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Complex effect of recess depth in nozzle design on the discharge coefficient

We used contemporary computational fluid dynamics techniques to evaluate how the geometric parameters of a recessed nozzle affect the perfection of flow processes. We verified our numerical simulation and obtained acceptable agreement between numerical and experimental investigation results in terms of specific impulse loss. We plotted the discharge coefficient as a function of the geometrical parameters of a recessed nozzle. Our numerical investigation forms the basis of certain guidelines we developed for designing arc-based recessed nozzles.

Keywords: gas-dynamics, rocket engines, jet nozzles, recessed nozzles, discharge coefficient, simulation.

Today, the main goal of design developments related to rocket engines is to increase aircraft speed, range, and manoeuvrability. The specified parameters can be improved, for example, by increasing discharge characteristics of the engine.

One of the parameters indicating the perfection of nozzle flow processes is the discharge coefficient. According to the study [1], the discharge coefficient is an integral coefficient that is a product of different components such as gas-dynamic and two-phase components, as well as those accounting for throat area changes due to burning and thermal deformations of parts and for variations of thermophysical properties of the flow passing through the nozzle throat area. Then these components are divided into sub-components, for example, the gas-dynamic component can be divided into sub-components which account for pressure loss, irregularity of parameters in the nozzle, friction in subsonic nozzle section [1]. The component accounting for pressure loss comprises the pressure loss in the channel and in the near-nozzle area.

Gas-dynamic losses are the main component of all losses. As a rule, they are determined by the following geometric parameters:

- ratio of the minimum nozzle cross-section area to the chamber cross-section area at the nozzle inlet $F_{кр} / F_{вх}$ (nozzle-contraction area ratio) [2];
- nozzle inlet angle $\theta_{кр}$ [2, 3];
- ratio of inlet rounding radius R_2 to minimum cross-section radius $R_{кр}$ [1, 4];
- presence of a cylindrical section in the minimum nozzle cross-section of length $L_{ц}$ [5].

The distinctive design feature of many solid-propellant rocket engines (SPRE) is that they have a recessed (flush mounted) nozzle. Using this nozzle design allows to reduce overall dimensions of the rocket engine and therefore to avoid an increase in loss due to chemical non-equilibrium and dissipation of flow. Improving the SPRE design configuration leads to extra losses [6] that are definitely lower than those mentioned but need to be taken into account as well.

The studies [3, 6] provide data indicating specific impulse losses caused by the SPRE recessed nozzles. In case the recess depth changes in the range of 0.3…0.5, the specific pulse loss varies between 0.22 %...1.4 % with the aluminium content in the propellant 5…21.5 %. Hereinafter the recess depth is the ratio of the length of the nozzle’s recessed section to charge length $\bar{L}_{ут} = L_{ут} / L_{з}$.

The known studies [3, 6] lay emphasis on the trend of increasing specific impulse losses with increasing recess depth. With the fixed recess depth, the specific pulse losses are getting higher as the percentage of the condensed phase content in combustion products grows. Maximum losses correspond to a rocket engine discussed here, having the maximum recess depth $\bar{L}_{ут} = 0.75$ and high content of aluminium in propellant equal to 21.5 %. However, an engine

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with aluminium content of 16 % at $\bar{L}_{\text{ут}} = 0.26$ takes the second place in terms of the loss value. These facts question the said trend that the recess depth and the content of condensed phase in combustion products affects specific impulse losses without regard to the shape of a recessed nozzle, and in their turn, define the importance of the gas-dynamic component of the losses. Therefore, the given data indicates that we can investigate how the recess depth affects the perfection of flow processes when studying homogeneous environment.

The above-mentioned studies have been conducted with no regard to different components of losses, i.e. a complex effect of the nozzle recess depth on the perfection of flow processes has not been considered. In addition to the flow inhomogeneity and non-equilibrium at the nozzle inlet, the complex effect also implies the dependence of losses on different geometric parameters of the recessed nozzle section similarly to the specified parameters of “classic design” axis-symmetrical nozzles (Fig. 1).

