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Method of correction factor calculation for linear reference systems

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The paper represents the method of correction factor calculation for linear reference systems along with the algorithm developed for real-time computation of the main task. The correction factor computation algorithm uses the principle of superposition of component functions with each of them represented as an approximation dependence.

Keywords: method, algorithm, main task, correction factor, correlation matrix, numerical data

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Introduction

Development of electronic technologies triggered introduction of special capabilities of aircraft system control based on airborne position sensor signal measurements and on calculation of composite function values during the aircraft's atmospheric flight phase, using the measured data.

Aircraft flight is accompanied with possible deviations of motion parameters from the set values, causing deviations from the required response time of the aircraft system (Fig. 1).

One of the methods intended to improve the accuracy of aircraft system's response is the feasibility to compensate the aircraft deviation from the calculated trajectory during its autonomous flight.

In aircraft engineering practice, developers use linear reference systems (LRS), which, based on the measured mismatch of the observed composite function and the predetermined correction

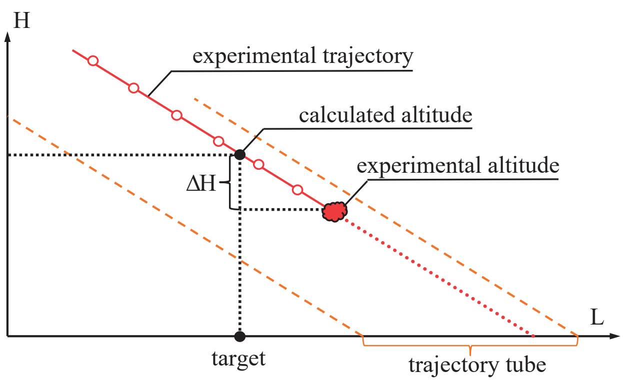

Fig. 1. Response altitude variation in case of deviations of motion parameters

factor, make it possible to compensate deviations by shortening the distance between the system’s response coordinate and the preset target coordinate. Based on the measured mismatch and predetermined correction factor K, the LRS allows to correct the system response time – any deviation is partially compensated; the LRS response accuracy is increased.

To make the LRS ready for timely command generation at the selected trajectory point, the main task (MT) is programmed into airborne equipment. The main task is computed before flight and LRS functioning. The MT includes correction factor K, which is to be calculated using a special computing system (SCS).

Development of the correction factor calculation method to be used for LRS command generation at the selected trajectory point defines the relevance of our work.

Study objectives and tasks

The task is to develop the correction factor calculation method. The method was developed with account for studied theoretical issues regarding composite function correction for linear reference systems.

We completed the following works to solve the problem.

1) Correction factor variation ranges for the domain of influencing variables have been analysed. We have found out that the correction factor depends on many variables: aircraft’s ballistic parameters (V, θ, H), instrumental errors of the response system ($ИО^{ССр}$), errors of external systems ($ИО^{СУ}$), $K \sim f(V, \theta, H, ИО^{СУ}, ИО^{ССр})$.

2) Limitations associated with software-based implementation of the correction factor computation algorithm in special computing systems have been considered.

3) Several methods of correction factor computation for linear reference systems have been considered.

4) A finite analytical expression is proposed as a method of correction factor computation for linear reference systems.

Purpose of work

Development of the algorithm that meets the algorithmic implementation requirements for real-time MT computations.

Requirements to the algorithm

The real-time computation algorithm shall provide the following:

– required accuracy of MT computation;

– minimum algorithmic, information, and time inputs.

The algorithm shall be simple and illustrative.

Since the correction factor is meant to be used for determining system response parameters, the correction factor computation error will affect the system’s response accuracy. In this respect, the requirements to computation accuracy are established before selection of the correction factor computation algorithm.

The method applied as the basis for elaborating requirements to the correction factor computation accuracy is intended to estimate the system’s response accuracy with a variable correction factor error [1]. Further, based on the established requirements to the correction factor computation accuracy, the problem is solved with respect to establishing requirements to the calculation error of individual components of the correction factor by solving the inverse problem of error theory – determination of components by a given resultant value [2].

The calculation error includes several components defined by different processes:

$$\Delta K_{РАСЧ} = \sqrt{\Delta K_{МЕТ}^{ДОП2} + \Delta K_{ВЫЧ}^{2} + \Delta K_{ЦЕР}^{2}}, \quad (1)$$

where $\Delta K_{МЕТ}^{ДОП}$ – allowable method error associated with simplification of the computation algorithm,

$\Delta K_{ВЫЧ}$ – correction factor computation error in special computing systems,

$\Delta K_{ЦЕР}$ – error of reducing the correction factor to the least significant digit for its representation in a code form.

