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Fractional step method in problems of hydromechanical processes in piston-cylinder unit of axial piston swash plate hydraulic machines

Having analyzed the piston-cylinder unit kinematics, we obtained an equation for the clearance height in the piston-cylinder unit for the case of low speeds, the equation being the basis for Reynolds equation for the lubricant layer of the piston mechanism. By a numerical experiment using the fractional step method, we built a pressure field for two different cases of the piston mechanism kinematics, and compared the bearing capacity of the hydrodynamic force. It was revealed analytically and with the help of a numerical experiment that when the piston rolls in the edges of the guide bushing, the total hydrodynamic force significantly exceeds the force created when the piston slides in the bushing.

Keywords: hydraulic machine, dynamic response, swash plate pump, fluid friction

The most critical assembly in an axial-piston swash-plate hydraulic machine is the piston mechanism. Performance characteristics of hydraulic machine as a whole, such as efficiency factor (EFF), static and dynamic response, depend on its functioning. The dynamic response of hydraulic piston machines depends on a dead zone occurring during hydraulic machine reversal due to high values of friction forces and volumetric leaks.

Axial-piston hydraulic machines are widely represented in drives where high pressure is required at high rotation speeds of the output shaft. Such hydraulic machines ensure high energy-output ratio which makes them unmatched for use as hydraulic power drive in various machinery: drives for tipping part raising, levelling drives, power takeoff drives, and others.

Hydraulic machines are also in demand in energy-intensive high-precision drives. Most notably, these are the drives that must meet certain static and dynamic response requirements: drives for aerial vehicles, positive-displacement hydromechanical transmissions for various-purpose ground machinery providing energy transfer from the driving motor to actuating devices [1, 2].

Axial-piston swash-plate hydraulic machines (APSPHM) have such advantages as relatively simple design and more favourable layout characteristics as compared with axial-piston swash-block hydraulic machines (APSBHM). However, a topical problem for such machines is adequate assurance of performance at low RPM and in the breakaway mode, which can be achieved by reducing friction forces in the piston-cylinder unit.

The objective of this paper is reduction of friction forces between piston and guide bushing. This task can be solved through implementation of the liquid friction mode [3] occurring under sufficiently great upward force produced by piston movement relative to bushing. To determine the value of hydrodynamic force conditioning the liquid friction mode, it is necessary to calculate hydrodynamic pressure in the piston-cylinder unit [4].

Piston axis, while under the action of a transverse force, is turned relative to the guide bushing axis to the maximum possible angle, which is determined by the value of radial clearance, thus creating favourable conditions for ‘oil-film wedge’ formation. However, during breakaway and at low speeds of piston movement relative to the inner surface of the guide bushing, upward force from the oil film side is insufficient to support the liquid friction mode [5].

Piston surface is in direct contact with the outer and inner edges of the guide bushing, which accounts for presence of high friction level in the piston-cylinder unit (Fig. 1).

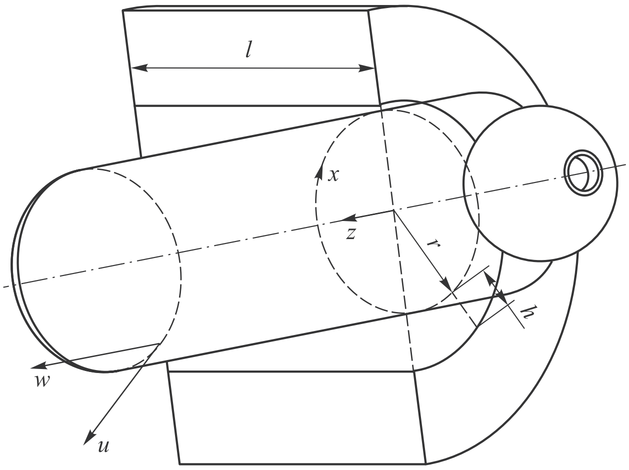

Fig. 1. Basic parameters of a piston-cylinder unit

The following designations are used in Fig. 1:

u – linear velocity of a point on piston surface during its rotation in the bushing;

w – translational velocity of a point on piston surface relative to the bushing;

l – guide bushing length;

r – piston radius;

h – clearance between piston and inner surface of the bushing.

