https://doi.org/10.38013/2542-0542-2022-3-22-31
UDC 621.391.96

A comparative analysis of methods for multiplexing of the multi-channel matched filter

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The paper presents a comparative analysis of the methods for multiplexing of the Doppler channels of the compression filter for resolvable targets with different distances, moving at different radial velocities. As an example, we consider signals with non-linear frequency modulation and signals with phase noise modulation sensitive to the Doppler frequency shift. To reduce losses, the intra-period processing of such signals requires a multi-channel arrangement related to this parameter. Simulation results prove that application of various methods for multiplexing is more efficient for signals with the intra-pulse modulation laws considered.

Keywords: multi-channel matched filter, non-linear frequency modulation, phase noise modulation, multiplexing, signal expansion

For citation: Elagina K. A. A comparative analysis of methods for multiplexing of the multi-channel matched filter // Journal of "Almaz – Antey" Air and Space Defence Corporation. 2022. No. 3. P. 22–31. https://doi.org/10.38013/2542-0542-2022-3-22-31

Для цитирования: Елагина К. А. Сравнительный анализ способов объединения каналов многоканального согласованного фильтра // Вестник Концерна ВКО «Алмаз – Антей». 2022. № 3. С. 22–31. https://doi.org/10.38013/2542-0542-2022-3-22-31

Submitted on 28.10.2021 *Reviewed on* 20.11.2021 *Approved on* 18.04.2022 *Published on* 24.06.2022

Introduction

A multi-channel Doppler frequency matched filter (MCMF) is required for signal detectors that detect targets moving at unknown radial velocity and are highly sensitive to the Doppler frequency shift. MCMF is also used to reduce losses during intra-period processing. In order to generate pre-threshold statistical data, multiplexing is normally conducted by selecting a maximum signal. In case of post-threshold multiplexing, output signals of threshold devices are combined by logical "OR" operation.

For signals with linear frequency modulation (LFM), for which nearly full invariance of the matched filter is enabled in relation to the Doppler frequency shift of the input signal, a single-channel Doppler frequency (radial velocity) compression filter is normally used. As a rule, such a filter is adjusted to the zero radial velocity. In this case, no multiplexing is required. If signals sensitive to the Doppler frequency shift (for example, signals with non-linear frequency modulation (NLFM) or with noise modulation (NM)) are used for detecting targets moving in a wide rage of radial velocities, it is impossible to leave out the urgent issue of compression filter multiplexing.

Without additional protection means, in particular, for signals with NLFM, in case of normal multiplexing of MCMF channels by applying a maximum selection pattern at each discrete value of range (range resolution element), there will be unwanted expansion of the resulting signal, plus spurious side lobes (SL) that are actually maximum signals from other MCMF channels. Such signals can be interpreted as false targets.

As a whole, protection from unwanted signal expansion together with generation of spurious SLs during multiplexing is important not only for MCMF, but also for other multi-channel devices. As an example, we can mention multi-channel target signal migration compensators. Such devices need protection from signal expansion and spurious SLs not only for NLFM signals sensitive to the Doppler shift [1], but also for signals with LMF [2]. As an example, the author uses a target signal migration compensator being integrated and arranged out of the compression filter [1, 2] in order to analyse the method of its multiplexing with protection from signal expansion. It is proposed to redefine the efficiency of the method for resolvable targets with different distances, moving at different radial velocities if MCMF is implemented without migration compensation when using signals with NLFM and phase NM sensitive to the Doppler shift. As a whole, the analysis described herein is intended to update the materials given in [1, 2], regarding the MCMF multiplexing performance assessment when using sounding signals with different intra-pulse modulation laws.

The purpose of the study is to conduct a comparative analysis and to select the most efficient method of MCMF multiplexing for NLFM signals and signals with phase NM under conditions either with single signals or with signals reflected from short-range targets moving at different radial velocities.

