure, we developed an algorithm for calculating construct dynamic response factors needed for further determination of kinematic excitation of construct foundations. During a numerical experiment intended to compare the results obtained by solving the linear dynamic problem with the results obtained by solving the same problem using the method of computational and experimental analysis represented as a linkage of individual constructs of different levels, we have found out that the results are practically identical.

Conclusion

We have developed a method of computational and experimental analysis to reduce the dimension of the problem. This method allows:

• to simplify and accelerate structural strength analyses that require high computation capacity;

• to take into account the influence of the object carrier (where the product is to be mounted);

• to determine coefficients of vibration acceleration transmission from the structure foundation to individual equipment units in order to estimate their strength under the effect of mechanical factors;

• to carry out the stress-strain analysis of the structure to be developed using dynamic overload coefficients.

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О разработке методики для оценки динамики и прочности конструкций модульных фазированных антенных решеток

Разработан метод расчетно-экспериментального анализа для снижения размерности задачи, позволяющий упростить и ускорить прочностные расчеты конструкций. При его использовании можно учитывать влияние жесткости объекта-носителя (на который будет установлено изделие), определять коэффициенты передачи виброускорений от основания конструкции на отдельные блоки аппаратуры для оценки их прочности и устойчивости при воздействии механических факторов, а также проводить анализ напряженно-деформированного состояния разрабатываемой конструкции с использованием коэффициентов динамических перегрузок. В настоящее время разработанный метод используется при разработке нескольких перспективных проектов с применением модульных фазированных антенных решеток как морского, так и наземного базирования.

Ключевые слова: модульные фазированные антенные решетки, математическое моделирование, динамика и прочность, деформации фазированных антенных решеток.

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