In this case, hydrodynamic processes in the piston – guide bushing pair until the moment of piston ‘floating up’ are considered. To build a hydrodynamic pressure field, an involute of piston surface was used, referenced to the Cartesian coordinate system with axes x and z. Since the radius of curvature of the piston surface exceeds clearance value by two orders of magnitude, it is allowed to use a rectangular coordinate system [4]. Coordinate y is determined by the hydraulic fluid layer thickness.

Linear velocity of a point on piston surface during its rotation in the bushing can be calculated by the formula

$$u = \omega r, \tag{1}$$

and translational velocity of a point on piston surface relative to the bushing –

$$w = \omega R \tan(\gamma)\sin(\alpha). \tag{2}$$

Here, ω – hydraulic machine shaft rotation speed;

R – cylinder block radius;

γ – back plate tilt angle;

α – piston angular position during a working cycle.

Formula (1) is applicable to piston surface sliding along the bushing edges. Equations of velocities on the piston surface for the kinematics case of piston rolling in the guide bushing edges, corresponding to formula (1), are given in paper [6].

Fig. 2 shows the basic geometric parameters of the clearance between the piston and the guide bushing. The clearance value cross-section-wise is

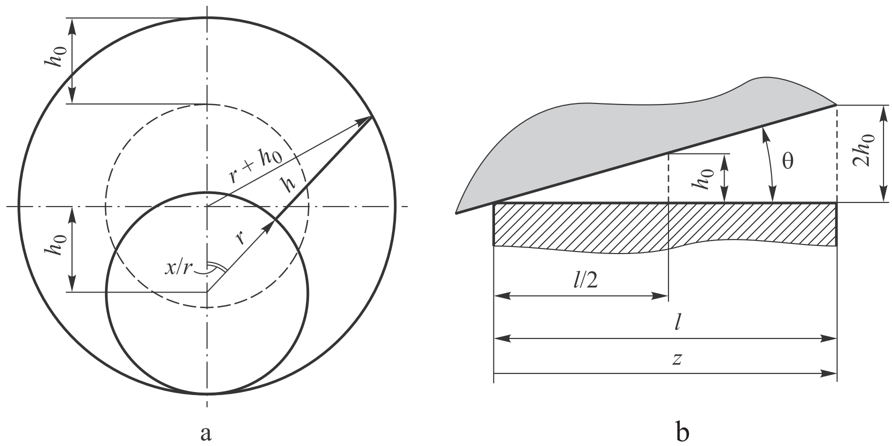

Fig. 2. Basic geometric parameters for determining lubricant layer in piston-cylinder unit:
θ – piston tilt angle

calculated by means of the law of sines and given in equation (3). Along the bushing length, the clearance at coordinate x, which corresponds to direct contact of the piston and the bushing, changes linearly from 0 to a value equal to twice the nominal clearance h_0:

$$h=(r+h_0)\frac{\sin\left(\pi-\frac{x}{r}-\arcsin\left(\frac{h_0-z\tan\theta}{r+h_0}\sin\frac{x}{r}\right)\right)}{\sin x/r}r. \quad (3)$$

Formula (3) is less universal than the one used in the Pelosi paper [7]; however, in case of constant tilt angle θ, clearance height is much easier calculated by means of it. When using formula (3), it is possible to simplify pressure field calculations in accordance with RANS equations.

Let us write down Reynolds equation for the piston mechanism kinematics, when piston makes a full revolution relative to the guide bushing inner surface within one revolution of the shaft and preserves the maximum tilt angle [6, 7]:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\mu u\frac{\partial h}{\partial x}+6\mu w\frac{\partial h}{\partial z}, \quad (4)$$

where p – hydrodynamic pressure in the hydraulic fluid layer;

μ – coefficient of fluid dynamic viscosity.