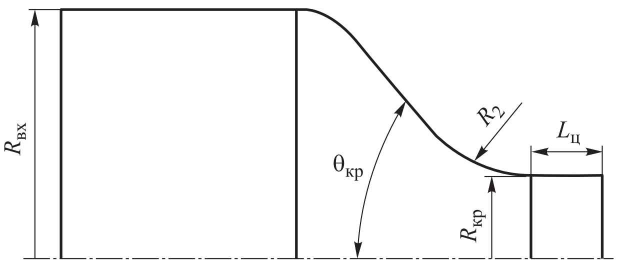

Fig. 1. Geometric characteristics of a “classic design” nozzle affecting the discharge coefficient

There are a lot of various geometric shapes of nozzle recessed sections, but, first and foremost, when analysing how its geometric parameters affect the perfection of flow processes and comparing it with the existing dependencies, it is reasonable to consider its radial shape. Studying the effect of the recess depth and geometric parameters of the radial recessed section of the nozzle upon the gas-dynamic loss component is a process of interest intended to enhance the performance of rocket engines being developed and improved.

This paper describes contemporary computational fluid dynamics techniques used for estimating the effect of geometric parameters on the perfection of flow processes. We have used the *ANSYS Fluent* software for simulation in an axis-symmetrical approximation with the ideal gas-adiabatic conditions of the steady-state problem. Objects of research are axis-symmetrical supersonic recessed nozzles in the rocket engine combustion chamber.

Based on the existing experimental results of the study of specific impulse losses due to recessed nozzle design [3, 4, 6], we can verify the numerical simulation method [7] for estimating the effect of the nozzle recess depth upon specific impulse losses, and respectively, to apply this method for calculating the discharge coefficient for recessed nozzles.

The computational model has been verified according to the diagram shown in Fig. 2. We have studied the rocket engine with minimum cross-section diameter $D_{\text{кр}} = 200$ mm, with a cylindrical charge featuring a conical section near the rear bottom, the length of which is $L_{\text{з}} = 2400$ mm. The nozzle recess depth has varied within $\bar{L}_{\text{ут}} = 0.09 \ldots 0.34$. The working medium is the air supplied from the charge surface at a temperature corresponding to the operating conditions.

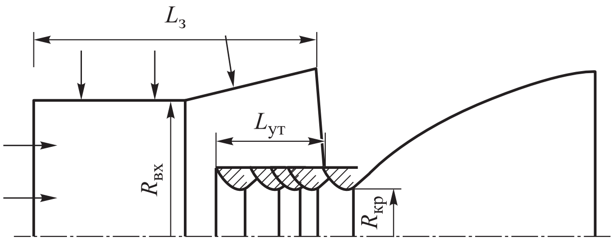

Fig. 2. Geometric layout of computational model

In addition to the combustion chamber and the nozzle, the geometric layout for computations comprised an extra volume for simulation of the jet discharge into free space. This allowed to skip the determination of boundary conditions at the nozzle outlet. The total number of computational grid cells was about 180,000, while non-dimensional number y^{+} calculated by the near-wall grid step and the dynamic velocity was not greater than 35 at the nozzle inlet.

The following boundary conditions of simulation have been established for the study: the surface of the assumed solid propellant has uniform distribution of the working medium, its temperature and flow turbulence parameters; constant atmospheric pressure at the outlet of the additional volume; the combustion chamber walls and nozzle walls are smooth with no-slip and impermeability conditions for the working medium.

As turbulence models, we studied a two-parameter $RNG\ k-\varepsilon$ model with a typical set of model constants verified for this type of problems [7].