Methods of MCMF multiplexing

For clarity, below we will analyse the case with single-pulse probing. When a burst of pulses is processed to reduce losses in target detection, MCMF Doppler channels are subject to multiplexing after inter-period processing.

If MCMF multiplexing is implemented by the traditional method based on the maximum selection pattern, we should select the channel where the signal amplitude at the current *t*-th discrete value of range reaches its maximum:

$$y_t = \max_k \{|x_{tk}|\},\ k = 1 \ldots M, \tag{1}$$

where $|x_{tk}|$ – signal amplitude on the output of the MCMF *k*-th channel at the *t*-th discrete value of range, M – optimal number of MCMF channels to ensure a low and uniform side lobe level (SLL) within the entire range of Doppler frequencies with admissible losses in the dips of the MCMF's amplitude-frequency response (AFR).

The method of MCMF multiplexing with protection from expansion of the resulting signal is represented as follows [1–2]:

$$\begin{gathered} y_t = |x_{tK}| \\ K = k,\ \max_k \left\{ \max_l \{|x_{lk}|\} \right\} = \max_l \{|x_{lk}|\}, \\ l = t - N_w \ldots t + N_w,\ k = 1 \ldots M. \end{gathered} \tag{2}$$

According to (2), the output at the *t*-th discrete value of range receives the signal amplitude from the optimal channel of MCMF during MCMF multiplexing. Optimal channel means the *K*-th channel of MCMF with the maximum signal amplitude in the range-related sliding window. The window size $t - N_w \ldots t + N_w$ in relation to one coordinate corresponds to the maximum displacement of the peak of the ambiguity function of a signal (correlation peak) on the time (range) axis with a variable Doppler frequency, taking into account its both signs; in relation to the second coordinate, it corresponds to the number of MCMF channels. Number of samples N_w shall cover the range of shifts of range-related correlation peaks in all MCMF channels.

The method of MCMF multiplexing (2) can be applied to both narrowband signals and broadband signals (BBS). The main difference is the size of range-related sliding window N_w that depends on given signal parameters and the maximum radial velocity of a target being detected. Also, incoherent integration of range signal envelopes is additionally required for BBS. For clarity, we assume BBS is reflected from one bright point of the target.

According to the author's calculations, the correlation peak shift (expressed in discrete values of range with account for rounding) for the NLFM signal [1] with Doppler frequency F_d exceeds $[f_d \times T \times F_d / W]$ approximately by 2 times, that is why for certainty we assume the maximum shift of range correlation peak N_w equal to $2 \times [f_d \times T \times F_d^{MAX} / W]$, T – signal duration, F_d^{MAX} – maximum Doppler frequency, f_d – sampling frequency, W – spectrum width; square brackets designate the rounding operation. The shift of the peak of LFM signal matched filter's response envelope with the Doppler shift of input signal is equal to $[f_d \times T \times F_d / W]$ [3].

Figure 1 shows the block diagram of a signal detector path section for a signal with NLFM and NM reflected from a point reflector moving at an unknown radial velocity, with the maximum selector unit and with/without protection from signal expansion. The noise-triggered false alarm level stabilization (FALS) circuit and the threshold device (TD) are not shown in Figure 1.

Modelling results

As an example of the sounding signal, we used the NLFM signal discussed in [1]. The phase of the NLFM signal reflected from a moving object is represented as follows:

$$\varphi(t) = \frac{W}{T \times k_2} \times \left(\frac{t^2 \times k_2 \times \pi - t \times T \times k_2 \times \pi}{1 + 2 \times k_1 \times \mathrm{tg}(0.5 \times k_2 \times \pi)} \right) + \frac{W}{T \times k_2} \times$$

$$\times \left(\frac{k_1 \times T^2 \times \ln\left(1 + \mathrm{tg}\left(0.5 \times k_2 \times \pi \times \frac{(-2 \times t + T)}{T}\right)^2\right)}{1 + 2 \times k_1 \times \mathrm{tg}(0.5 \times k_2 \times \pi)} \right) \pm$$

$$\pm 2 \times \pi \times F_d \times t, \quad 0 \le t \le T$$

where F_d – Doppler frequency, k_1, k_2 – FM nonlinearity coefficients.