Correction factor computation method

During our studies, we analysed the following methods of correction factor determination:

– search method;

– method based on the finite analytical expression.

The search method is intended to determine the optimal value correction factor K in a given domain of influencing variables by searching a local minimum deviation of composite functions from nominal values in case of variable active disturbances. The method is sufficiently labour-consuming because it requires a large amount of computations using the Monte Carlo method [3].

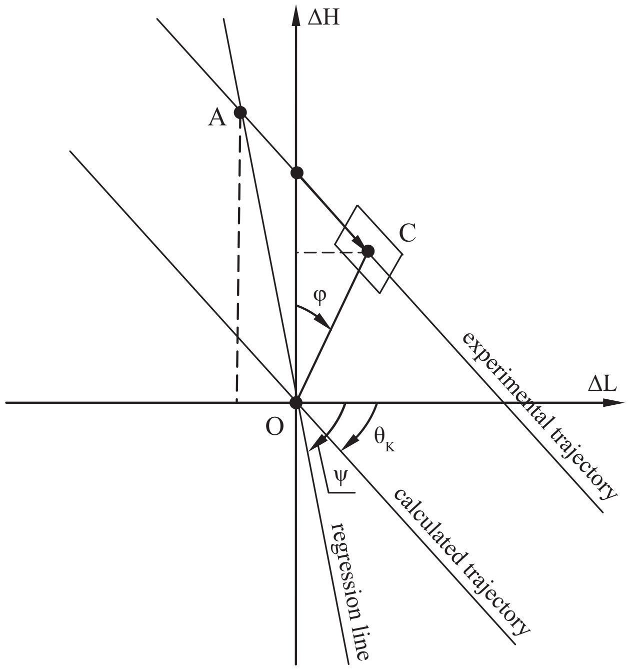

Fig. 2. Correction in the optimal plane:
– coordinate system $\Delta L\ O\ \Delta H$,
– point "O" corresponds to the aircraft position on the calculated trajectory,
– point "A" corresponds to the aircraft position on the real trajectory,
– ΔH, ΔL – altitude and range deviation from the calculated trajectory,
– $\psi = arctg \frac{1}{\rho_{L/H}}$ – regression line slope angle,
– $\rho_{L/H} = \frac{\sigma_L}{\sigma_H} \cdot K_{LH}$ – regression factor,
– K_{LH} – correction factor,
– θ – velocity vector angle of slope to the horizon,
– φ – angle specifying the position of the optimal plane relative to the vertical plane.

The method based on the finite analytical expression uses the regressive relationship between deviations of system response characteristics (altitude ΔH, flight range ΔL, distance ΔR) from a given response coordinate. The pair of parameters (ΔH, ΔL) have the normal law of distribution with the following characteristics: mathematical expectation – $ME(\Delta H)$, $ME(\Delta L)$, root-mean-square error (RMSE) $\sigma(\Delta H)$, $\sigma(\Delta L)$ and correlation factor $\rho_{\Delta H \Delta L}$. The regression equation allows to estimate the mean values of range deviation ΔL from the calculated (set) point with the known altitude deviation ΔH using the optimal value of correction factor K.

To select the correction factor computation algorithm, three correction cases were analysed:

– case 1 – response point correction with minimization of altitude deviation (factor K_H);

– case 2 – response point correction with minimization of range deviation (factor K_L);

– case 3 – response point correction in the optimal plane (factor K_R).

The following procedures were completed for each case:

– ballistic calculations;

– estimation of spread of LRS response coordinates;

– minimization of the LRS response error using the correction factor determined by different methods.

Calculations allow to determine probabilistic characteristics of aircraft motion parameters and system response parameters by using the Monte Carlo method, based on simulation models of aircraft spatial motion and LRS function for a selected system of disturbing factors.

Fig. 2 shows a graphic interpretation of response point correction in the optimal plane.

For the analysed cases of correction implementation, with account for taken assumptions and definitions (normal law of distribution, application of the dispersion operator to current altitude, range and distance deviations), formulas for correction factor calculation were determined by solving the appropriate equation.

