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Svetlov G. V., Sumenkov N. A., Kostrov B. V., Fokina N. S.

The theory of discrete signals determined at finite intervals and its application for aerospace image processing

The purpose of the work was to generalise the well-known and proved by the authors ideas allowing us to expand the methods of applying orthogonal transformations in the field of aerospace image processing. A theoretical and methodological basis for application of Vilenkin – Krestenson's function systems has been built in a non-trigonometric, minimally possible form. According to the provisions and findings resulting from the theory of discrete signals at finite intervals, we offer the most viable option for constructing the basic functions system from the entire variety of Vilenkin – Krestenson's functions for which the signal shift is defined as the bitwise addition of numbers by a certain modulus. The theoretical and methodological provisions obtained are supported by the development of the algorithms for filtering and correlation analysis of aerospace images.

Keywords: aerospace images, orthogonal function systems, spectral analysis, correlation analysis, quasi-two-dimensional filtration algorithms, correlation alignment algorithms.

Introduction

An image can be regarded as a discrete signal defined at the finite interval of coordinate measurements on a plane. At the present time, algorithms built in spatial coordinates proved most successful in the field of aerospace image processing. In so doing, the spatial and frequency properties of the images are not accounted for. Information on the use of a classical Fourier analysis for image processing can be found in study [1]. However, computational complexity of Fourier transform implementation restrains widespread application of these methods in the aerospace image processing practice.

To solve the problems of spectral analysis, in a general case, any systems containing a required number of orthogonal functions can be used. Selection of a system of functions will be determined by the requirements of computational convenience and, ultimately, labour input with regard to the algorithms of the sought transform implementation. For application of alternative systems of basis functions, a methodological comprehension of feasibility of their application for solving the posed problems is required.

This paper generalises the ideas, well-known and proved by the authors, that make it possible to expand the methods of applying orthogonal transformations in the field of aerospace image processing. In this way, a theoretical and methodological base is created for application of the systems of Vilenkin – Krestenson's functions (VKF) in a non-trigonometric, minimally possible form of their construction as Walsh functions. The obtained theoretical and methodological provisions are supported by examples of development of the algorithms for filtration and correlation analysis of aerospace images.

Basic investigation results

Based on the analysis of the principles and means of aerospace image (ASI) generation [2], all image-capture sensors of radio engineering systems can be divided into five groups:

- sensors built on the basis of charge-coupled devices (CCD strips);
- single-beam sensors with conical or planar scanning;
- scan-type sensors;
- radar facilities of various stationing and operating principle;
- thermal imaging and television devices and systems employed, as a rule, on board atmospheric aerial vehicles or ground-based stations.

The imaging methods and systems are used for generating video data that can be recorded or saved by any known technique. All of them have certain geometric structure, represent informative part in the form of certain photographic density of video tone, and can be transmitted via any known data transfer system to a certain distance, i.e. possess a certain commonality, which permits to refer to them using a common term “ASI”.

In accordance with the physical principles of image generation and image transmission conditions, the generated ASIs have specific distortions, which include:

• synchronous distortions, conditioned by variation of transfer characteristics of the pixel brightness forming path and rigidly linked to the ASI scanning law (the distortions are manifested in the form of characteristic row- or column-wise “synchronous banding”);

• distortions in the form of disturbance characterised by absolutely destructive effect (manifested by “dead pixels”, normally grouped along the ASI row), associated with losses in the communication channel;

• non-synchronous distortions, not associated with the process of pixel brightness forming and the scanning law (manifested in the form of characteristic periodical “non-synchronous banding” with random angle of inclination to the ASI columns). Elimination of such disturbances is a partial problem in filtering images with periodical disturbances and is not considered in this paper.

Elimination of the aforementioned distortions is the main task in pre-processing of ASIs for creating the conditions for their use according to the intended purpose.

The mathematical apparatus applicable to the operations of image processing depends on whether the image exists in digital form. It is, first of all, discrete transforms defined at finite intervals. Introduction of the notion of “finiteness” makes it possible to avoid contradictions arising in the use of Fourier transform for spatial spectral analysis of images that are described in a general case by a non-stationary random process.

In this respect, use of the basic provisions and findings resulting from the theory of discrete signals at finite intervals seems reasonable. This theory is based on selection of the most acceptable option for constructing a system of basis functions out of the entire multitude of VKFs for which the signal shift is defined as the bitwise addition of numbers by a certain modulus. The VKF notion embraces, as a special case, the system of Walsh functions based on binary arithmetic. The methodology of the Walsh system is based on the following properties.

