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Improving the adequacy of aircraft manoeuvre simulation by determining the positions of the instantaneous axes of its rotational motion

A. A. Piskovatsky

Public Joint-Stock Company "Dolgoprudny Scientific Production Enterprise", Dolgoprudny, Russian Federation

The potential of developing a system for automatic control over manoeuvres at critical stall and sideslip angles, and aircraft (AC) automatic recovery from emergency flight modes, is limited by the presence of the control system's dynamic errors, which, in case their critical level is exceeded, lead to a failed manoeuvre and may result in the loss of AC in an unstable flight situation. One of the causes of such errors are the measurement-method errors occurring due to the fact that synthesis of the control laws (CL) takes place on the basis of mathematical models (MM) of those AC motion dynamics that are not accurate enough. The presence of the measurement-method control errors, as determined by the principle of flight mathematical simulation, will not allow to determine CL corresponding to these flight processes and use them for implementation of such manoeuvres in the automatic flight modes.

The objective of the study is to improve the adequacy of the MM of AC manoeuvring through development of new principles of its configuration using a criterion with more stringent requirements to determining the equivalence of a simulated and a real manoeuvre (hereinafter SM and RM) of the AC.

By way of proving three theorems, the paper formulates the conditions for assuming AC's SM and RM as equivalent, and substantiates a possibility to reduce control system's dynamic errors in high-speed processes of AC manoeuvring through the use of a novel principle of mathematical modelling of such a flight.

Despite the theoretical and investigative orientation of the paper, the expected outcomes from the improvement of MM adequacy due to a more accurate prediction of the AC manoeuvre parameters and generation of control signals under such manoeuvring that are free from the measurement-method errors will be of great importance for practical application.

Keywords: model adequacy, equivalence of states, trajectory coincidence, instantaneous axis, acceleration distribution

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Definitions and abbreviations

RB – rigid body

AC rotational motion – aircraft's rotational motion to be considered irrespective of its translational motion

AEF – action of external forces affecting aircraft motion dynamics

Acting distribution of aircraft forces (influencing mass distribution) with respect to its BCS coordinates – distribution of aircraft forces (mass) with respect to its BCS coordinates after linearisation of its values

AC – aircraft

MM – mathematical model of aircraft manoeuvring

IAAF – instantaneous axis of action of external force torque that has an effect on a change in parameters of aircraft's rotational motion

IAAV – instantaneous axis of angular velocity to be determined as a set of spatial points with zero velocity values determined for aircraft's rotational motion

IAAA – instantaneous axis of angular acceleration to be determined as a set of spatial points with zero velocity values determined for isolated rotational motion of the aircraft

NF – non-uniform forces that form aircraft's angular acceleration

Type 1 MLCS – moving local coordinate system bound to aircraft's IAAV

Type 2 MLCS – moving local coordinate system bound to aircraft's BSC position at the initial moment of simulation step

Resultant distributed force – the vector that may substitute the action of a distributed force. The vector modulus is equal to the distribution volume while the vector itself bisects the force distribution cross-section areas

Distribution of forces (mass) with respect to aircraft's BCS coordinates – distribution of non-uniform loads of external forces (aircraft mass) with respect to aircraft's BCS coordinates

Distributed forces – loads acting on a certain surface

Aircraft velocity distribution – distribution where a set of distribution boundary points is determined by aircraft points (distribution bases) in accordance with velocity vectors of these points

RM – aircraft's real manoeuvre

UF – uniformly balanced forces that form aircraft's linear acceleration

DANF – distribution of acceleration of non-uniformly balanced forces

DAUF – distribution of acceleration of uniformly balanced forces

CS – coordinate system

BCS – coordinate system bound to the origin at the aircraft nose

SM – aircraft's simulated manoeuvre

PARF – point of application of the resultant force applied to aircraft

PARV – point of application of resultant vector

Aircraft velocity manoeuvre Φ_D^{Vi} for AC being in position D (lower-case index) – indivisible set of points that characterises the boundary of volumetric distribution of the aircraft points velocity at moment of time t_i. This is determined by possible aircraft movement per time unit [4]

Aircraft acceleration manoeuvre Φ_D^{ai} for aircraft being in position D – indivisible set of points that characterises the boundary of acceleration distribution of the all aircraft points at moment of time t_i [4]

CM – aircraft's centre of mass

Equivalent transformation – transformation of geometric and analytical representations of aircraft motion whose output parameters allow to characterize the motion as equivalent to the initial motion of aircraft

EP – aircraft's elementary particles

Typical sections – aircraft sections that belong to its BCS axes

$[\ ]_{Ox}$ ($[\ ]_{\mathrm{ATT}}$) – sign intended to attribute the action to a typical section that belongs to the axis Ox (to the whole aircraft being determined by RB)

Introduction

Fast-pace development of complex manoeuvres (including aerobatics) made by modern aircraft opens doors to promising applications in combat. To be performed, such manoeuvres require critical stall and sideslip angles. That is why only experienced stunt pilots are able to do this. Extensive execution of such manoeuvres in automatic modes of aircraft and UAV control systems in order to improve their operational performance is inapplicable due to the existing dynamic errors of control systems. One of the reasons for such errors consists in using insufficiently accurate aircraft flight MMs when synthesizing their control laws (CLs). For aircraft manoeuvring in the automatic mode, such control errors shall be minimized to extremely low values as appropriate, due to non-return aircraft responses which occur once the key manoeuvre parameters are exceeded. The solution to the problem would allow to solve the problem of aircraft automatic recovery from an unstable flight area in case of an emergency.

Better understanding and, therefore, prediction of in-flight aircraft dynamic processes enables a compromise between stability and controllability as well as a better quality of transient processes of AC flight control and flight parameter stabilization.

Moreover, using the existing aircraft flight MMs featuring the above errors for analysing aircraft manoeuvres results in considerable errors in the aircraft manoeuvring computational experiment, misinterpretation of results obtained and, eventually, restricts the possibility to always make accurate conclusions on changes in ongoing aircraft flight dynamic processes.

Errors being determined by the method of analytical description of aircraft flight are one of the causes of dynamic errors in control. Improving the adequacy of MMs describing aircraft manoeuvring allows to considerably reduce measurement-method errors caused by the CL synthesis based on insufficiently accurate MM of aircraft motion dynamics data.

It is a challenging problem to improve the adequacy of the aircraft flight MM for the existing conditions that allow to consider the aircraft SM equivalent to the RM. A simpler solution is to develop the principle of mathematical modelling using a more rigid criterion for considering the aircraft SM equivalent to the RM.

The objective of the study is to improve the adequacy of the aircraft manoeuvre MM by justifying the principles of mathematical simulation that involve a more rigid criterion for considering the aircraft SM equivalent to the RM.

Research conditions and assumptions

1. The aircraft is assumed to be a rigid body (RB) with non-uniform mass distribution to be determined in relation to the coordinate system bound to the aircraft. All the elementary particles (EPs) of the aircraft are somehow exposed to the action of distributed external forces.

2. Motion dynamics is analyzed for the aircraft determined by the free body [3, p. 10] or the RB that can make rotational motion evolutions with the instantaneous axis of rotation, IAAV, IAAA, and IAAF being in any position.

3. For each step of mathematical simulation of aircraft manoeuvre, we assume that the time interval required for the simulation step is so small that the values of all aircraft motion parameters (except angular and linear displacements) remain unchanged as determined at the initial moment of the specific simulation step. These parameters change only at the initial moment of the next simulation step.

4. AC manoeuvring is analyzed by distributions of aircraft velocity, angular velocity, acceleration, angular acceleration as well as by forces acting on the aircraft and determined relatively to the axes of its bound coordinate system (BCS) [3, pp. 58–61]. The selected origin of the BCS is located at the aircraft nose because the role of the aircraft's centre of mass (CM) is not so important in the studies under consideration compared with its role when flight parameters are reduced to the centre.

Errors in measurement (calculation) of rotational motion parameters can be eliminated only if they are determined relative to the instantaneous axis, in relation to which the parameter is changed in the RM. That is why it is reasonable to determine the aircraft's angular velocity and acceleration as well as the action of NFs relative to IAAV, IAAA and IAAF, respectively.

Therefore, aircraft's rotational motion is determined in a moving local coordinate system (Type 1 MLCS) (not bound to the position of other objects) that is determined for each simulation step at its initial moment of time, while the coordinate system maintains this position within this step. The selected origin of the MLCS is pole *O* to be determined as the interception point of the IAAV and the plane where the angular movement of the missile nose takes place. Positions of Type 1 MLCS axes: axis *Ox* – relative to IAAV, with the direction to be determined by the left-hand rule (for aircraft rotation direction); axis *Oy* – from centre *O* to the aircraft nose, the position of which is determined at the initial moment of the simulation step; axis *Oz* expands axes *Ox* and *Oy* to the right-hand CS.