As shown in paper [6], formula (4) is only applicable to the kinematics under which the rotational movement of piston in bushing is of the sliding nature on both edges. Also, the paper [6] contains Reynolds equation for the kinematics case when the piston rolls in the outer and inner edges of the bushing. However, this equation is too cumbersome and contains terms of the second order of infinitesimals or higher. To compare the latter, an expression containing characteristic values is associated with each term:

$$12\mu v'\sim 12\mu v_0;\ 6\mu u'\frac{\partial h}{\partial x}\sim 6\mu u_0\frac{h_0}{r};\ 6h\mu\frac{\partial u'}{\partial x}\sim 6h_0\mu\frac{u_0}{r};$$

$$18\mu v'\left(\frac{\partial h}{\partial x}\right)^2\sim 18\mu v_0\left(\frac{h_0}{r}\right)^2;\ 6\mu h\frac{\partial v'}{\partial x}\frac{\partial h}{\partial x}\sim 6h_0\mu\frac{v_0 h_0}{r^2};$$

$$6\mu hv\frac{\partial^2 h}{\partial x^2}\sim 6\mu h_0 v_0\left(\frac{h_0}{r}\right)^2;$$

$$6\mu w'\frac{\partial h}{\partial z}\sim 6\mu w_0\frac{h_0}{l};\ 18\mu v'\left(\frac{\partial h}{\partial z}\right)^2\sim 18\mu v_0\left(\frac{h_0}{l}\right)^2;$$

$$6\mu hv'\frac{\partial^2 h}{\partial z^2}\sim 6\mu h_0 v_0\left(\frac{h_0}{l}\right)^2.$$

Here, v_0, u_0, w_0 – maximum values for the selected rotation speed of hydraulic machine shaft.

The nominal clearance has a magnitude of the order of 10 μm, therefore, terms having two or more multipliers h_0 can be excluded. Characteristic magnitudes of velocities v_0 and u_0 have the same order of infinitesimals, the value of velocity w_0 can exceed these components by no more than one order, and radius and length of the guide bushing take the values of the order of 10^{-2} m. For that reason, addends having even one multiplier h_0 will be, as a minimum, by three orders of magnitude less than the addend without this multiplier.

The summary formula for the kinematics case of piston rolling in the guide bushing edges is given in formula

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=12\mu v'. \quad (5)$$

To solve equations (4) and (5), a relaxation method is used, in particular, the method of fractional steps (also called Yanenko method) [8]. Its use ensures sufficiently high accuracy of calculations. The method of fractional steps implies application of double-sweep method in each half-step by time.

Let us write down a scheme for the first and second half-steps:

$$\frac{p_{i,j}^{k+\frac{1}{2}}-p_{i,j}^{k}}{\Delta\tau}=$$

$$=\lambda\left(\frac{\partial h_{i,j}^3}{\partial z}\frac{p_{i,j+1}^{k+\frac{1}{2}}-p_{i,j-1}^{k+\frac{1}{2}}}{2\Delta z}+h_{i,j}^3\frac{p_{i,j+1}^{k+\frac{1}{2}}-2p_{i,j}^{k+\frac{1}{2}}+p_{i,j-1}^{k+\frac{1}{2}}}{\Delta z^2}\right)-$$

$$-\lambda\left(6\mu u\frac{\partial h}{\partial x}+6\mu w\frac{\partial h}{\partial z}\right);$$

$$\frac{p_{i,j}^{k+1}-p_{i,j}^{k+\frac{1}{2}}}{\Delta\tau}=$$

$$=\lambda\left(\frac{\partial h_{i,j}^3}{\partial x}\frac{p_{i+1,j}^{k+1}-p_{i-1,j}^{k+1}}{2\Delta x}+h_{i,j}^3\frac{p_{i+1,j}^{k+1}-2p_{i,j}^{k+1}+p_{i-1,j}^{k+1}}{\Delta x^2}\right)-$$
$$-\lambda\left(6\mu u\frac{\partial h}{\partial x}+6\mu w\frac{\partial h}{\partial z}\right).$$

Here, $\Delta\tau$ – step by time;

λ – coefficient determining task-solving speed;

k – step number by time.