Formulas for correction factor calculation for three correction cases described above are represented by the following ratios:

$$K_H = \frac{\left(\sigma H_k^*\right)^2 + \rho\left(H_k^*, H_v^*\right)\sigma H_k^* \sigma H_v^*}{\left(\sigma H_k^*\right)^2 + 2\rho\left(H_k^*, H_v^*\right)\sigma H_k^* \sigma H_v^* + \left(\sigma H_v^*\right)^2}; \quad (2)$$

$$K_L = K_H - \frac{\rho\left(L_к, H_к^*\right)\sigma L_к \sigma H_к^* + \rho\left(L_к, H_v^*\right)\sigma L_к \sigma H_v^*}{\left(\sigma H_к^*\right)^2 + 2\rho\left(H_к, H_v^*\right)\sigma H_к^* \sigma H_v^* + \left(\sigma H_v^*\right)^2} tg\theta_к^{ном}; \quad (3)$$

$$K_R = K_H - \frac{\rho\left(R_к, H_к^*\right)\sigma R_к \sigma H_к^* + \rho\left(R_к, H_v^*\right)\sigma R_к \sigma H_v^*}{\left(\sigma H_к^*\right)^2 + 2\rho\left(H_к, H_v^*\right)\sigma H_к^* \sigma H_v^* + \left(\sigma H_v^*\right)^2} \cos\theta_к^{ном}. \quad (4)$$

RMSE and correlation factors of observed composite functions, and ballistic parameter θ, i.e. flight-path angle, are the components of correction factors with minimization by altitude K_H, range K_L and in optimal plane K_R.

Formulas (2), (3), and (4) are the basis for developing the correction factor computation algorithm to be used for real-time computation of the MT in the SCS.

Correction factor computation algorithms

Taking into account recommendations given by developers of special computing systems, various correction factor computation algorithms were developed regarding a particular system.

First algorithm – for computing system "A".

Second algorithm – for computing system "B".

Computing system "A"

The algorithm is developed and represented in the table form with the values of correction factors depending on its constitutive parameters (velocity, flight-path angle, altitude, external systems operation modes). For this purpose, values of factors K_H and K_L are pre-calculated by formulas (2), (3), (4), and then saved to the special computing system's memory. The sought value of the correction factor is calculated during real-time computation of the MT based on table values of K_H and K_L.

Computing system "B"

The developed correction factor computation algorithm is based on approximated finite relations.

The principle of independence of components that define RMSE of deviations (Δ*H*, Δ*L*) is used for algorithm development.

$$\begin{vmatrix} \sigma\Delta L & K_{\Delta L\Delta H} \\ & \sigma\Delta H \end{vmatrix}_{\Sigma} = \begin{vmatrix} \sigma\Delta L & K_{\Delta L\Delta H} \\ & \sigma\Delta H \end{vmatrix}_{СУ} + \begin{vmatrix} \sigma\Delta L & K_{\Delta L\Delta H} \\ & \sigma\Delta H \end{vmatrix}_{ATM}. \quad (5)$$

Inferior indices "СУ" and "АТМ" are related to groups of disturbing factors affecting an aircraft, СУ – from external systems, АТМ – from disturbances acting on the atmospheric segment of flight trajectory.

This principle allowed to distribute the zone of responsibility for calculating the components that define deviations Δ*H*, Δ*L*, on the one hand, and predetermined the possibility to represent each component in formulas in the form of the sum of independent components for correction factor calculation. The principle of superposition of factor components was used for the correction factor computation algorithm [6].

Moreover, according to [5], when multiplying approximated values, errors are added up; in this case, to reduce the correction factor computation error, it is reasonable to go from root-mean-square errors and correlation factors to correlation moments in formulas (2), (3). Based on the above, the following ratios are obtained:

$$q_H = \frac{\sigma H_\Sigma^2 + K_{HV}}{\sigma H_\Sigma^2 + \sigma H_V^2 + 2 \cdot K_{HV}}; \quad (6)$$

$$q_L = q_H - \frac{K_{LH\Sigma}}{\sigma H_\Sigma^2 + \sigma H_V^2 + 2 \cdot K_{HV}} \cdot tg\theta_K. \quad (7)$$

Each component (6), (7) is represented by the approximating polynomial [6], the arguments of which are ballistic parameters (V, θ, H).