Specific impulse losses ξ are determined as the ratio of the difference between the theoretical value of the specific impulse and its actual value reduced to the theoretical specific impulse [8]:

$$\xi = (I_{\text{ид}} - I)/I_{\text{ид}}.$$

To compare the obtained results of simulation with the known data [3, 6], specific impulse losses ξ can be expressed through the effective flow velocity and calculated using the following formula

$$\xi = \left[\omega_{\text{эф}}^{\text{ид}} - \left(\omega_a + \frac{p_a - p_H}{\rho_a \omega_a}\right)\right] \Big/ \omega_{\text{эф}}^{\text{ид}},$$

where $\omega_{\text{эф}}^{\text{ид}}$ – effective ideal flow velocity;

ω_a – calculated values of flow velocity at the nozzle exit;

p_a – calculated values of pressure at the nozzle exit;

p_H – pressure inside the additional volume simulating the environment;

ρ_a – flux density at the nozzle exit.

The share of specific impulse losses due to nozzle recess depth $\xi_{\text{ут}}$ is determined as the ratio of the difference between losses with recess $\xi_{\bar{L}_{\text{ут}}}$ and without recess $\xi_{\bar{L}_{\text{ут}}=0}$ to the losses for a protruded nozzle:

$$\xi_{\text{ут}} = \frac{\xi_{\bar{L}_{\text{ут}}} - \xi_{\bar{L}_{\text{ут}}=0}}{\xi_{\bar{L}_{\text{ут}}=0}}.$$

Fig. 3 shows a wide range of specific impulse loss variations depending on the nozzle recess depth and aluminium content in propellant. Calculated values of losses obtained for homogeneous environment are in good agreement with the experimental data at the minimum content of the condensed phase in combustion products. Satisfactory verification results allow to justify the possibility for studying the discharge coefficient (only by the gas-dynamic component with no regard to the condensed phase) of the SPRE recessed nozzle depending on geometric characteristics by the gas-dynamic component for a computational model with homogeneous working medium.

Let us proceed to study the effect of geometric characteristics of an arc-based recessed nozzle of the combustion chamber. Based on simulation results, we can calculate the gas-dynamic

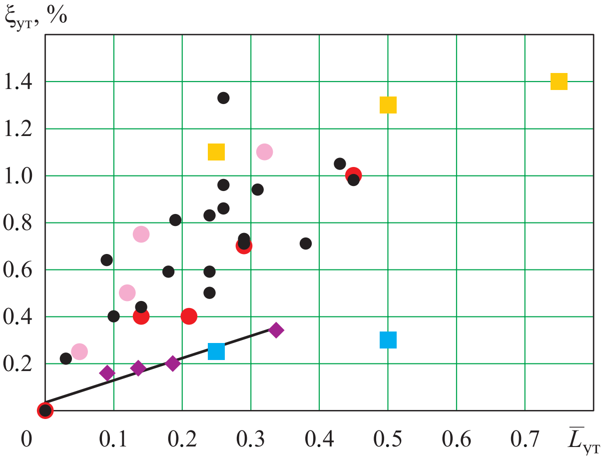

Fig. 3. Effect of nozzle recess depth on specific impulse losses:
🔴 – high-altitude SPREs with recessed nozzles [3];
○ – domestic experimental SPRE using propellant with aluminium content of 18 % [6]; 🟦 – foreign-made experimental SPRE using propellant with aluminium content of 5 % [6]; 🟨 – foreign-made experimental SPRE using propellant with aluminium content of 21.5 % [6];
● – experimental data indicating specific impulse losses due to SPRE recessed nozzle design (USA) [6];
◆ – computational model of a rocket engine with homogeneous working medium studied herein

component of the discharge coefficient using the following formula

$$\mu_c = \dot{m} \Big/ \left(\frac{A(k) p_{oc} F_{\text{кр}}}{\sqrt{RT_{oc}}} \right),$$

where $\dot{m}$ – actual flow rate value;

$A(k)$ – gas-dynamic function;

k – thermal capacity ratio;

p_{oc} – pressure;

R – equilibrium value of gas constant across the minimum nozzle cross-section;

T_{oc} – stagnation temperature at nozzle inlet.