With the aircraft's angular velocity unavailable, its rotational motion parameters are calculated within the Type 2 MLCS, which is determined depending on the BCS position at the initial moment of the simulation step and remains in this position until the simulation step is complete.

5. The action of external forces (AEF) affecting the aircraft motion dynamics is characterized by non-uniformly acting forces (NF) and uniformly balanced forces (UF). These components of AEF are determined based on the condition that the UFs form aircraft's linear acceleration $a_{\text{ЛА}}$ while NFs form aircraft's angular acceleration $\varepsilon_{\text{ЛА}}$. Thus, for each simulation step, the AEF is characterized by the parameters of resultant external forces and torque (acting relative to IAAF).

Preventing the aircraft's rotational motion from substitution for its translational motion

The requirements for applicable CSs and transformations (determined by the simulation principle) in development of the aircraft flight MM include not only the possibility for easy representation of forces acting on the aircraft, but also the possibility to ensure perfect match of force values in the RM. The synthesis of CLs based on the MM whose output parameters differ from the RM parameters results in aircraft control with dynamic errors. In the author's opinion, one of the causes of such errors is the application of the principle of force reduction to the certain centre where aircraft's rotational motion is partially substituted for its translational motion.

Let us analyse the adequacy of such substitution.

The parameters of rotational motion of aircraft points (angular acceleration and angular velocity) [6] are associated with the range of action of these parameters. By comparison, the translational motion parameters (acceleration and velocity of aircraft points) are not associated with distances to any centres of aircraft motion.

Considerable differences in trajectories of AC translational and rotational motion and respective motion parameters allow to make a conclusion that aircraft's translational and rotational motion are radically different modes of aircraft motion. Therefore, their action shall be determined avoiding substitution of one mode of motion for another.

In terms of triggering events and a strong transition depending on the IAAV position (in case of transitional motion, the IAAV is at infinity), the difference in these modes of aircraft (being a rigid body) motion defines them as a complete group of disjoint events.

The principle of reduction of forces applied to the RB, to the certain centre

Louis Poinsot, a peer of France, was the inventor of the existing principle of the rigid body's motion dynamics analysis. In 1803, Poinsot formulated the principle of transformation of vectors acting on a rigid body by reducing vectors to a selected centre [1]. Later, Poinsot applied the method to solve the dynamics problems.

N. E. Zhukovsky studied the motion of a rigid body and determined an instantaneous centre of rotation, an instantaneous centre of velocity (ICV) and an instantaneous centre of acceleration (ICA) [2, pp. 242–256]. Due to the simplicity of the principle of reducing the vectors to a certain centre [2], Zhukovsky probably applied this exact principle to solve aircraft flight dynamics problems. Nowadays it remains the main principle for determining the motion parameters of all rigid bodies.

According to the principle described by L. Poinsot, "a system of forces acting on a rigid body could be resolved into a single force passing through a random point and into a couple... (describing the action of NFs. – *Author's note*)" [1].

To determine the influence of AEF on a manoeuvring aircraft, the principle determines the following:

– finding the principal vector by determining the resultant force and by transiting the point of application of resultant force (PARF) to the centre of reduction;

– finding the principal moment by determining the action of couple that is formed by resultant force $\vec{F}^{*}_{\text{PC}}$ and oppositely directed vector $\vec{F}^{**}_{\text{PC}}$. In this case, vector $\vec{F}^{*}_{\text{PC}}$ is applied to the PARF as per the force reduction rule, and vector $\vec{F}^{**}_{\text{PC}}$ is applied to the centre.

Let us analyse this transformation as the distributed action of forces applied to the aircraft with uniform mass distribution.

The AEF on a manoeuvring aircraft makes itself evident in forming UF and NF. Figure 1a shows these forces as distributions of UF $F_{\text{PC}}(x)$ and NF $F_{\text{HC}}(x)$ determined for coordinate values on axis Ox in the aircraft's BCS.

Let us analyse variations in the distribution of NFs $F_{\text{HC}}(x)$ after reducing forces to centre $O_{\text{прив}}$. After forces are reduced to centre $O_{\text{прив}}$, the distribution of NFs $F_{\text{HC}}(x)$ is divided into two components which also describe non-uniform $F_{\text{HC}}^{\text{прив}}(x)$ and uniformly balanced action $F_{\text{PC}}^{\text{прив}}(x)$ of the distribution $F_{\text{HC}}(x)$ (Fig. 1b).

As a result of such reduction of forces to centre $O_{\text{прив}}$, the action of NF F_{HC} is determined only by the component of NF $F_{\text{HC}}^{\text{прив}}$. The second component of NF $F_{\text{PC}}^{\text{прив}}$ that determines the action of NF in the section from the PARF $O_{\text{ТПР}}$ to centre of reduction $O_{\text{прив}}$ is considered to be the UF distribution. We can observe the NF distribution partially substituted for the UF distribution.

Besides, as the aircraft's spatial motion is reduced to centre $O_{\text{прив}}$, angular and linear velocities (as well as acceleration and linear acceleration) are also resolved relative to the centre of reduction, irrespective of the position of IAAV (IAAA). In this case, reduction errors are determined through the distances from $O_{\text{прив}}$ to the PARF, from $O_{\text{прив}}$ to the point where acceleration and angular acceleration are resolved, and from $O_{\text{прив}}$ to the point where angular and linear velocities are resolved. The greater the distances, the greater the errors in determination of the action of NFs, angular acceleration and angular velocity as a result of reduction. In classical mechanics, the principle description states that the rotational component is independent of pole selection [3, p. 128].

The factor which compensates this disagreement is the selection of the aircraft's CM as the centre of reduction. For the aircraft being stable

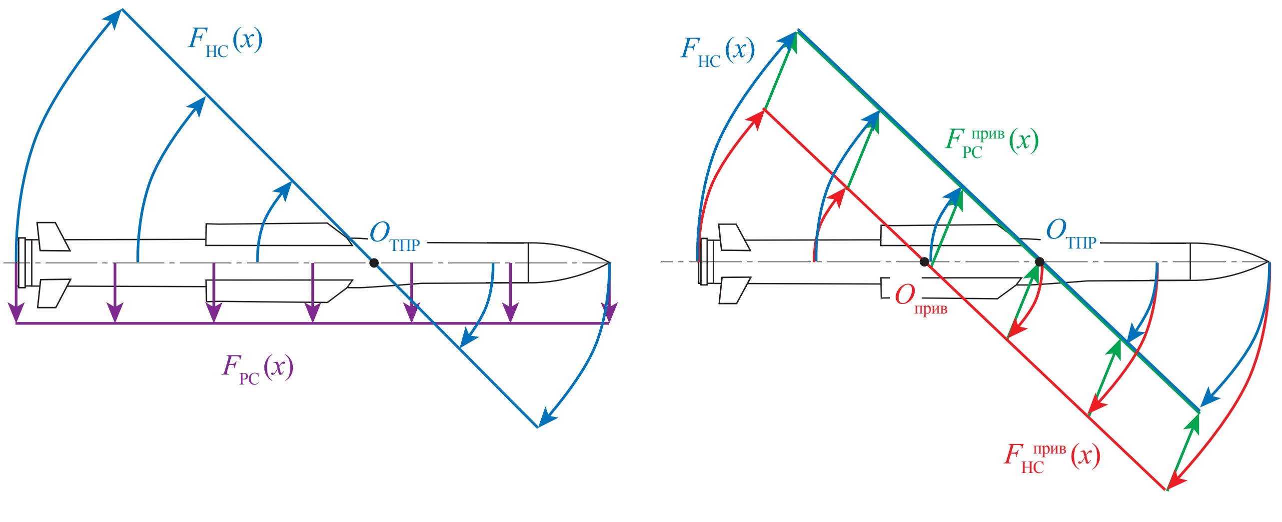


Fig. 1. Substituting some NF distribution for UF distribution

in terms of the angle of attack at low values of the control force, the PARF moves at a short distance away from its CM. That is why, in this case reduction of forces to the CM in the SM will not lead to a considerable disagreement with the aircraft’s RM parameters.