Optimal values k_1, k_2 are obtained by the brute force method based on the SLL minimum criterion, with the equal width of main lobes (ML) of compressed NLFM and LFM signals with intra-period processing (weighted with the help of the Hamming window with a relative pedestal of 0.08).

Sounding signal parameters: T – 80 μs, spectrum width W – 2.5 MHz, sampling frequency f_d – 9 MHz, Doppler frequency F_d – 12 kHz (radial velocity – 600 m/s). MCMF channels were uniformly distributed in a wide Doppler frequency range of –160…160 kHz (for carrier frequency of 3 GHz, target radial velocities of –8000…8000 m/s).

Figures 2 and 3 show the signal envelopes in MCMF channels and the result of channel multiplexing according to (1), respectively. For clarity, Figure 2 shows 9 MCMF channels only, Figure 3 – for 9, 17, and 33 channels. Numbers of formulas of MCMF multiplexing methods are given in brackets in the legend as shown in the figures.

According to Figures 2 and 3, in MCMF without additional protection the resulting signal

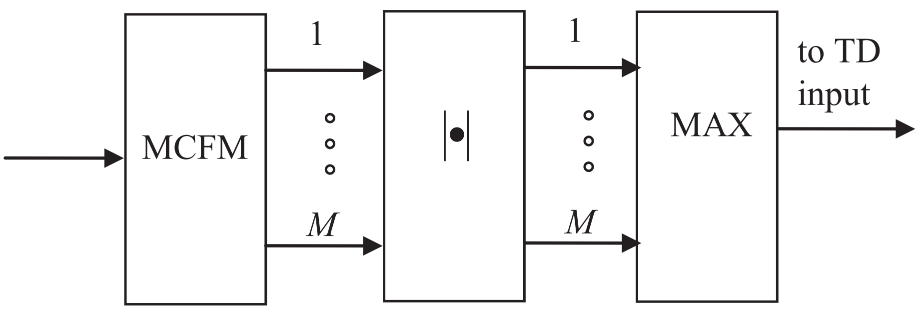

Fig. 1. Block diagram of the signal detector with MCMF:
MCMF – multi-channel matched filter, MAX – maximum selector unit with/without protection against signal expansion according to (1)–(2)

envelope actually describes signal envelopes in each MCMF channel individually, since the maximum selection pattern switches between channels at each discrete value of range.

Figure 4 illustrates the method efficiency (2), showing MCMF output signal envelopes after multiplexing by means of the maximum selection pattern without (1) and with (2) protection from signal expansion. 33 Doppler channels were used to provide the low SLL within the entire Doppler frequency range with admissible average losses in the dips of the MCMF’s amplitude-frequency response (AFR) (not more than 0.5 dB).

For a more comprehensive check of the MCMF multiplexing methods efficiency, the ML width was estimated (expressed in discrete values, by the level of –3 dB and –15 dB) of the compressed NLFM signals with the previously analysed parameters after multiplexing 33 MCMF channels in accordance with (1) and (2). The obtained relationship between the ML width and the Doppler shift of the input signal in the range of –160...160 kHz (with the increment of 1 kHz (50 m/s)) is shown in Figure 5.

Figure 5 shows that for the method (2) the width of the compressed signal’s ML is almost constant within the entire Doppler frequency range. The signal expansion effect is neutralized after MCFM multiplexing.