1. All functions of the system are real-valued functions in the definition interval $N = 2^n$.

2. The functions of the system take values +1 and –1 only, therefore, when expansion by the Walsh system is used, the basic operations are addition and deduction.

3. The system of Walsh functions is orthogonal at the definition interval N, whereas Hadamard matrix constructed by the Walsh functions is symmetrical.

4. The Hadamard matrix has dimension $N \times N$, therefore it comprises N orthogonal functions and cannot be added to with a new orthogonal function. Such system of functions is complete and can be used for constructing unitary transforms of non-harmonic spectral analysis. The computational complexity of such transform will be minimal, since all operations are substituted for the operations of adding real numbers, unlike the discrete exponential functions, where all the numbers are complex [3].

The VKFs can be represented through Rademacher functions, i.e. complex functions set at the same interval $N = m^n$:

$$R_i(x) = e^{j(2\pi/m)x_i}.$$

Here, i – order of Rademacher function.

Then the VKFs can be written as

$$F(p,x) = \prod_{i=0}^{n-1} \left[R_i(x)\right]^{<P_i>}.$$

For the case of $m = 2$, the Rademacher functions can be set as follows:

$$r_i(x) = (-1)^{<x_i>},$$

where $<x_i>$ – the i-th place of binary representation of variable x.

Under such setting, all Rademacher functions are real-valued and odd at the interval N. A system composed of them is not complete.

After complementing it to a complete one, the system of Walsh functions can be written

$$\mathrm{wal}(w,x)=\prod_{i=1}^{n}\left[r_j(x)^{<w_i>}\right],$$

where $<w_i>$ – value of the i-th place of Rademacher function number represented in the Gray code; $i = 1, 2, 3, ..., n$.

If there is just one unity present in the Rademacher function number, then Walsh function coincides with Rademacher function with the corresponding number.

This way, at the definition interval $N = 2^n$ the system of Walsh functions can be divided into n groups. In this case, zero-order function is not considered. If these groups are designated by numbers $k = 1, 2, 3, ..., n$, then each group will start with Rademacher function r_{n+1-k}. Each group includes 2^{n-k} functions, with the Rademacher function accounted for. There and then, the system of Rademacher functions is a sort of "frame" around which the Walsh system is built. This feature can be used in spectral analysis of signals.

The methodology of such transform application is based on a number of theorems, differing from the theorems of classical spectral analysis [3]. Assertions following from the theorems considered below allow to construct efficient algorithms for filtration and correlation analysis of aerospace images.

1. Theorem on dyad convolution. If $\{X_n\}$ and $\{Y_n\}$ are digit sequences set at interval N, then convolution (correlation) sequence

$Z_s = \dfrac{1}{N}\sum_{n=0}^{N-1} X_n Y_{n\oplus s}$ will be defined as follows:

$$Z_s = \sum_{u=0}^{N-1} N(C_u^X \times C_u^Y).$$

Here, C_u^X – spectral coefficients of sequence $\{X_n\}$;

C_u^Y – spectral coefficients of sequence $\{Y_n\}$.

When using the dyadic convolution theorem, the mechanism of signal elements formation in spectral space after filtering can be perceived. Unlike the existing classical spectral analysis theorem, in this one no extremum is formed in the coincidence point when computing the autocorrelation function. Moreover, it cannot be applied for constructing algorithms of automatic alignment of image fragments.

2. Theorem on real-dyad convolution. If $\{X_n\}$, $\{Y_n\}$ are digit sequences set at interval N, then convolution (correlation) sequence $\{Z_s\} = \dfrac{1}{N}\sum_{n=0}^{N-1} X_n Y_{n-s}$ can be found as

$$Z_s = \sum_{u=0}^{N-1} C_u^X (C_u^Y)_s.$$

Here, $(C_u^Y)_s$ – spectral coefficients computed at a shift of sequence $\{Y_n\}$.

By means of this theorem it is possible to construct complex spectral filters and efficient correlation analysis algorithms; it can as well be used for constructing image alignment systems. The theorem has no analogue in the classical spectral analysis.

3. Theorem on invariance of thinned basis. Let us assume that $\{V_n\}$ is a digit sequence obtained from sequence $\{X_n\}$ as a result of dyad shift by l (i.e. $V_n = X_{n\oplus l}$), and

$$C^v = \frac{1}{N}\mathbf{H}_w\mathbf{V},\ C^x = \frac{1}{N}\mathbf{H}_w^0\mathbf{X},$$

where C^v – Walsh spectrum coefficients for sequence $\{V_n\}$;

$\mathbf{H}_w$ – initial Hadamard matrix;

C^x – sequency spectrum coefficients of sequence $\{X_n\}$;

$\mathbf{H}_w^0$ – randomly thinned Hadamard matrix.