In [4] and [5], the author presents a hypothesis alternative to L. Poinsot’s principle. According to the hypothesis, at any time the aircraft motion has unambiguous instantaneous centres of rotational motion, relative to which the angular displacement, velocity, acceleration and, respectively, the external torque can be determined without errors. Also, the author [5] proves that while analysing the flight of an aircraft moving at angular acceleration, omitting of the couple transition and displacement of the force vector along its line of action will allow to avoid measurement-method errors related to these transformations and to improve the accuracy of prediction of aircraft flight parameters.

Choosing the criterion of aircraft motion transformation equivalence

To determine the adequacy of transformations of forces and aircraft flight parameters, we choose the criterion to determine aircraft’s SM and RM equivalence. Such an assumption shall not raise any doubts whether it is true or not in order to take it for granted.

Time-dependent distribution of motion trajectories of aircraft points is an ideal “integral” criterion for any variations of velocity, acceleration and distributed forces that act on the said aircraft points. This is due to the fact that for any point of the RB the time-dependant distribution of its trajectory is a result of double integration of its acceleration and integration of its velocity.

The following is taken as the criterion:

Aircraft motions are considered equivalent if such aircraft move in sync, following the same trajectories $L_1(x_1, y_1, z_1, t)$ and $L_2(x_2, y_2, z_2, t)$ at any scaling:

$$L_1(x_1, y_1, z_1, t) \equiv L_2(x_2, y_2, z_2, t). \qquad (1)$$

The properties of RB motion dynamics were used for proving. Due to a rigid link between adjacent EPs in the RB, these EPs cannot have a considerable difference in motion velocities and accelerations, therefore:

– velocities and accelerations of points located on the same straight line may only change proportionally to variations in their coordinates within the aircraft BCS;

– once the RB makes a single turn, the sections interconnecting the initial position and the end position of points taken at any radius of the turn form a number of parallel lines.

Using the proofs given in [4] and [5], we can formulate the basic assertion described herein.

For any moment of the aircraft manoeuvre, resolving its angular velocity, angular acceleration and external torque acting on the aircraft relative to any axes (centres) of reduction which do not coincide with the axes of action of these parameters (IAAV, IAAA and IAAF, respectively) in a real manoeuvre is not an equivalent transformation. The parameters of rotational motion of a manoeuvring aircraft can be accurately determined only in relation to the axes of rotation.

Let us prove this assertion for each axis of rotation of the aircraft listed above.

Theorem 1

Formulation. The aircraft’s angular velocity can be accurately determined at any moment of aircraft motion only in relation to the IAAV, the axis the aircraft rotates around at the actual moment time. Reducing the aircraft rotational motion relative to all the remaining centres (axes) is not an equivalent transformation.

Problem statement. To prove that the parameters of aircraft’s rotational motion are changed when reduced to a certain centre, let us analyse the displacement of aircraft’s individual points. The analysis is conducted for two cases when the aircraft rotates in the Type 1 MLCS from position D to position E within time interval Δt, which is so small that angular velocities of aircraft’s points can be assumed as constant values (Fig. 2a). In case of initial motion, the aircraft rotates relative to centre O_2^* (to be analysed in the Type 1 MLCS bound to the centre).

In case of reduced motion, the aircraft moves away from the same initial points to the same end points, but with the points' velocity vectors reduced to centre O_1^*. For each mode of motion, time interval Δt is divided into m^* of equal areas (where $m^* \geq 10$) and trajectories (cross-connections) of points movement O_1^* and O_2^*.

Proof 1. Let us compare cross-connections of initial and reduced motion of aircraft points O_1^* and O_2^* at simulation interval Δt in an expanded time scale with $\Delta t_p = 0.1\Delta t$ (Fig. 2a).

For the motion reduced to centre O_1^*, the reduction means that rotational motion is considered within the Type 1 MLCS bound to O_1^*, as if it takes place relative to this centre while centre O_1^* itself makes translational motion only. The trajectory of point O_2^* in this mode of motion is shown in Figure 2b (the trajectory is shown in scale $\Delta t_p = 0.1\ \Delta t$). In case of initial motion, this is the motion trajectory consisting of point O_2^*.

For point O_1^* being in plane-parallel motion reduced to this point, it is the translational motion by S [m] to point O_1^{**}. In case of rotational motion, it is the rotational motion of point O_1^* relative to centre O_2^*.

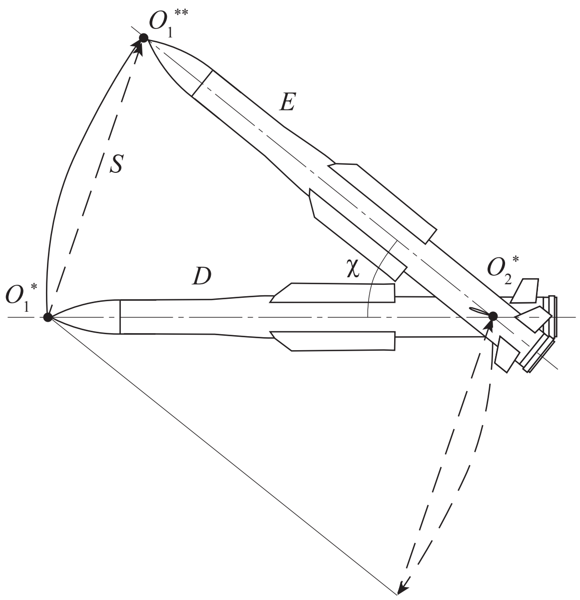
Conclusion. Trajectories of aircraft's initial and reduced motion do not align, but their misalignment is not associated with discreetness variation. The difference is in the nature of cross-connections of aircraft's individual points. As per criterion (1) within the time interval where aircraft's flight parameters remain constant, motions under consideration from the same initial points to the same end points of the aircraft are considered non-equivalent.

Proof 2. Let us compare the parameters of aircraft's plane-parallel motion shown in Figure 2a to the parameters of similar motion reduced to point O_1^*.

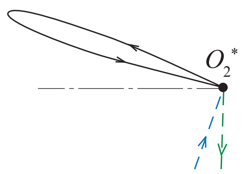
In case of initial motion, point O_1^* rotated at angular velocity $\chi\ /\ \Delta t$ [rad/s] without linear velocity.

In case of reduced motion, the angular velocity of point O_1^* relative to point O_1^* is equal to zero, and the linear velocity of the point is s / Δt [m/s].

The difference in linear and angular velocities of aircraft's individual points in two cases under consideration indicates that due to reduction of initial motion to a new centre, the obtained reduced motion differs from the aircraft's initial motion. As these points of the aircraft move at velocities that differ from initial angular and linear



a



b

Fig. 2. Cross-connections of points of aircraft's initial and reduced motion

motion velocities, the initial and reduced motions are considered non-equivalent.

Note. We should note that in terms of the assumptions of the existing dynamics, the described aircraft's plane-parallel motions being considered within a single simulation interval with rotational and translational motions separated relative to different poles are assumed to be equivalent motions with different centres of reduction.

Conclusion. Under the classical description of aircraft motion it is assumed that the trajectories of rotational motion of aircraft's points can be represented as trajectories of translational motion of these points for the same case under consideration. As a result, values of aircraft's linear and angular velocities change. Note that this is one of the disadvantages of the principle of reduction of forces to a the certain centre because the dependence of parameters of the same aircraft motion on the centre of reduction shall not be observed.

Thus, any aircraft manoeuvre can be viewed as rotational motion relative to the IAAV that dynamically changes its position as a result of aircraft's translational motion. As an example, Figure 3 shows cross-connections of IAAV displacement for a manoeuvring fighter [4] (in this case, the IAAVs are shown as instantaneous centres of velocities of the fighter's rotational motion).

Figure 3 also shows the directions of instantaneous velocity vectors of aircraft's CM $\vec{V}_1 \div \vec{V}_{10}$, to be determined as parameters of instantaneous angular velocities of the centre's motion at different moments of time as the following expression $\vec{\omega}_i = \vec{V}_i / r_i$ (where $i = \overline{1.10}$).

Figure 3 shows that the IAAV sometimes tends to infinity and the aircraft goes to the state of translational motion. Such states are determined by the points of intersection of straight lines *b*, *c*, *d*, *e* and the aircraft trajectory, as shown in the figure.