The complex effect of nozzle recess depth on the discharge coefficient is determined by the ratio

$$\mu_{\text{ут}} = \frac{\mu_{\bar{L}_{\text{ут}}=0} - \mu_{\bar{L}_{\text{ут}}}}{\mu_{\bar{L}_{\text{ут}}=0}},$$

where $\mu_{\bar{L}_{\text{ут}}=0}$ – discharge coefficient with a protruded nozzle;

$\mu_{\bar{L}_{\text{ут}}}$ – discharge coefficient with current recess depth.

The combustion chamber of the rocket engine with a cylindrical charge, a supersonic conical nozzle and a radial recessed inlet has been used as the computational geometric model (Fig. 4). Recess depth varies in the range of 0…0.35. The charge length is $L_з = 400$ mm, the minimum cross-section diameter is $D_{\text{кр}} = 40$ mm. The simulation boundary conditions, working medium, and turbulence model are similar to the previous computational model. The total number of computational grid cells is about 150,000; nozzle inlet y^+ is not greater than 35, similarly to the previous computational model. Geometric parameters under study:

- relative radius of recessed nozzle inlet to minimum cross-section $\bar{R}_2 = R_2 / R_{\text{кр}}$;
- relative radius of recessed nozzle section inlet $\bar{R}_в = R_{\text{кр}} / R_в$ at constant value $\bar{R}_2$;
- relative radius of recessed nozzle section outer edge $\bar{R}_3 = R_3 / R_{\text{кр}}$.

When investigating the effect of the relative radius of recessed nozzle inlet to minimum cross-section upon the discharge coefficient, we have analysed variation $\bar{R}_2 = 0.1 \ldots 1.0$ for different nozzle recess depths. We have found out that $\bar{R}_2$ has the similar effect on the discharge coefficient both for recessed nozzles discussed herein and for "classic design" nozzles [2, 4] (Fig. 5). The paper [4] gives summarized results for supersonic nozzles with small angles $\theta_{\text{кр}}$. That is why such nozzles have higher values. The results represented in the paper are in good agreement with the data for nozzles with a conical supersonic section at $F_{\text{кр}}/F_{\text{вх}} = 0.25$ and $\theta_{\text{кр}} = 45°…90°$ [2]. In computational model (see Fig. 4) $F_{\text{кр}}/F_{\text{вх}} = 0.11$.

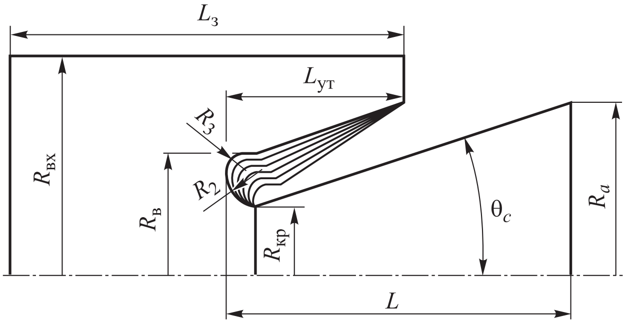

Fig. 4. Computational geometric model diagram: θ_c – supersonic nozzle section expansion angle