In this case, time is introduced as a dummy parameter, and each iteration step during computing of the summary pressure value in each point does not show a real temporal change of hydrodynamic pressure. In connection with this, coefficient λ is selected with account of the orders and dimensionalities of the terms. Coefficient λ for the first and second half-steps is selected in accordance with the coefficients of the double-sweep method:

- for the first half-step

$$\lambda=\frac{l^2}{h_{i,j}^3};$$

- for the second half-step

$$\lambda=\frac{(\pi d)^2}{h_{i,j}^3}.$$

Here, d – piston diameter.

Steps by spatial coordinates $\frac{l}{200}$ m and $\frac{\pi d}{200}$ m, step by time $\Delta\tau=\frac{1}{40\,000}$ s are selected, and the difference between steps for count completion is selected as a difference of 0.0001 of the function value.

Given in Fig. 3 and 4 is an involute of a lubricant layer between the piston and the bushing during formation of hydrodynamic pressure in it, produced by piston movement in the bushing. Pressure field on the piston surface under sliding kinematics for piston position (see Fig. 3) for the following parameters: $\alpha = 0$; $\omega = 100$ rad/s; $r = 10$ mm; $h_0 = 12.5$ µm. Thus, axis x corresponds to the involute by circumference, and axis z – by bushing axis. Pressure peaks lie close to the points of the least clearance, i. e., the least layer thickness, but they have a certain offset, as described in papers [4, 5].

The computing results shown in Fig. 3 are supported by the experimental data given in the paper [7]. A comparison between pressure values in Fig. 3, 4 confirms the results obtained earlier in paper [6] by a different numerical method and without excluding the terms of higher order of infinitesimals.

According to the Korovchinsky's monograph [4] and equations (4) and (5), the bearing