Qualitative and quantitative differences in distributions of accelerations of aircraft's translational and rotational motion determine separate consideration of forces which form them. That is why, below we will determine the distribution of aircraft's angular acceleration (aircraft's rotational motion acceleration) in relation to the coordinates of aircraft's BCS as the distribution of acceleration of non-uniform forces acting on the aircraft (DANF). Accordingly, the distribution

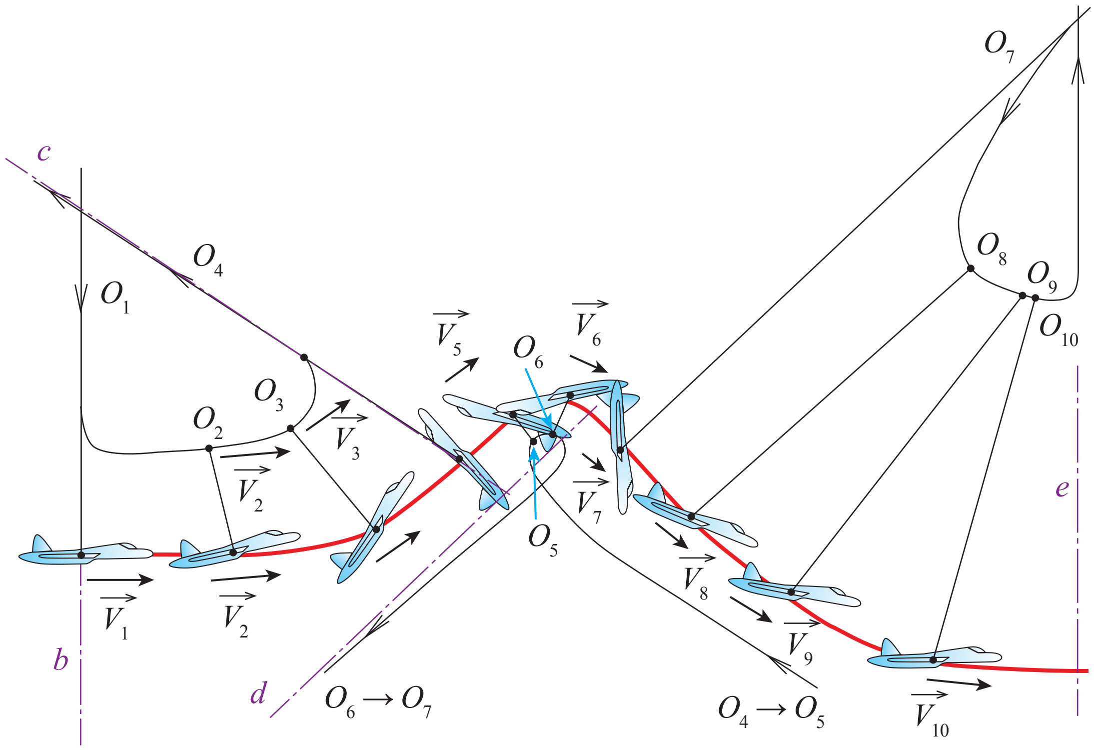


Fig. 3. Cross-connections of aircraft's IAAV projection on the plane of rotation while making a manoeuvre [4]

of acceleration of translational motion in relation to its BCS coordinates is determined as the distribution of aircraft acceleration of uniformly balanced forces (DAUF) acting on the aircraft.

For a clearer representation of distributions of velocities (accelerations) of aircraft points, in addition to their boundaries in the form of velocity (acceleration) manoeuvres, for example, $[\Phi^{V1}_{A}]_{ATT}$, we will show the boundaries of distributions of velocities (accelerations) for the typical section of the aircraft $[\Phi^{V1}_{A}]_{Ox}$, that belongs to axis Ox of the aircraft's BCS.

To determine the DANF calculation algorithm, let us consider a simpler example for determining such a distribution – determination of the DAUF boundary $[\Phi^{a1}_{A}]_{Ox}$ in the Type 2 MLCS for the aircraft performing translational motion only (Fig. 4a) at moment of time t_0. According to initial data, at moments of time t_0, t_1, t_2 the aircraft is located in position corresponding to the manoeuvres F, G, H, respectively, where $t_2 - t_1 = t_1 - t_0 = \Delta t$.

Based on the fact that the acceleration of the aircraft point can be determined

$$a_{y1}(x) = \lim_{\Delta t \to 0} \frac{V_{y2}(x) - V_{y1}(x)}{\Delta t},$$

using the DAUF condition for its typical section at moment of time t_0 when it is determined using the Euler method (for this purpose, velocity values shall be used in scale $-1/\Delta t$) for the aircraft being in position F is determined by the expression (Fig. 4b):

$$[\Phi^{a1}_{F}]_{Ox} = \frac{1}{\Delta t}([\Phi^{V2}]_{Ox} - [\Phi^{V1}]_{Ox}).$$

Theorem 2

Formulation. For any moment of time of aircraft motion, resolving the aircraft's angular acceleration relative to any centres (axes) of reduction which do not align with the IAAA in the RM is not an equivalent transformation. The angular acceleration of a manoeuvring aircraft can be accurately determined only relative to the IAAA.

Problem statement. With the aircraft (missile) in three positions (manoeuvres A, B and C, Fig. 5a), recorded at moments of time t_0, t_1 and t_2, for moment of time t_0 the aircraft acceleration is determined in relation to the aircraft's BCS coordinates in the Type 1 MLCS.

For unambiguous determination of aircraft's rotational motion according to Theorem 1 using the method [2, p. 240] in order to find the position of the Type 1 MLCS for t_0 and t_1 we determined the positions of aircraft IAAV that pass perpendicularly to planes of rotation α and β intersecting at points O_1 and O_2, respectively. Typical sections of manoeuvres A, B and C in Figure 5a are shown as sections KN, K_1N_1, K_2N_2.

We should prove that the aircraft's angular acceleration being constant at time interval $\Delta t = t_1 - t_0$ can be determined without error only relative to the single IAAA.

Proof. As possible motion of aircraft points as shown in Figure 5 is presented as rotation relative to the certain centre, the velocity distributions in question are not considered linear distributions. In this case, the term "sector distributions of aircraft's rotational motion" is more preferable. Hereinafter we will designate these

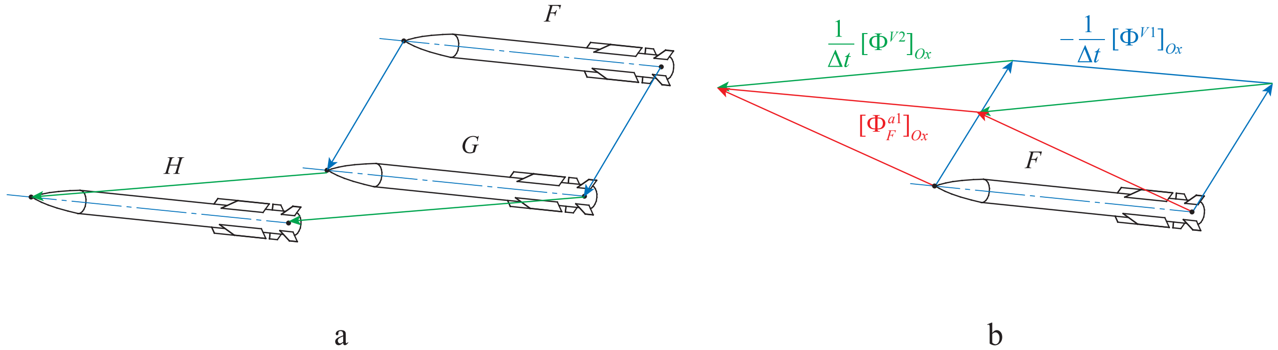


Fig. 4. Representation of the action of aircraft's linear acceleration in the form of DAUF

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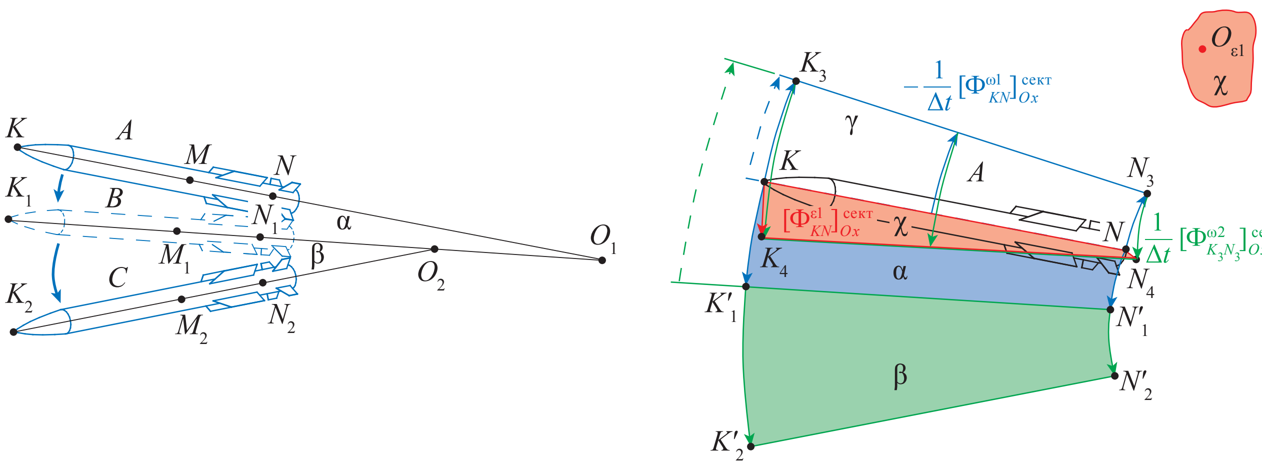

a b

Fig. 5. Representation of the action of aircraft’s angular acceleration in the form of DANF

distribution using the upper-case index $^{\text{сект}}$, for example $[\Phi^{\omega1}_{KN}]^{\text{сект}}_{Ox}$.