Fig. 6 shows the results represented in the

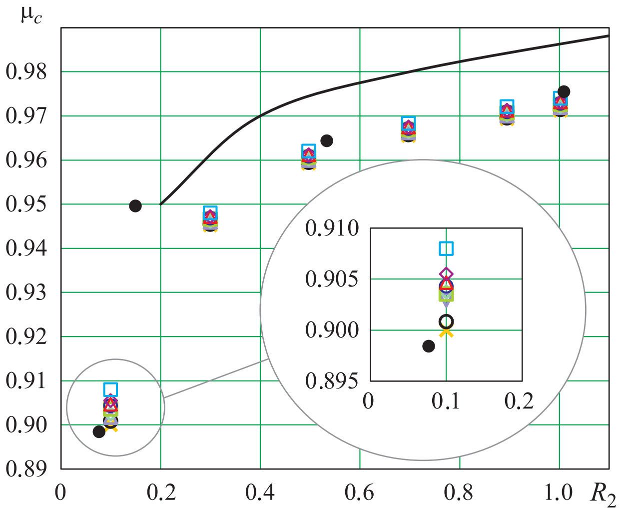

Fig. 5. Effect of $\bar{R}_2$ on the discharge coefficient for known [4] (——), [2] (●) at $F_{\text{кр}}/F_{\text{вх}} = 0.25$, $\theta_{\text{кр}} = 45 \div 90$ and studies described in the paper: □ – $\bar{L}_{\text{ут}} = 0$; ◇ – $\bar{L}_{\text{ут}} = 0.025$; △ – $\bar{L}_{\text{ут}} = 0.05$; ○ – $\bar{L}_{\text{ут}} = 0.1$; □ – $\bar{L}_{\text{ут}} = 0.15$; ◇ – $\bar{L}_{\text{ут}} = 0.2$; △ – $\bar{L}_{\text{ут}} = 0.25$; ○ – $\bar{L}_{\text{ут}} = 0.3$; ✕ – $\bar{L}_{\text{ут}} = 0.35$

paper regarding the recess depth. It is shown that for the same recess depth the discharge coefficient may considerably depend on $\bar{R}_2$. We should note that the higher $\bar{L}_{\text{ут}}$, the lower the discharge coefficient for all the discussed $\bar{R}_2$.

The component that takes into account the effect of recess depth relative to a protruded nozzle with the relevant relative radius and inlet nozzle section angle $\theta_{\text{кр}} = 90°$, changes nonuniformly for recessed nozzles with different $\bar{R}_2$ (Fig. 7). For nozzles with $\bar{R}_2 = 1 \ldots 0.5$, a recessed nozzle in the combustion chamber has the similar effect on losses. The effect of recess depth in nozzle design is in the range of 0…0.2 %, while for nozzles with relative radius $\bar{R}_2 < 0.5$ this parameter may reach 0.88 %. We may conclude that nozzles with $\bar{R}_2 > 0.5$ can be recessed deeper with no risk of severe degradation of discharge characteristics with other geometric conditions being equal. When simulating nozzles with $\bar{R}_2$ less than 0.5, we should take into account a considerable increase in loss if the nozzle recess is getting deeper.

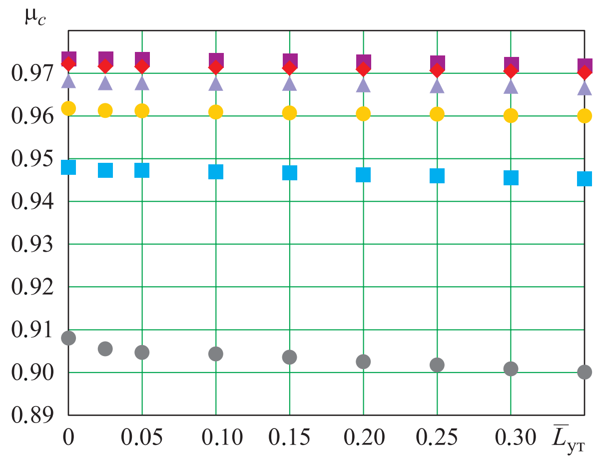

Fig. 6. Effect of the minimum nozzle cross-section inlet radius on the discharge coefficient:
■ – $\bar{R}_2 = 1$; ◆ – $\bar{R}_2 = 0.9$; ▲ – $\bar{R}_2 = 0.7$;
● – $\bar{R}_2 = 0.5$; ■ – $\bar{R}_2 = 0.3$; ● – $\bar{R}_2 = 0.1$

The effect of the relative radius of the recessed nozzle inlet section on the discharge coefficient has been investigated for a geometric model, which has the nozzle inlet with constant radius R_2 and at $R_3 = 0$ (Fig. 8). When studying the effect of $\bar{R}_2$, radius R_2 has been simulated as the quadrant, the initial and end points of which are located in horizontal and vertical planes, respectively. As the nozzle inlet section radius was becoming smaller, the circumference sector reduced accordingly. Calculation has been conducted for maximum relative radius $\bar{R}_2 = 1$.