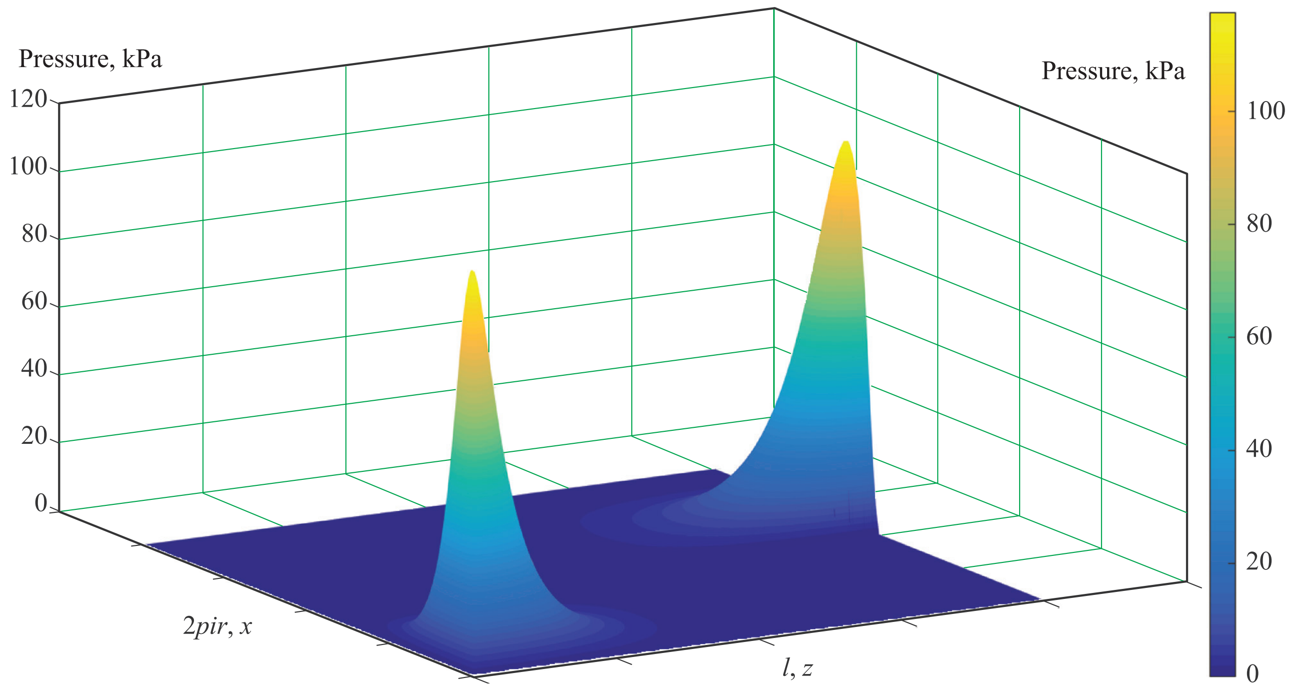

Fig. 3. Pressure field in hydraulic fluid layer for the kinematics of piston sliding in bushing

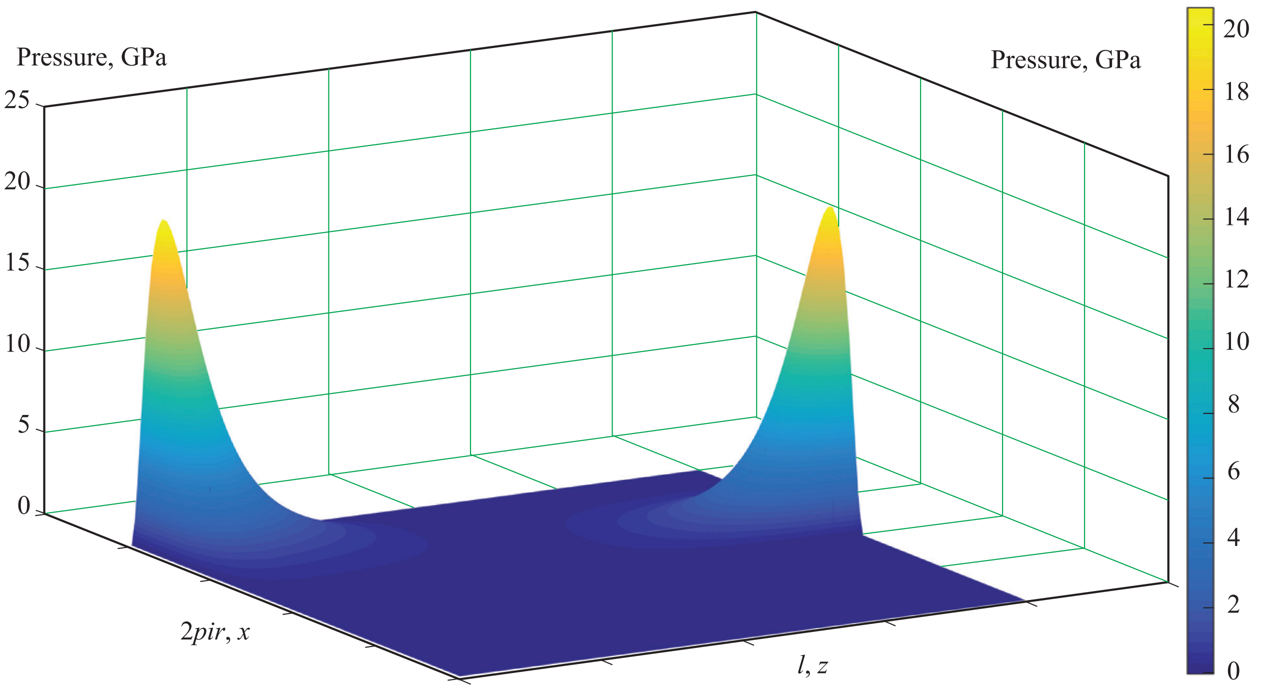

Fig. 4. Pressure field in hydraulic fluid layer for the kinematics of piston rolling in bushing edges

capacity of oil film hydrodynamic force is directly proportional to the rotation speed of the lip in the bearing (in our case, the piston in the bushing). To compare the values of hydrodynamic force for the sliding and rolling kinematics without application of a numerical experiment, a comparison was drawn between speed values in a point where, according to [4], hydrodynamic pressure is to reach its maximum. For the specified conditions, such point lies at 14° from the location of the least clearance between the bushing and the piston.