That is why, similarly to the notions of distributions of aircraft’s velocity and acceleration, velocity and acceleration manoeuvres [4], we introduce the notions of distribution of aircraft’s angular velocity and angular acceleration, and manoeuvres of angular velocity and angular acceleration of the aircraft.

The distribution of aircraft’s angular velocity is the sector distribution of aircraft’s angular movement, according to which the spatial set of distribution boundary points is defined for the points of actual position of the aircraft (base) according to their possible angular movement per the time unit.

The distribution of aircraft’s angular acceleration is the sector distribution of aircraft’s angular movement, according to which the spatial set of distribution boundary points is defined for the points of actual position of the aircraft (base) according to the predicted increment of aircraft’s angular velocity per the time unit.

Angular velocity manoeuvre $\Phi^{\omega i}_{H}$ with base H (lower-case index of symbol $\Phi^{\omega i}_{H}$) at moment of time t_i is the boundary of aircraft’s angular velocity distribution as an indivisible set of points which characterizes the aircraft position per the time unit after the predicted angular movement.

Angular acceleration manoeuvre $\Phi^{\varepsilon i}_{D}$ with base D at current moment of time t_i is the boundary of aircraft’s angular acceleration as an indivisible set of points which characterizes the predicted increment of aircraft points per the time unit.

To prevent the substitution of aircraft’s rotational motion parameters for translational motion parameters, the motion analysis by resolving the acceleration vector of the aircraft point into tangential acceleration a_τ and normal acceleration a_n of this point [3, p. 110] is not used. For the analysis, the described sector distributions of aircraft’s angular velocities and accelerations are used.

By difference in distributions of angular velocities $[\Phi^{\omega1}_{KN}]^{\text{сект}}_{Ox}$ and $[\Phi^{\omega2}_{KN}]^{\text{сект}}_{Ox}$ taken in scale $1 / \Delta t$ (where Δt is the time interval between the determination of these angular velocities), let us determine an analytical expression for DANF $[\Phi^{\varepsilon1}_{KN}]^{\text{сект}}_{Ox}$ (Fig. 5b):

$$[\Phi^{\varepsilon1}_{KN}]^{\text{сект}}_{Ox} = \frac{[\Phi^{\omega2}]^{\text{сект}}_{Ox} - [\Phi^{\omega1}]^{\text{сект}}_{Ox}}{\Delta t},$$

where $[\Phi^{\varepsilon1}_{KN}]^{\text{сект}}_{Ox}$ – DANF with section KN at the base (with the typical section along the Ox-axis of the aircraft being in position A at moment of time $t_0 = 1$).

Let us determine the DANF spatial position $[\Phi^{\varepsilon1}_{KN}]^{\text{сект}}_{Ox}$.

1. In plane α in angular velocity scale $1 / \Delta t$, let us determine the distribution $-[\Phi^{\omega 1}_{KN}]^{\text{сект}}_{Ox} / \Delta t$ as the the distribution opposite to angular velocity distribution $[\Phi^{\omega 1}_{KN}]^{\text{сект}}_{Ox}$ (shown with a blue arrow in Fig. 5b). The obtained distribution boundary is designated as section K_3N_3.

2. By turning in the clockwise direction we reduce the angular velocity distribution $[\Phi^{\omega 2}_{K_1N_1}]^{\text{сект}}_{Ox}$ determined in scale $1 / \Delta t$ until its base is aligned with section K_3N_3 in plane γ at γ || β (shown in green colour in Figure 5). We obtain sector distribution $[\Phi^{\omega 2}_{K'_3N'_3}]^{\text{сект}}_{Ox} / \Delta t$.

3. By summing sector distributions of angular velocity $-[\Phi^{\omega 1}_{KN}]^{\text{сект}}_{Ox} / \Delta t$ and $[\Phi^{\omega 2}_{K'_3N'_3}]^{\text{сект}}_{Ox} / \Delta t$ in plane χ we obtain distribution $[\Phi^{\varepsilon 1}_{KN}]^{\text{сект}}_{Ox}$ by the values of the aircraft's BCS coordinates with base *KN* (the distribution sector is shown in red colour in Figure 5).

4. Using the method described in the theorem [2, p. 240] for t_0, we determine the position of DANF IAAA $[\Phi^{\varepsilon 1}_{A}]^{\text{сект}}_{\text{ЛА}}$ with base *A*. For this case of aircraft motion (Fig. 5a) the axis under consideration, which passes through point $O_{\varepsilon 1}$, is located perpendicularly to plane χ (Fig. 5b).

For proof by contradiction we assume that $[\Phi^{\varepsilon 1}_{A}]^{\text{сект}}_{\text{ЛА}}$ can be resolved into distributions of aircraft's linear acceleration $[\Phi^{a1}]^{*}_{\text{ЛА}}$ and aircraft's angular acceleration $[\Phi^{\varepsilon 1}]^{\text{сект}*}_{\text{ЛА}}$ relative to a random centre:

$$[\Phi^{\varepsilon 1}_{A}]^{\text{сект}}_{\text{ЛА}} = [\Phi^{\varepsilon 1}]^{\text{сект}*}_{\text{ЛА}} + [\Phi^{a1}]^{*}_{\text{ЛА}}. \quad (3)$$

However, based on the condition that $[\Phi^{\varepsilon 1}_{A}]^{\text{сект}}_{\text{ЛА}}$ can be resolved into components,

$$[\Phi^{\varepsilon 1}_{KN}]^{\text{сект}}_{Ox} = \frac{[\Phi^{\omega 2}]^{\text{сект}}_{Ox} - [\Phi^{\omega 1}]^{\text{сект}}_{Ox}}{\Delta t},$$

its components $[\Phi^{\omega 1}_{A}]^{\text{сект}}_{\text{ЛА}}$ and $[\Phi^{\omega 2}_{A}]^{\text{сект}}_{\text{ЛА}}$ can be resolved into the similar sums (3) as well:

$$[\Phi^{\omega 1}_{A}]^{\text{сект}}_{\text{ЛА}} = [\Phi^{\omega 1}_{A}]^{\text{сект}*}_{\text{ЛА}} + [\Phi^{V1}_{A}]^{*}_{\text{ЛА}}$$
$$\text{and } [\Phi^{\omega 2}_{A}]^{\text{сект}}_{\text{ЛА}} = [\Phi^{\omega 2}_{A}]^{\text{сект}*}_{\text{ЛА}} + [\Phi^{V2}_{A}]^{*}_{\text{ЛА}}. \quad (4)$$

However, as shown in theorem 1, resolving aircraft's angular velocities $[\Phi^{\omega 1}_{A}]^{\text{сект}}_{\text{ЛА}}$ and $[\Phi^{\omega 2}_{A}]^{\text{сект}}_{\text{ЛА}}$ into components of transitional and rotational motion will lead to misalignment of trajectories of transformed motion (4) and initial trajectories. Therefore, expression (3) defines the aircraft motion with a change of the initial motion trajectory. This comes into conflict with the criterion (1) and proves theorem 2 by proof by contradiction.

Similarly to conclusion 2 for theorem 2, we can show a variation in parameters of aircraft's initial and reduced manoeuvres.