Then

$$(C^v)^2 = (C^x)^2 .$$

The theorem allows to eliminate some spectral components by implementing certain filtration types in the process of obtaining spectral representation, which is not characteristic of other systems of basis functions. The theorem proof can be found in the paper [4].

4. Theorem on limiting of non-trigonometric spectrum. Let us assume that n is the order of a system of Rademacher functions, k – number of a group of functions in the Walsh system, and spectrum limitation is provided at the level of Rademacher functions with number $n+1-k$. Then in the newly formed image, brightness of the obtained elements is

$$b_{ij}^{[N_s]} = \frac{1}{S^2}\sum_{g=1}^{s}\sum_{p=1}^{s} b_{gp},$$

where $N_s = \dfrac{N}{S}$;

$s = 2^{k-1}$;

$i, j = \overline{1, N_s}$.

For notation compactness, dependence $i(s)$; $j(s)$; $g(i)$; $p(j)$ is not shown in the formula.

The theorem on limiting of non-trigonometric spectrum has no analogue in the classical spectral analysis. It can be used in constructing image alignment systems. Intuitive use of this property allows to construct a two-level alignment algorithm, using which makes the computational costs tens of times lower. An implementation example of such approach is given in [1].

5. Theorem on energy completeness of quasi-two-dimensional spectral representation. If $\mathbf{b}$ – digital image matrix with dimension $M \times N$, C – coefficients of its quasi-two-dimensional spectral representation, then the following equality holds true:

$$\frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\mathbf{b}_{ij}^2 = \frac{1}{M}\sum_{i=0}^{M-1}\sum_{v=0}^{N-1}C_{iv}^2.$$

This expression is a Parseval equality for quasi-two-dimensional representation of two-dimensional signals. To prove it, no special explication is required, since it follows from the known spectral analysis properties. Consideration of the theorem on energy completeness of quasi-two-dimensional spectral representation has an important methodological value, since it serves as substantiation of the use of one-dimensional spectrum of the image under two-dimensional filtration, without a loss in image energy value. It allows to reduce the computational costs two times and have some other benefits from elimination of certain distortion kinds.

6. Theorem on steady component of the spectrum. If it is assumed that $b(x, 0) = 0$ is a particular value of image element in each row, then there is no need to transmit the steady component of an image over the communication channel, as it can always be reconstructed by the formula

$$C_0 = -\sum_{v=1}^{N-1} C(x, v-1)(-1)\sum_{i=1}^{n}\langle (v-1)_i x_i \rangle .$$

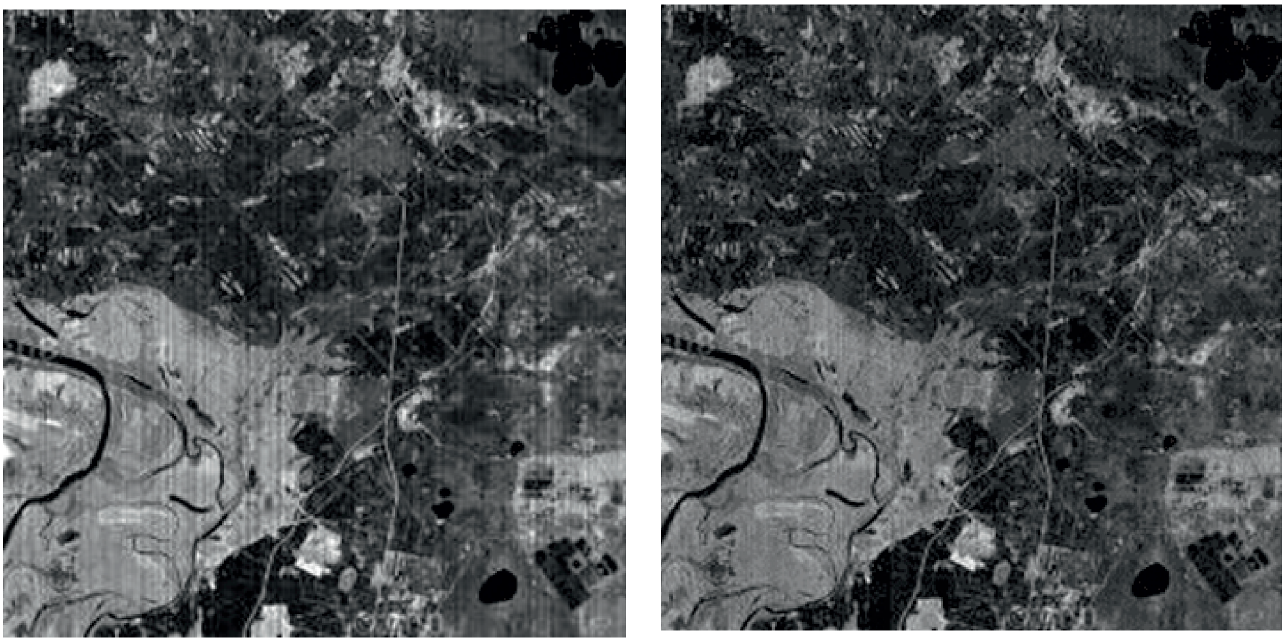
In this case the transmitted spectrum has no steady component [5]. The theorem is characteristic of the given type of transforms only, resulting from their algebraic structure.