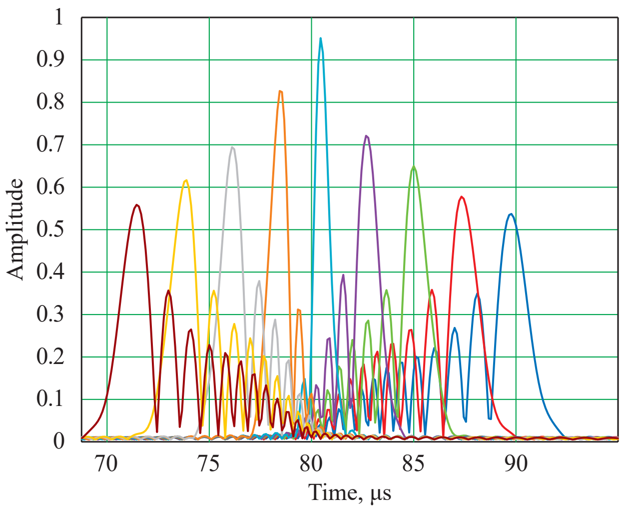

Fig. 2. Signal envelope

– 1st channel; – 2nd channel; – 3rd channel; – 4th channel; – 5th channel; – 6th channel; – 7th channel; – 8th channel; – 9th channel

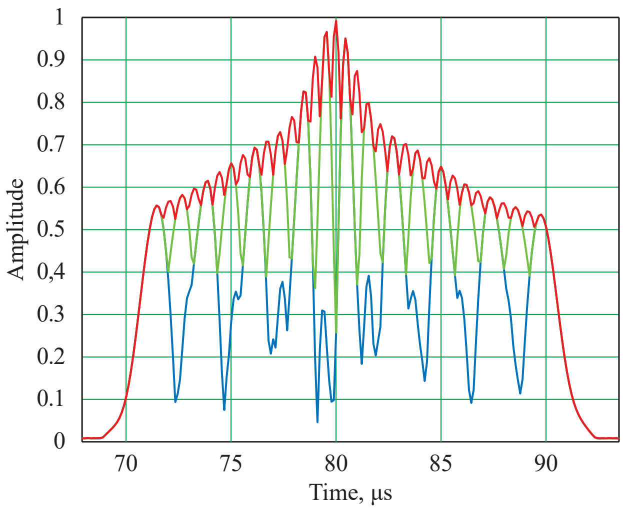

Fig. 3. Signal envelope

– (1) MCMF (9 channels); – (1) MCMF (17 channels); – (1) MCMF (33 channels)

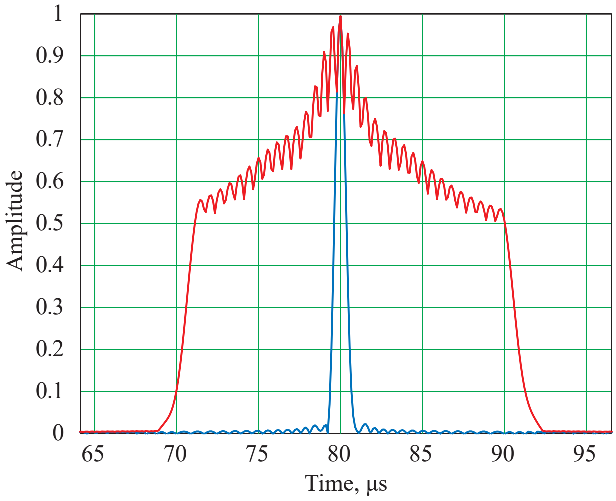

Fig. 4. Signal envelope

– (2); – (1)