Let us take a randomly selected DANF (for example, $[\Phi^{\varepsilon 1}_{A}]^{\text{сект}}_{\text{ЛА}}$) of a manoeuvring aircraft to the centre of reduction that does not belong to the IAAA of the DANF (for example, according to expression (3)). As a result, we will obtain distributions of linear and angular acceleration in relation to the aircraft coordinates. The resulted and reduced states of aircraft motion being determined relative to different IAAAs have different values of angular accelerations of aircraft points ($\varepsilon^{\text{тек}}_{\text{ЛА}} \sim$ volume $[\Phi^{\varepsilon 1}_{A}]^{\text{сект}}_{\text{ЛА}}$ and $\varepsilon^{\text{тек}*}_{\text{прив}} \sim$ volume $[\Phi^{\varepsilon 1}_{A}]^{\text{сект}*}_{\text{ЛА}}$). During angular movement from similar initial positions to similar end positions of the aircraft, this defines that the states of aircraft motion characterized by these positions are not considered equivalent. Reduction of the DANF to the centre of reduction, which does not belong to the IAAA of the DANF, leads to a change of the aircraft motion state, thus proving theorem 2.

Thus, any reduction of the DANF relative to the selected centre of reduction that does not belong to the axis, relative to which the angular acceleration of aircraft points is acting in the RM at a given moment of time (i.e. relative to the current IAAV), leads to non-equivalent resolving of the DANF.

As the obtained DANF shown in Figure 5b (the DAUF shown in Fig. 4b) is obtained by single rotation (single parallel translation), we may conclude that for any moment of time the boundaries of these distributions in relation to the base (the aircraft manoeuvre) remain unchanged in terms of sizes, and therefore, in terms of shape.

Maintaining constant sizes of manoeuvres of linear and angular velocities as well as linear and angular accelerations suggests that all values of distributions of velocities and acceleration for aircraft points located on the same straight line can be described as a direct proportion of the base coordinates. In particular, the boundaries of aircraft’s DAUF determined for typical sections of the aircraft can be characterized by the position of straight lines $a_y^{\text{рс}}(x)$, $a_z^{\text{рс}}(y)$, $a_x^{\text{рс}}(z)$ that contain the said boundaries:

$$a_y^{\text{рс}}(x) = a_{y0}^{\text{рс}};\ a_z^{\text{рс}}(y) = a_{z0}^{\text{рс}};\ a_x^{\text{рс}}(z) = a_{x0}^{\text{рс}}, \qquad (5)$$

where $a_{y0}^{\text{рс}}$, $a_{z0}^{\text{рс}}$, $a_{x0}^{\text{рс}}$ – values of linear acceleration of the distribution border at all points of this distribution.

Similarly, the DANF boundaries determined for typical sections of the aircraft can be characterized by the position of straight lines $a_y^{\text{нс}}(x)$, $a_z^{\text{нс}}(y)$, $a_x^{\text{нс}}(z)$ that contain the said sections:

$$\begin{aligned} a_y^{\text{нс}}(x), &= k_{y(x)}^{\text{нс}}\,(x - x_0) + a_{y0}^{\text{нс}},\\ a_z^{\text{нс}}(y), &= k_{z(y)}^{\text{нс}}\,(y - y_0) + a_{z0}^{\text{нс}},\\ a_x^{\text{нс}}(z), &= k_{x(z)}^{\text{нс}}\,(z - z_0) + a_{x0}^{\text{нс}}, \end{aligned} \qquad (6)$$

where $k_{y(x)}^{\text{нс}}$, $k_{z(y)}^{\text{нс}}$, $k_{x(z)}^{\text{нс}}$ – factors of proportionality of straight lines containing the DANF boundaries of typical sections; $a_{y0}^{\text{нс}}$, $a_{z0}^{\text{нс}}$, $a_{x0}^{\text{нс}}$ – values of DAF boundaries at aircraft points with zero values of the BCS x-axes; x_0, y_0, z_0 – values of straight line shift from the position of the distribution boundary above its base.

Before presenting the next lemma, we will give some explanations related to the used terms and designations.

According to the lemma, the linearisation of parameter distribution with respect to the aircraft’s BCS coordinates is the distribution transformation that defines smoothing of their values by values to linear functions that are determined from the aircraft’s BCS coordinates while maintaining the parameters of initial distribution acting on aircraft dynamics.

Maintaining the parameters acting on aircraft dynamics means maintaining the direction and modulus of the resultant force for uniformly balanced distribution of the parameter along the axis of the aircraft’s BCS as well as the modulus, plane and axis of action of the parameter along the axes.

For further investigation of UFs and NFs on the aircraft, we represent Newton’s second law in [m/s^2] for uniform and non-uniform actions of external forces on the aircraft ($F_{Ox}^{\text{рс}}(i)$ and $F_{Ox}^{\text{нс}}(i)$) to be determined at the i-th moment of time:

$$a_{\text{ЛА}}^{\text{рс}}(i) = \frac{F_{\text{ЛА}}^{\text{рс}}(i)}{m_{\text{ЛА}}(i)};\ \ a_{\text{ЛА}}^{\text{нс}}(i) = \frac{F_{\text{ЛА}}^{\text{нс}}(i)}{m_{\text{ЛА}}(i)}. \qquad (7)$$

where $a_{\text{ЛА}}^{\text{рс}}(i)$, $a_{\text{ЛА}}^{\text{нс}}(i)$ – aircraft accelerations caused by the action of UF and NF at the i-th moment of time, $m_{\text{ЛА}}(i)$ – aircraft mass recorded at the i-th moment of time.

We should note that spatial distributions can be used to represent not only DAUF (2) and DANF (4), but also distributions of aircraft’s UF, NF and mass. The only difference is that the boundaries of such distributions cannot always be represented in the form of solid indivisible figures.

For the aircraft located in the position of manoeuvre T at the i-th moment of time this allows to represent (7) in the form of DAUF $[\Phi_T^{a\,i}]_{Ox}$ and DANF $[\Phi_T^{a\,\text{нс}\,i}]_{Ox}^{\text{сект}}$ with a typical section at the base (expressed in [m/s^2]):

$$[\Phi_T^{a\,i}]_{Ox} = \frac{[\Phi_T^{F\,i}]_{Ox}}{[\Phi_T^{m\,i}]_{Ox}};\ \ [\Phi_T^{a\,\text{НС}\,i}]_{Ox}^{\text{сект}} = \frac{[\Phi_T^{F\,\text{НС}i}]_{Ox}^{\text{сект}}}{[\Phi_T^{m\,i}]_{Ox}}. \qquad (8)$$

where $[\Phi_T^{F\,i}]_{Ox}$, $[\Phi_T^{F\,\text{нс}\,i}]_{Ox}^{\text{сект}}$, $[\Phi_T^{m\,i}]_{Ox}$ – distributions of UFs, NFs and mass of the aircraft with a typical section at the base, determined at the i-th moment of time.

Lemma

For any random moment of time i of NF distribution $[\Phi_T^{F\,\text{нс}\,i}]_{\text{ЛА}}^{\text{сект}}$ and UF distribution $[\Phi_T^{F\,i}]_{\text{ЛА}}$ by aircraft coordinates, as well as aircraft mass distribution $[\Phi_T^{m\,i}]_{\text{ЛА}}$ by these aircraft coordinates upon the influence on the aircraft manoeuvre dynamics determined in relation to a rigid body, are linearised depending on the functions of these distributions.

Proof. Distributions of angular and linear accelerations by the aircraft’s BCS coordinates $[\Phi_T^{a\,\text{нс}\,i}]_{Ox}^{\text{сект}}$ and $[\Phi_T^{a\,i}]_{Ox}$ with the boundaries characterized by linear functions (6) and (5) as per (8), are the quotients. This defines the proportionality of distributions of the numerator and the denominator, i.e. the proportionality of NF and UF distributions and aircraft mass distribution.

The paradox of relations between the right members in expression (8) is the obviousness that non-uniform UF distribution $[\Phi_T^{F\,i}]_{ЛА}$ and non-uniform NF distribution $[\Phi_T^{F\,нс\,i}]_{ЛА}^{сект}$ at any time during manoeuvring cannot be proportional to non-uniform aircraft mass distribution $[\Phi_T^{m\,i}]_{ЛА}$.

This paradox can be explained if we understand how for a non-uniform AEF acting on the body of a manoeuvring aircraft the boundaries of its DANF and DAUF for any moment of time maintain their shape and sizes unchanged in relation to their base.

The properties of the aircraft represented as a rigid body may serve as a rationale for this discrepancy. In this case, this is the property of linearisation of the action on the aircraft's NF and UF as well as linearisation of the inertial effect of the non-uniform aircraft mass distribution with respect to its BCS coordinates with coordinate-related initial distributions unchanged. The property can be applied to both NF and UF at any moment of time of aircraft motion under consideration, which was to be proved according to the lemma conditions.