Algorithms for filtration, transmission, and correlation analysis of aerospace images

The result of using theorems 1 and 5 for filtration of an image with synchronous disturbances [6] is given in Fig. 1.

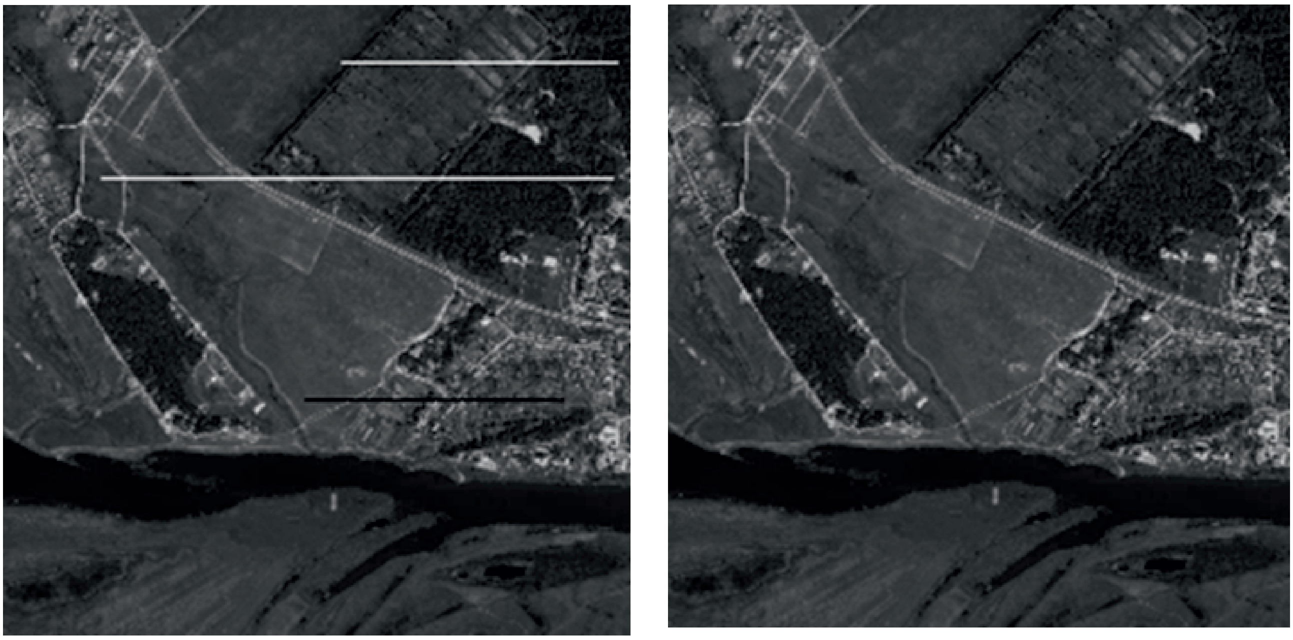
The algorithm idea is based on application of an averaging filter along image horizontal axis. In accordance with theorem 1, a portion of the energy of some spectral components is lost in the process, which violates the requirements of the theorem on energy completeness. The steady components are reinserted by introducing a correction factor based on comparison between image steady components column-wise before and after filtering.