Basic calculation ratios that define the correction factor computation algorithm are represented by the following formulas:

1) The following functions are calculated:

$$\sigma H_V = \sum_{i=0}^{2}\sum_{j=0}^{2}\sum_{d=0}^{2} h1_{ijd}\cdot \bar{V}^I\cdot \bar{\theta}^J\cdot \bar{H}^D, \quad (8)$$

$$\sigma H^{ATM} = \sum_{i=0}^{2}\sum_{j=0}^{2}\sum_{d=0}^{2} h2_{ijd}\cdot \bar{V}^I\cdot \bar{\theta}^J\cdot \bar{H}^D, \quad (9)$$

$$K_{HV} = \sum_{i=0}^{2}\sum_{j=0}^{2}\sum_{d=0}^{2} h3_{ijd}\cdot \bar{V}^I\cdot \bar{\theta}^J\cdot \bar{H}^D, \quad (10)$$

$$K_{LH}^{ATM} = \sum_{i=0}^{2}\sum_{j=0}^{2}\sum_{d=0}^{2} h4_{ijd}\cdot \bar{V}^I\cdot \bar{\theta}^J\cdot \bar{H}^D. \quad (11)$$

where $h1_{ijd}$, $h2_{ijd}$, $h3_{ijd}$, $h4_{ijd}$ – approximation factors;

$\bar{V}^I\cdot \bar{\theta}^J\cdot \bar{H}^D$ – normalised ballistic parameters.

2) Total RMSE is calculated as follows:

$$\sigma H_{K\Sigma} = \sigma H^{CУ} + \sigma H^{ATM}, \quad (12)$$

where $\sigma H^{CУ}$ is included in input data.

3) Total moment K_{LH_Σ} is calculated by the following formula:

$$K_{LH_\Sigma} = K_{LH}^{CУ} + K_{LH}^{ATM}, \quad (13)$$

where $K_{LH}^{CУ}$ is included in input data.

4) Correction factors q_0 and q_1 are calculated

$$q_0 = \frac{\sigma H_\Sigma^2 + K_{HV}}{\sigma H_\Sigma^2 + \sigma H_V^2 + 2\cdot K_{HV}}. \quad (14)$$

$$q_1 = q_0 - \frac{K_{LH_\Sigma}}{\sigma H_\Sigma^2 + \sigma H_V^2 + 2\cdot K_{HV}}\cdot tg\theta_K. \quad (15)$$

Results of estimation of correction factor computation algorithm accuracy

A criterion of correctness of the developed algorithm is fulfilment of the condition:

$$\Delta K_{MET} < \Delta K_{MET}^{ДОП}, \quad (16)$$

or, in designations of the correction factor computation algorithm:

$$\Delta q_{MET} \le \Delta q_{MET}^{ДОП}, \quad (17)$$

where $\Delta q_{MET}^{ДОП}$ – allowable method error of the correction factor computation algorithm (the first term in formula 1);

Δq_{MET} – method error of correction factor determination as per the developed algorithm.

In its turn, Δq_{MET} is calculated as follows:

$$\Delta q_{MET} = q^{PAC} - q^{TOЧ}, \quad (18)$$

where q^{PAC} – correction factor value based on the developed algorithm;

$q^{TOЧ}$ – correction factor value calculated at minimum composite function deviation (when searching through factors in the selected variation range) when conducting ballistic calculations and determining probabilistic characteristics of LRS response.

The characteristics given in Tables 1 and 2 prove that the developed correction factor computation algorithm meets the specified requirements:

– approximation errors of correction factor components (Table 1, relative values);

– value of allowable (relative) and estimated method errors of factor Δq_{0MET}, Δq_{1MET} in the domain of argument determination (Table 2).

According to Table 2, the method error of the correction factor computation algorithm (q_0, q_1) meets the requirement to non-exceedance of the allowable error, thus proving that the developed algorithm is relevant and correct.

The accuracy (relative error) of linear reference system response was estimated with account for the correction factor computation algorithm error. Assuming that the system response error is equal to 1 ($\sigma^{TOЧ} = 1$), in case we use the correction factor calculated by applying an accurate method, we will find out that the system response error can increase by 1.5 %, when calculating the correction factor based on the developed algorithm. This will cause a minor variation in the total system response error: $\sigma_{AHAЛ} = 1.000112$.

For analysis, development, and testing of MT computation algorithms for different response

Table 1

Characteristics of algorithm components

Characteristics	$\overline{\sigma H_V}$	$\overline{\sigma H^{ATM}}$	$\overline{K_{HV}}$	$\overline{K_{LH}^{ATM}}$
Components variation range	0.3333–1.0000	0.2–1.0	0.5–1.0	–(0.4–1.0)
Approximation error	0.0095	0.00312	0.00133	0.00849

Table 2

Characteristics of correction factor computation algorithm

Correction factor variation range	$\Delta q_{MET}^{ДОП}$	Δq_{0MET}	Δq_{1MET}
0–1	0.1	0.0041	0.0055

systems, the authors have preliminarily developed a software package that allows to form the structure of MT computation algorithms, calculate numerical characteristics of the algorithm, simulate MT computation algorithms in special computing systems, and check the correctness of MT computation.