NF and UF distributions determined with regard to their linearisation with respect to the aircraft's BCS coordinates will be referred to as acting NF and UF distributions of the aircraft. The aircraft mass distribution with regard to this process will be referred to as the influencing mass distribution of this aircraft.

The property of linearisation (alignment) of force distribution values with respect to the aircraft's BCS coordinates can be explained by means of mutual compensations and redistributions of non-uniformly acting external forces on the aircraft body.

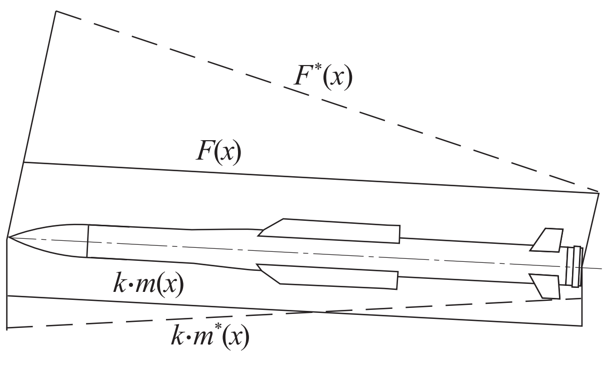

Fig. 6. Acting distribution of aircraft's UFs and influencing aircraft mass distribution

Solving the problem of dynamics for non-uniform distributions of NF, UF and mass in RM is a challenging task. Using the property of distribution linearisation facilitates the determination of the aircraft's DAUF and DANF when solving the problem.

Using the aircraft motion that takes place in plane *Oxy* as an example, we represent the obtained results in geometric interpretation (Figs. 6, 7).

The following designations in Figures 6 and 7: $k \bullet m(x)$, $k \bullet m^*(x)$ are influencing aircraft mass distributions presented for different cases. The functions values are scaled for clarity, where k – scale of functions $m(x)$ and $m^*(x)$; $F^{нс}(x)^{сект}$ and $F^*(x)$ – sector and linear distributions of acting forces, with the aircraft's typical section at the base.

The event that triggers only linear acceleration of the aircraft with mass distribution $m(x)$ uniformly acting with respect to these coordinates is uniform force distribution $F(x)$ acting with respect to these coordinates (Fig. 6). With non-uniformly influencing mass distribution $m^*(x)$ with respect to aircraft coordinates, such an event is the acting force distribution $F^*(x)$ proportional by values of distribution $m^*(x)$ (Fig. 6). We should note that in this case $F^*(x)$ is not a sector distribution but linear distribution because it forms a linear acceleration.

The action of any NF is a necessary condition for transition from the motion with linear acceleration to the motion with aircraft's angular acceleration with uniformly influencing mass distribution with respect to aircraft coordinates. For example, for the aircraft with uniformly influencing distribution of mass $m(x)$ along the BCS axis Ox, as shown in Figure 6, it may be the action of NF with acting distribution $F^{нс}(x)^{сект}$ (Fig. 7).

One of the ways for transition from the motion with aircraft's angular acceleration $a(x)^{сект}$ to the motion with linear acceleration with the aircraft exposed to the same force distribution $F^{нс}(x)^{сект}$ is a change of the aircraft mass distribution to the uniformly influencing distribution $m^*(x)$ with respect to aircraft coordinates (Fig. 6).

Values of influencing aircraft mass distribution shall be proportional to the corresponding values of acting force distribution $F^*(x)$. Thus, the aircraft motion with angular acceleration may degenerate, i.e. change to the special case with the IAAA being at infinity. Such a sector distribution will be referred to as the degenerated sector distribution of the aircraft's rotational motion parameter.

Using aircraft's velocity (angular velocity) distributions allows to clearly demonstrate complex transitions of the flight dynamics with the velocities changing. Figure 8 shows the result of the action of angular acceleration $\vec{\varepsilon}_2$ (relative to centre $O_{\varepsilon 2}$) as a transition from the motion with angular velocity (relative to initial centre $O_{\omega 2}$) to the aircraft motion with degenerated distribution of aircraft's angular velocity $\vec{\omega}_2$, i.e. to the aircraft motion with linear velocity $\vec{V}_3$.

If we give no consideration to the sphericity of NF distribution and consider only the position of the distribution boundary, there is misinterpretation that a constant component can be singled out in this distribution. This is the key factor that defines the classical dynamics assumption [3, 6] that the action of NFs can be represented as a simultaneous action of forces that form linear and angular aircraft acceleration (i.e. NF and UF). However the properties of sector distribution significantly differ from linear distribution, thus excluding the said possibility.

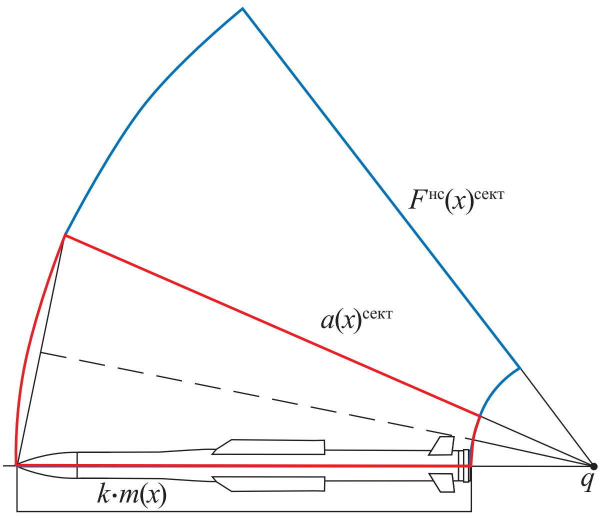

Fig. 7. Acting distribution of aircraft's NFs with uniform influencing mass distribution

We should note that resolving the AEF into the action of NF and UF is possible only for the known aircraft mass distribution at the current moment of time. With the aircraft mass changing, the NFs may change to the UFs and vice versa.

Theorem 3.

Formulation. The action of NF on a manoeuvring aircraft with current mass distribution m (x, y, z) on the axes of its BCS at any moment of time takes place only relative to a single IAAF.

Problem statement. For the aircraft motion with non-uniform mass distribution m (x, y, z) on the axes of its BCS and with the known acting NF distribution at the current moment of time, it is necessary to prove that the IAAF for these forces may exist in one position only.

For proving, we can use the properties of the DANF considered under theorem 2 or the properties of forces acting on the RB.

Proof 1. To prove that only one IAAF may exist for every moment of time during aircraft manoeuvring, we can use the property of the action of its DANF relative to the single IAAV for this moment of time. According to expression (8), the only distribution that may change the IAAF position is the aircraft mass distribution.

Considering different variants, it is easy to prove that for uniform influencing aircraft mass distribution, the IAAF aligns with the IAAA. Proportional change of the lengths of the arcs of acting distribution of NF and DANF that interconnect the base points and the boundaries of these sector distributions results in mutual alignment of the IAAV and IAAF positions (Fig. 7). This may be proven analytically by determining a

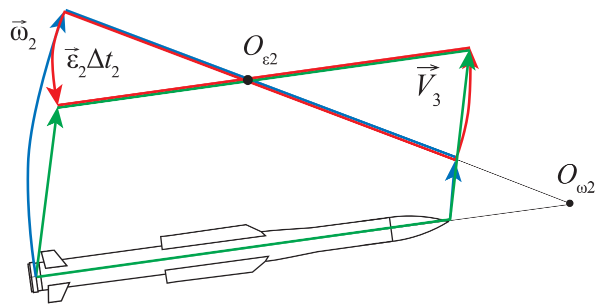

Fig. 8. Aircraft transition to motion with linear velocity (expressed in [rad/s])

proportional decrease in the length of the straight line that contains the boundary of acting NF distribution and the straight line that contains the DANF boundary of the aircraft's typical section, to the point with the y-coordinate equal to 0, being common for both straight lines.

In case of non-uniform influencing mass distribution, the resulted force distribution (8) is determined not only by acceleration distribution (angular acceleration values) "expanded" along the ordinate axis, but also by proportional shift of the boundary of this variation along the abscissa axis. That is why, for this distribution the sections connecting the base points and distribution boundaries remain parallel to each other. Consequently, this proves the existence of a single IAAF q^* for this distribution, not coinciding with IAAV q. For distribution of the selected aircraft's NF acceleration $a(x)^{\text{сект}}$ shown in Figure 7, Figure 9 shows sector distribution of aircraft NF $F^{\text{нс}*}(x)^{\text{сект}}$,determined for the aircraft with non-uniform distribution of the influencing mass $k \cdot m'(x)$.