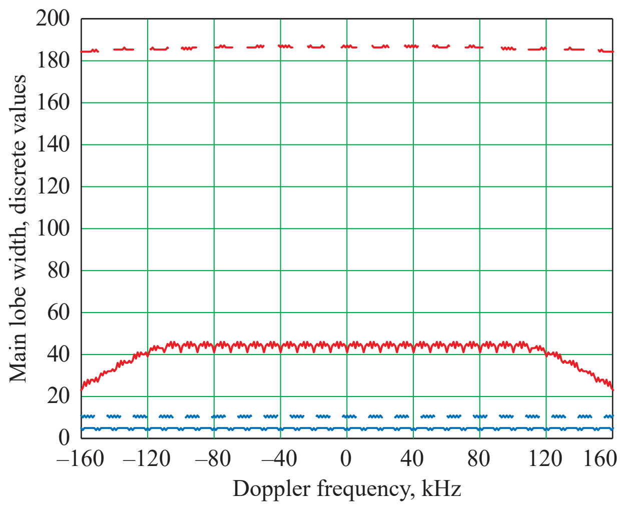

Fig. 5. ML width – Doppler frequency relationship

– (1) (–3 dB); – (1) (–15 dB); – (2) (–3 dB); – (2) (–15 dB)

Although the method of MCMF multiplexing (2) seems to be effective, it has a certain drawback – possible losses when detecting a weak signal from one of the short-range targets moving at different radial velocities. With a larger signal spacing by range, this effect is neutralized. To illustrate both cases with different signal spacing by 46 and 107 discrete values (N_w – 92 discrete values), Figure 6 and 7 show an example of signal envelopes after compression and multiplexing of 33 MCMF Doppler channels for time-dependant overlapping NLFM signals reflected from short-range targets moving at different radial velocities (12 and 60 kHz). For simplicity, calculations are given for two signals of the same amplitude, irrespective of the noise. With signal amplitudes being equal, the signal is considered the maximum one if it matches with one of the MCMF Doppler channels to the maximum extent.

According to Figures 6 and 7, with input signal spacing by 46 discrete values as a result of processing in the unmatched MCMF channel (which is optimal for the signal from the second target), the signal from the first target for (2) is considerably attenuated and distorted. This effect is neutralized with signal spacing by 107 discrete values. For short-range targets with near values of radial velocities for (2) with enabled protection from signal expansion, losses in signal detection caused by mismatching of Doppler frequencies of the optimal MCMF channel and the input signal will be minimized.

For a more comprehensive analysis of the efficiency of the detector (Figure 1) that detects a signal reflected from a point target moving at an unknown radial velocity, and 33-channel NLFM signal MCMF, we plotted target detection curves and estimated signal-to-noise threshold ratios corresponding to the probability of correct detection equal to 0.5. The cases of non-fluctuating signal (Swerling 0 model) and fluctuating signal according to the Swerling target model (Swerling 1 – Swerling 4 models) are considered. For single-pulse probing and methods of MCMF multiplexing (1)–(2), Figures 8 and 9 show simulation-based target detection curves (input signal Doppler frequency – 0 Hz) and dependencies of signal-to-noise threshold ratios averaged within the entire range of target radial velocities, on the number of signal fluctuation model, respectively. The probability of false alarm (PFA) caused by noise was equal to 10^{-6} (for the example shown in Figure 8) and 10^{-6}...10^{-4} (for the example shown in Figure 9), the number of experiment repetitions for plotting detection curves used to read out signal-to-noise threshold ratios was equal to 10^3. For comparison, Figure 8 also shows the results for the single-channel compression filter.

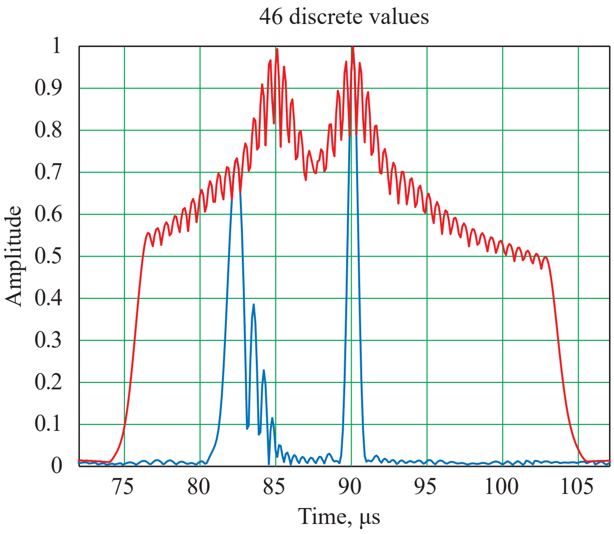

Fig. 6. Signal envelope
– (2); – (1)