For the point of peak pressure, the speed of the piston sliding in the bushing will remain equal to $u = \omega r$, whereas rotation speed component $v' = \omega r \cdot \sin 14°$.

Then the speed ratio

$$\frac{v'}{u} = 0,242.$$

The involute of piston-cylinder unit clearance height in the cross-section is close in form to sine curve, therefore the value of clearance height change by circumference will have the same order of infinitesimals as the clearance height value.

Having compared the right parts of equations (4) and (5) of the obtained speed ratio, and taking into account the aforementioned estimation of the order of infinitesimals of the clearance height derivative by coordinate x, we have obtained a ratio

$$12\mu v' \Big/ 6\mu u \frac{\partial h}{\partial x} \sim 10^4. \qquad (6)$$

It means that in the case of rolling kinematics the bearing capacity of hydrodynamic force is at least by four orders of magnitude greater than in the case of piston sliding.

Shown in Fig. 5 are the values of total hydrodynamic force at the guide bushing outer edge for the two cases of piston mechanism kinematics, obtained through calculations after computing the pressure field values.

Resulting from comparison of the graphs, it was established that the hydrodynamic force created during rolling exceeded that created during sliding by at least five orders of magnitude. In this way, if such kinematics is ensured, it can be possible to switch to the liquid friction mode at lower RPM of the hydraulic machine shaft, which will improve performance at low RPM and at breakaway. However, this kinematics type is only feasible at comparatively low friction forces in the piston – footplate pair, which is difficult to achieve with the available design of hydraulic machine.

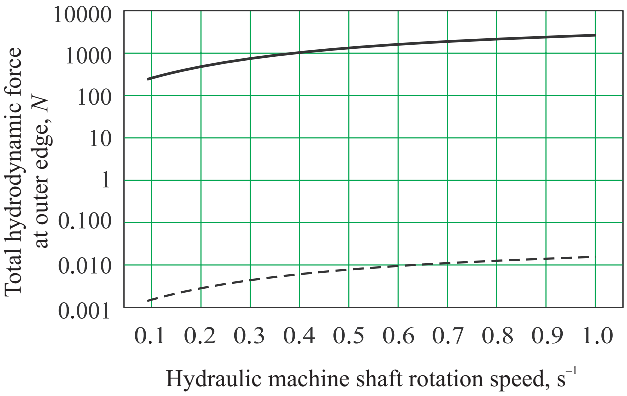

Fig. 5. Dependence of total hydrodynamic force at the outer edge on hydraulic machine shaft rotation speed for the kinematics cases of piston sliding (– – –) and rolling (——) in bushing

The study performed has yielded the following results:

- a simplified formula for oil film thickness in the APSPHM piston-cylinder unit;
- an overall equation for pressure in the layer for the case of piston rolling in the guide bushing edges.

A numerical experiment was carried out to compute pressure field for the two cases of kinematics, performed by the fractional step method. It has been shown that the kinematics of piston rolling in the bushing edges allows to create hydrodynamic force which is by five orders of magnitude greater than that created in the kinematics case when the piston slides in the bushing.

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Применение метода дробных шагов для решения задач гидромеханических процессов в поршневой паре аксиально-поршневых гидромашин с наклонным диском

На базе анализа кинематики поршневой пары получено уравнение для высоты зазора в поршневой паре для случая низких скоростей, являющееся основой для уравнения Рейнольдса для смазочного слоя поршневого механизма. С помощью численного эксперимента методом дробных шагов построено поле давления для двух различных случаев кинематики поршневого механизма, сопоставлена несущая способность гидродинамической силы. Выявлено аналитически и с помощью численного эксперимента, что при обкатывании поршнем кромок направляющей втулки суммарная гидродинамическая сила значительно превышает силу, создаваемую при скольжении поршня во втулке.

Ключевые слова: гидромашина, динамическая характеристика, наклонный диск, жидкостной режим трения.

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