The algorithm of quasi-two-dimensional filtration of images with disturbances in the form of "dead pixels" (Fig. 2) is executed in two stages [7]. At the first stage, based on detection of spectral rows having deviations in energy, as compared



a b

Fig. 1. Example of filtering an image with synchronous disturbance:
a – initial image at RMSE = 7.4184; b – filtration result at RMSE = 2.77



a b

Fig. 2. Application results of the algorithm of quasi-two-dimensional filtration of images with disturbances in the form of "dead pixels":
a – image with disturbance at RMSE = 14.06; b – filtration result at RMSE = 1.71

with the rest of the rows, a set of "dead pixels" is automatically generated. At the second stage, interpolation filtering is performed, based on accounting of the values of spectral components near the "dead pixels". After reinsertion in the image space, an image suitable for analysis is obtained.

The idea of the algorithm for transmitting images without a steady component [5] directly follows from theorem 6. If a column with a priori known values (0 or 256) is entered in the initial image, then transmission of spectral component corresponding to the steady component can be omitted, and the component value will be computed at the receiving side. Organising such a mode of transmission is feasible, since the field of vision of an optical system is always larger than the readout element dimensions, and there are the so-called shadow zones present in the image. Application of this algorithm practically does not affect the transmission quality (Fig. 3). However, additional possibilities emerge in this case for transmission process optimisation, including image compression. An example of compressed image transmission, with the image featuring an urban area, is given in Fig. 4.

a b

Fig. 3. Example of image transmission with steady component reinsertion:
a – initial image; b – transmitted image at RMSE = 0.012

a b

Fig. 4. Image transmission example:
a – initial image with dimension 512 × 512 pixels (size – 256 KB, transmission time – 210 ms);
b – after compression with compression rate 0.2 at RMSE = 7.0 (size– 21 KB, transmission time – 16 ms)

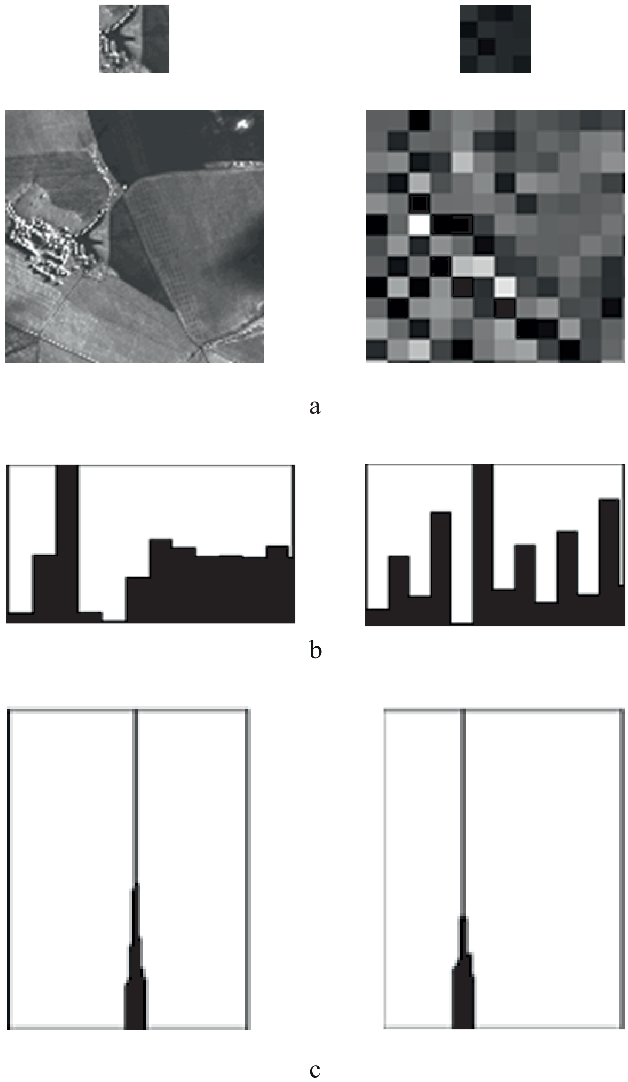


Fig. 5. Process of image alignment:
a – initial and master images; b – images of the first stage (theorem 4) and correlation function, horizontally and vertically, obtained at the first stage; c – correlation function, horizontally and vertically, obtained at the second stage

Additional image compression during transmission becomes possible after application of theorem 6 in the algorithm for transmitting images without the steady component. Resulting from this algorithm operation, spectral components are transmitted that correspond only to the changing part of the images. At the same time, not all of the spectrum components are significant in terms of energy, so they can be easily dismissed. Besides, the theorem provisions on energy completeness of a quasi-two-dimensional spectral representation are violated, therefore the image transmission protocol provides for fields for transmission of correctness, which ensure compliance with those provisions [8].