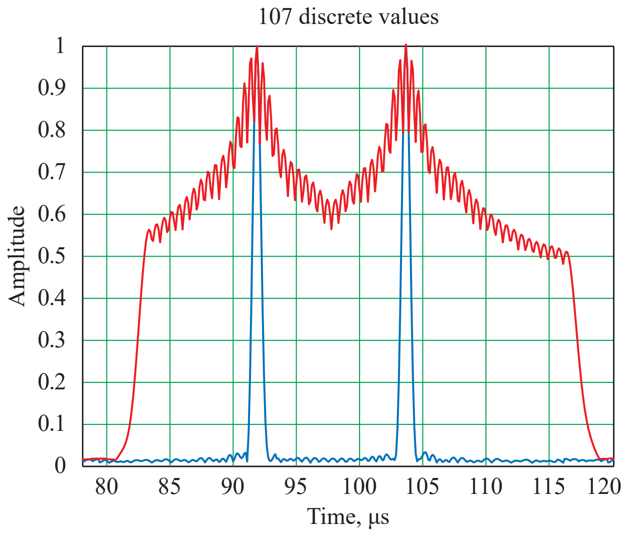

Fig. 7. Signal envelope
– (2); – (1)

| ISSN 2542-0542 Journal of "Almaz – Antey" Air and Space Defence Corporation | No. 3, 2022

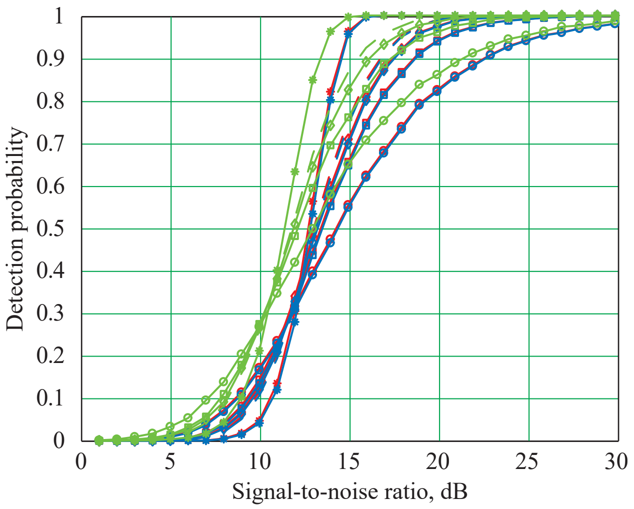

Fig. 8. Detection characteristics
– MF: Swerling 0; – MF: Swerling 1;
– MF: Swerling 2; – MF: Swerling 3;
– MF: Swerling 4; – MCMF: Swerling 0 (1);
– MCMF: Swerling 1 (1); – MCMF: Swerling 2 (1);
– MCMF: Swerling 3 (1); – MCMF: Swerling 4 (1);
– MCMF: Swerling 0 (2); – MCMF: Swerling 1 (2); – MCMF: Swerling 2 (2); – MCMF: Swerling 3 (2); – MCMF: Swerling 4 (2)