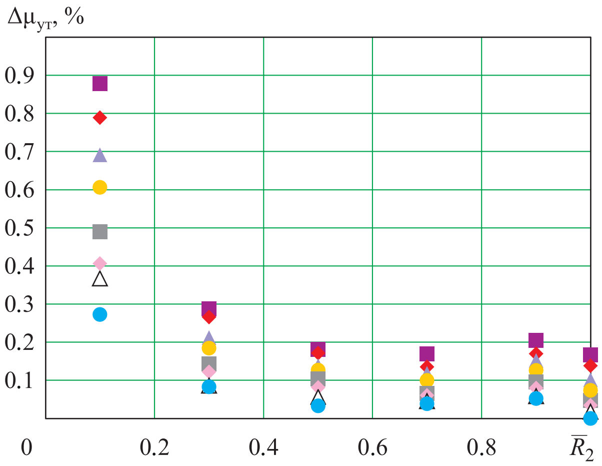

Fig. 7. Effect of the nozzle minimum cross-section inlet radius on the relative component of discharge coefficient regarding the recess depth:
■ – $\bar{L}_{\text{ут}} = 0.35$; ◆ – $\bar{L}_{\text{ут}} = 0.3$; ▲ – $\bar{L}_{\text{ут}} = 0.25$;
● – $\bar{L}_{\text{ут}} = 0.2$; ■ – $\bar{L}_{\text{ут}} = 0.15$; ◆ – $\bar{L}_{\text{ут}} = 0.1$;
△ – $\bar{L}_{\text{ут}} = 0.05$; ● – $\bar{L}_{\text{ут}} = 0.025$

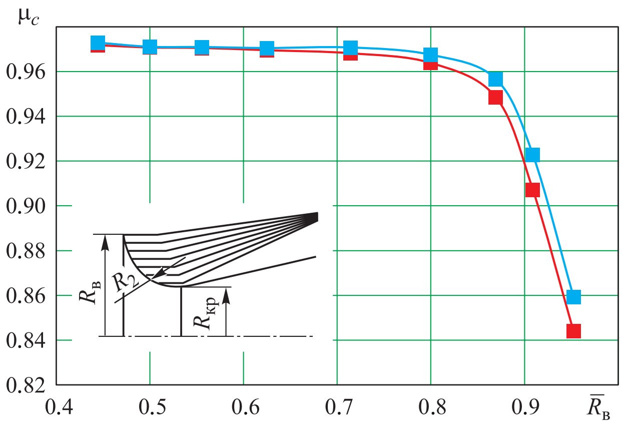

Fig. 8. Effect of the recessed nozzle section inlet relative radius on the discharge coefficient:
■ – $\bar{L}_{\text{ут}} = 0.35$; ■ – $\bar{L}_{\text{ут}} = 0.15$

Obtained results prove that the relative radius of the recessed nozzle section inlet shall be $\bar{R}_{\text{в}} \le 0.75$ irrespective of the recess depth (see Fig. 8). Further reduction of the inlet section relative radius severely affects discharge characteristics with other geometric conditions being equal. Degradation of the discharge coefficient with the inlet section radius reduced from 0.44 to 0.95 reaches 13.16 % at $\bar{L}_{\text{ут}} = 0.35$. When the recess depth is reduced, the effect on the discharge coefficient is reduced too. The paper [9] specifies the recommended range of $\bar{R}_{\text{в}} = 0.48...0.53$.