The software package was modified and improved in order to develop a new algorithm for computing the correction factor representing one of the MT computation algorithm components. Fig. 3 shows the software package block diagram.

Scientific novelty of the study

1) We developed a suite of methods which allow to develop and compile the computation algorithm.

2) The correction factor computation error model was determined.

3) We developed the method and specified the sequence of computations to develop the correction factor computation algorithm.

4) Correction factor computation algorithm was developed to be implemented in the SCS.

The software package for development, testing, and check of MT computation algorithms was improved. New blocks were implemented in the LRS

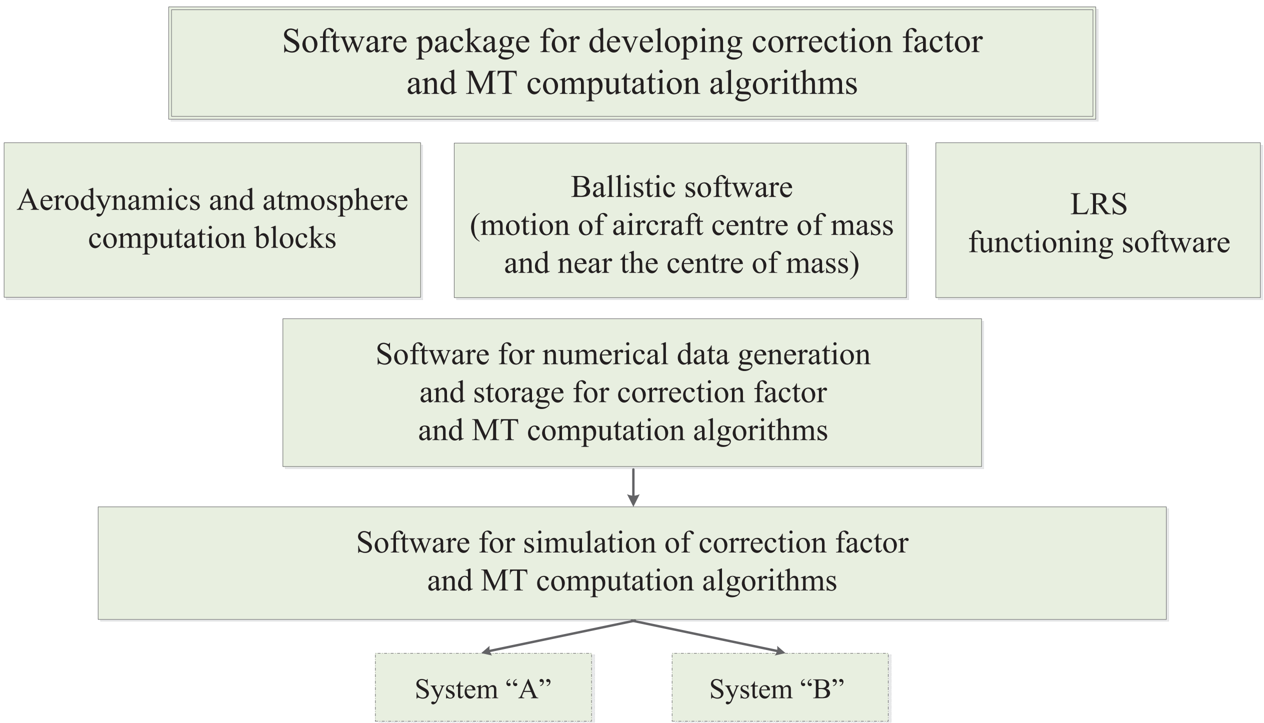

Fig. 3. Software package block diagram

functioning program, approximation program, and correction factor computation simulation program in the SCS.

Practical implementation

1. We proposed and implemented a new algorithm for computing the LRS correction factor intended for real-time MT computations. This algorithm allowed to rationally determine the MT diagram, structure and computation algorithms.

2. The developed correction factor computation algorithm provides real-time data preparation.

3. The correction factor value calculated as per the developed algorithm provides the required response accuracy of a linear reference system.

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Методика расчета коэффициента коррекции для линейно-корректируемых систем

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Представлена методика расчета коэффициента коррекции для линейно-корректируемых систем, разработан алгоритм расчета, предложенный для оперативного расчета основного задания. В алгоритме расчета коэффициента коррекции используется принцип суперпозиции составляющих функций, каждая из которых представляется аппроксимирующей зависимостью.

Ключевые слова: методика, алгоритм, основное задание, коэффициент коррекции, корреляционная матрица, числовой материал

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