Proof 2. The uniqueness of IAAF can be proved through step-by-step determination of processes of NF and UF formation from the cumulative AEF on a manoeuvring aircraft.

To identify the ongoing processes, we assume that the aircraft is a system of elementary particles (EP) which are so small that the density of each particle can be considered homogeneous. The whole initial non-uniform AEF on the aircraft formed by three types of differently directed forces of different nature can be divided into a system of vectors, each applied to the centre of aircraft's EPs and having the modulus characterizing the

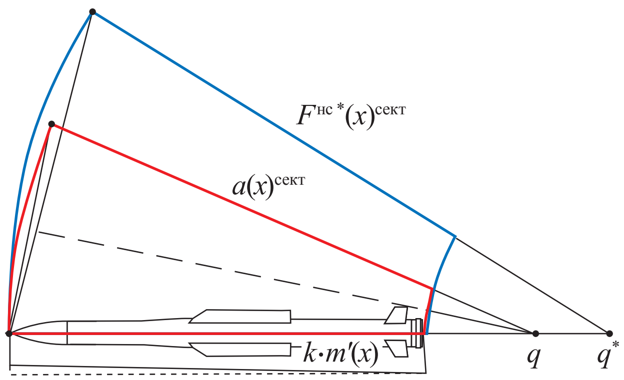

Fig. 9. Acting distribution of aircraft's NFs with non-uniform influencing mass distribution

cumulative action of the described forces on the EP.

Since there is a resultant action of forces of all the aircraft's EPs for each moment of time in different directions, it would be reasonable to assume that in one direction their resultant action will become greater than those in other directions, and its prevailing action will become the action that forms the resultant action of the forces described.

Another reasonable assumption is that the resultant cumulative action of forces in other direction will correct the parameters of the forming vector to match the parameters of resultant vector $\vec{F}_{\text{PC}}$ of the whole system of forces acting on the aircraft.

We should note that not every vector of force acting on the aircraft's EPs takes part in formation of resultant vector $\vec{F}_{\text{PC}}$, which will form the aircraft's linear acceleration at a given moment of time.

That is why, the expression

$$\vec{F}_{\sum} = \sum_{1}^{n} \vec{F}_i ,$$

where $\sum_{1}^{n} \vec{F}_i$ is the sum of all vectors of forces acting on the aircraft's EPs, cannot be used for determining its modulus.

Assuming that we know the parameters of resultant vector $\vec{F}_{\text{PC}}$, i.e. based on the inverse assumption, let us estimate which vectors of EP forces take part in its formation.

First of all, all the vectors being part of the straight lines with the same direction, which cross the point of application of resultant vector $\vec{F}_{\text{PC}}$ (PARV), take part in formation of resultant vector $\vec{F}_{\text{PC}}$.

Besides, such vectors should include a part of the system of vectors of forces directed similarly to the vector-containing straight lines, which do not cross the PARV. Such vectors take part in formation of resultant vector $\vec{F}_{\text{PC}}$ only when for each of them there is a vector of equal modulus that acts symmetrically to the current RARV $O_{\text{ТПРВ}}$ (for example, vectors applied to L and L_1 in

Figure 10), or a vector with the torque contra-directional to $O_{\text{ТПРВ}}$.

Another part of the above-mentioned system of vectors with the direction similar to that of the vector-containing straight lines that do not cross the PARV does not participate in formation of the resultant vector if their torque is not compensated by other vectors (i.e. those being part of NF). Figure 10 shows such a vector represented as the vector of force applied to point Q, which does not take part in formation of resultant vector $\vec{F}_{\text{РС}}$. Along with the direction of the vector-containing straight line which does not cross PARV $O_{\text{ТПР}}$, this vector does not have a paired vector (from the vectors shown in Figure 10) relative to centre of $O_{\text{ТПР}}$.

We should note that there are a lot of vectors that belong to the above-described type, which take part in formation of both non-uniform and uniformly balanced action of forces on the aircraft.

Since it is difficult to determine the paired relationship of all the vectors forming the action of UF based on the known distribution of AEF with the simultaneous action of NF, it is impossible to calculate the PARV position at once.

In this case, it is easier to initially determine the position of IAAF of the whole AEF (the initial system of differently directed forces) as well as the torque formed relative to the axis. Then we can determine the UF distribution (and, consequently, the parameters of resultant vector of forces $\vec{F}_{\text{РС}}$ aircraft) by subtracting the acting NF distribution from the total AEF distribution.

Let us prove that all NF vectors form the cumulative torque of the aircraft exactly relative to the PARV. Based on the fact that each NF vector can be transformed into the UF vector by applying an additional paired vector with the torque contra-directional to the PARV, to the aircraft, we may conclude that all the NF vectors act relative to the PARV. For example, Figure 10 shows the NF vector applied to T, which will become the UF vector if an additional paired vector applied to point T^* is applied to the aircraft.

Continuing the process description, we should note that the formed cumulative NF torque is not able to act relative to the point (RARV) only. At any moment of time under consideration, the action of NFs will have a greater torque in one of the planes in comparison with other planes. As a result, its action corrected by the action of NFs in other planes will form the resultant torque of all NFs. One of the parameters of the resultant torque is the position of its action plane, which contains the PARV and determines the IAAF position – the only axis of action of the torque.

The properties of NF and UF considered above indicate that a different nature of triggering events, plus different properties of aircraft's angular and linear acceleration caused by the action of these forces, prevent possible substitution of the action of some NFs for the action of some UFs when reduced to the axis (centre) of reduction (which does not coincide with the IAAF). As a result of such reduction, the obtained external torque will act relative to a new IAAF with the modulus not equal to the initial one. This indicates its non-equivalence to the initial external torque acting on the aircraft. Thus, the IAAF is the only axis of action of NFs for given conditions. Theorem 3 is proven.

The conclusions arising out of theorems 1–3 prove the main statement of the paper: aircraft's SM and RM can be considered equivalent as per criterion (1), and their parameters are identical time-dependant variables only if their IAAV, IAAA, and IAAF coincide at any time of aircraft motion.

The principle of aircraft flight dynamics simulation corresponding to this statement is as follows: aircraft manoeuvre simulation for every moment of time is performed not relative to the aircraft's CM, but with account for the positions of IAAV, IAAA, and IAAF of the aircraft motion.

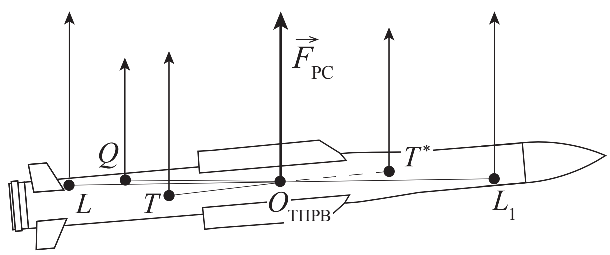

Fig. 10. NF and UF formation

Algorithmically, determination of the aircraft attitude per the simulation step with the known initial data (aircraft position, its IAAV position, distribution of external forces and mass with respect to its BCS coordinates) means:

– determination of aircraft's IAAF and NF torque relative to the axis;

– determination of acting NF and UF distributions, determination of influencing aircraft mass distribution;

– determination of DAUF and DANF parameters relative to the respective IAAA;

– determination of distributions of aircraft's linear and angular acceleration increments per the unit of simulation (consequently, determination of the IAAV position relative to the determined aircraft's angular velocity distribution);

– determination of the IAAV attitude during aircraft motion by moving the IAAV at the previous moment of time by the value of aircraft's linear velocity increment per the current simulation step;

– determination of the aircraft attitude during its motion using the following methods:

a) superimposition of the distribution of angular velocity increment per simulation step on the aircraft attitude during its motion with alignment of:

– determined spatial IAAV with the distribution IAAV;

– aircraft nose point attitude at the previous moment of time with the aircraft nose point in the distribution;

b) aircraft turn from the position of the alignment described above to the distribution of angular velocity increment per simulation step.

The fundamental difference of output data obtained using the proposed simulation method is that the obtained distributions of forces acting on the aircraft with respect to its BCS coordinates and, therefore, similar distributions of velocity and acceleration with respect to the aircraft coordinates, do not depend on the position of the centre of reduction of forces acting on the aircraft.

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Information about the author

Piskovatsky Andrey Anatolyevich – Candidate of Engineering Sciences, Design Engineer of the 1st category, PJSC "Dolgoprudny Research and Production Enterprise", Dolgoprudny, Moscow Region, Russian Federation.
Science research interests: flight dynamics, manoeuvrable aircraft control systems.