The effect of recessed nozzle outer edge's relative radius $\bar{R}_3$ on the discharge coefficient has been studied at constant radius R_2 for two recess depths. We have found out that there is an insignificant effect of less than 0.1 % for $\bar{R}_2 = 1$. However, for $\bar{R}_2 = 0.1$ at $\bar{L}_{\text{ут}} = 0.35$, the effect of nozzle recess depth on the discharge coefficient with variable $\bar{R}_3 = 0.05...0.45$ reaches 0.31 %, and for $\bar{L}_{\text{ут}} = 0.15$ – 0.29 % (Fig. 9). The effect of the component accounting for the recess depth has varied within $\bar{R}_2 = 0.1$ % for (with changing $\bar{R}_3$) and $\bar{L}_{\text{ут}} = 0.35$ varied within $\Delta\mu_{\text{ут}} = 0.8...1.1$ % and $\Delta\mu_{\text{ут}} = 0.36...0.64$ % for $\bar{L}_{\text{ут}} = 0.15$. It should be noted that the effect of $\bar{R}_3$ reduced as the recess depth decreased.

Therefore, for rocket engines with recessed nozzles, degradation of discharge characteristics mainly depends not only on the recess depth and the content of the condensed phase in combustion products with the ambiguity of their effect confirmed through the analysis of the existing experimental data and calculation results, but also on geometric parameters of the recessed nozzle inlet section. Verification of the numerical model proves that the gas-dynamic component of the discharge coefficient can be used for analysing the perfection of flow processes in rocket engines with recessed nozzles. Numerical studies prove that for nozzles with the similar recess depth the discharge coefficient may vary up to 13 % if the geometry of the inlet section changes.

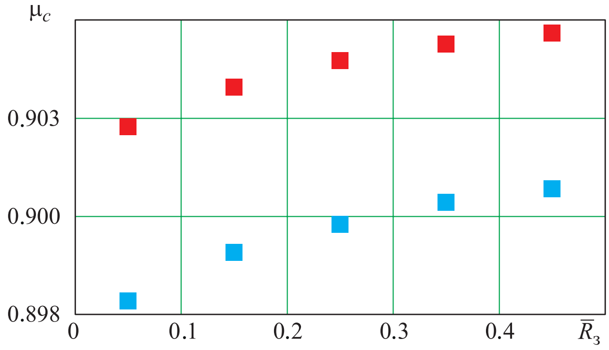

Fig. 9. Effect of the recessed nozzle section outer edge radius on the discharge coefficient:
🟥 – $\bar{L}_{\text{ут}} = 0.35$; 🟦 – $\bar{L}_{\text{ут}} = 0.15$

Based on research data for designing radial recessed nozzles, the following recommendations may be given:

- design of a nozzle with high relative radii $\bar{R}_2$ and $\bar{R}_3$ will allow to drastically increase the recess depth without loss growth;
- if nozzles with low $\bar{R}_2$ and $\bar{R}_3$ need to be designed, we should take into account extra losses due to deep recession of the nozzle into the combustion chamber;
- if a nozzle with small radius $\bar{R}_2$ needs to be designed, larger radius $\bar{R}_3$ is recommended;
- using relative radius of the recessed nozzle inlet section over $\bar{R}_{\text{в}} = 0.75$, is not recommended as losses dramatically grow with higher values.

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Многофакторность влияния степени утопленности сопла на коэффициент расхода

Современными методами вычислительной гидродинамики реализована оценка влияния геометрических параметров утопленного сопла на совершенство процессов истечения. Проведена верификация численной модели, получено удовлетворительное соответствие численных и экспериментальных результатов исследований потерь удельного импульса. Установлены зависимости коэффициента расхода от геометрических параметров утопленного сопла. По результатам численных исследований сформулированы некоторые рекомендации для проектирования радиусных утопленных сопел.

Ключевые слова: газодинамика, ракетные двигатели, реактивные сопла, утопленные сопла, коэффициент расхода, моделирование.

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