The approaches and advantages of correlation analysis for alignment of similar image fragments are considered in detail in [9]. In this paper an example is given of operation of a two-stage algorithm for finding a master image in the current image, generated in the course of Earth's surface survey (Fig. 5). A coarse but fast search is performed at the first stage and its results are elaborated on at the second.

Conclusion

Because of the large amounts of data handled in solving the tasks of ASI processing, it is necessary to continuously search for methods and means to accelerate this process, upgrading the mathematical apparatus applied for constructing respective algorithms, and first of all, those for discrete transforms defined at finite intervals. Introduction of the notion of finiteness makes it possible to avoid contradictions caused by the use of Fourier transform for spatial spectral analysis of images that are described in a general case by a non-stationary random process.

The considered examples allow to comprehend the methodology of applying Walsh transform for constructing algorithms for elimination of synchronous disturbances, as well as those associated with the loss of pixels during image transmission. The quality of the obtained images is quite acceptable for their further use (RMSE max. 7.0). The algorithms for image transmitting and compressing enable to reach time indices corresponding to video stream transmission at a rate of 50 frame/s. The algorithms for correlative alignment of images make it possible to solve the tasks of stitching together the images of adjacent flight paths of aerial vehicles and construct systems of independent navigation as per digital terrain maps. The application of a two-stage approach allows to reduce the time for solving the task of finding similar segments ten-fold and more.

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Svetlov Gennadiy Valentinovich – Candidate of Economical Sciences, General Director, Joint Stock Company “Ryazan Production and Technological Enterprise “Granit”, Ryazan.
Science research interests: systems for improving the quality of development and operation of complex systems.

Sumenkov Nikolay Aleksandrovich – Doctor of Engineering Sciences, Chief Engineer – Deputy General Director, Joint Stock Company “Ryazan Production and Technological Enterprise “Granit”, Ryazan.
Research interests: operation of complex radio engineering systems.

Kostrov Boris Vasilevich – Doctor of Engineering Sciences, Professor, Deputy Head of Automated Control System Department, Joint Stock Company “Ryazan Production and Technological Enterprise “Granit”, Ryazan.
Research interests: image processing, artificial intelligence, information technology.

Fokina Natalya Sergeevna – Adviser Director General, Joint Stock Company “Ryazan Production and Technological Enterprise “Granit”, Ryazan.
Research interests: IT and multimedia systems application in image processing.

Применение теории дискретных сигналов, определенных на конечных интервалах, для обработки аэрокосмических изображений

Обобщены известные и доказанные авторами положения, позволяющие расширить методологию применения ортогональных преобразований в области обработки аэрокосмических изображений. Создана теоретическая и методологическая основа применения систем функций Виленкина – Крестенсона в нетригонометрической, минимально возможной форме. На основе положений и выводов, следующих из теории дискретных сигналов на конечных интервалах, выбран наиболее приемлемый вариант построения системы базисных функций из всего многообразия функций Виленкина – Крестенсона, для которых сдвиг сигнала определяется как поразрядное сложение чисел по некоторому модулю. Полученные теоретические и методологические положения подкреплены разработкой алгоритмов фильтрации и корреляционного анализа аэрокосмических изображений.

Ключевые слова: аэрокосмические изображения, системы ортогональных функций, спектральный анализ, корреляционный анализ, алгоритмы квазидвумерной фильтрации, алгоритмы корреляционного совмещения.

Светлов Геннадий Валентинович – кандидат экономических наук, генеральный директор Акционерного общества «Рязанское производственно-техническое предприятие «Гранит», г. Рязань.
Область научных интересов: системы повышения качества разработки и эксплуатации сложных систем.

Суменков Николай Александрович – доктор технических наук, главный инженер – заместитель генерального директора Акционерного общества «Рязанское производственно-техническое предприятие «Гранит», г. Рязань.
Область научных интересов: эксплуатация сложных радиотехнических комплексов.

Костров Борис Васильевич – доктор технических наук, профессор, заместитель начальника отдела автоматизированной системы управления Акционерного общества «Рязанское производственно-техническое предприятие «Гранит», г. Рязань.
Область научных интересов: обработка изображений, искусственный интеллект, информационные технологии.

Фокина Наталья Сергеевна – советник генерального директора Акционерного общества «Рязанское производственно-техническое предприятие «Гранит», г. Рязань.
Область научных интересов: применение информационных технологий и мультимедийных систем в обработке изображений.