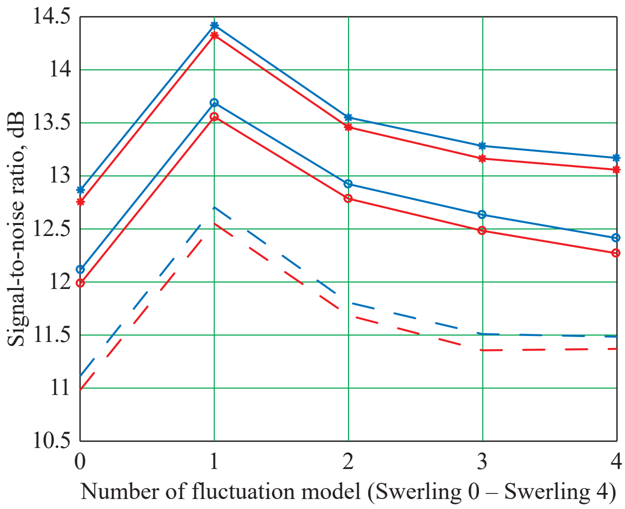

Fig. 9. Signal-to-noise threshold ratio – fluctuation number relationship
– 1e–6: (1); – 1e–5: (1); – 1e–4: (1)
– 1e–6: (1); – 1e–5: (1); – 1e–4: (1)

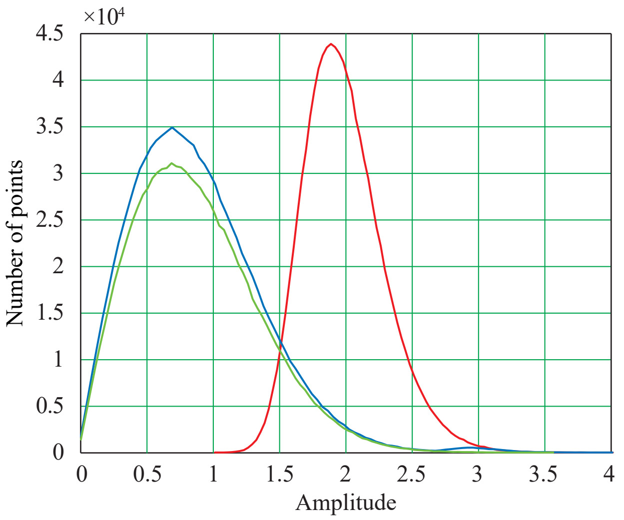

Fig. 10. Solving statistics distribution histograms
– MCMF and (1); – MCMF and (2); – MF

The analysis of Figures 8 and 9 proves that losses of the detection algorithms with the method of MCMF multiplexing (2) in comparison with (1) for any model of signal fluctuation do not exceed ~0.1…0.2 dB, with the resulting signal protected from expansion. A slight increase in the signal-to-noise threshold ratio with MCMF multiplexing according to (2) instead of (1) is related to the need to increase the amplitude threshold of the noise-related detection in order to provide the nominal PFA. In comparison to a single-channel MF, losses of the signal detector comprising the NLFM MCMF with the parameters considered above, regarding an increase in the signal-to-noise threshold ratios (~ 1 dB), is a kind of “price” to be paid for the capability to detect target signals in a wide range of targets’ radial velocities.

To illustrate the causes of increase in signal-to-noise threshold ratio, Figure 10 shows an example of input signal distribution histograms of the single-channel compression filter and the 33-channel MCMF with multiplexing according to (1) and (2), respectively, after calculation of envelopes (amplitudes) in the noise environment. The number of experiment repetitions for histogram plotting was equal to 10^6.

According to Figure 10, histograms for the MF with a different number of channels and different methods of multiplexing are different in shape and in mode values. Thus, for the method of MCMF multiplexing (1), selection of the maximum signal amplitude from all the MCMF channels at each discrete value of range leads to the shift of histogram mode toward higher values of signal amplitudes. On the contrary, for the method of MCMF multiplexing (2), a channel with the maximum signal amplitude is selected as the optimal one in the range-related 2D sliding window at each discrete value of range. As a result, after multiplexing the signal amplitude is represented by groups of samples of the signal envelope from one of the MCMF channels only (optimal channel for the corresponding discrete values of range). That is why, in this case the histogram mode corresponds to lower values of signal amplitudes. In comparison with the single-channel MF, the histogram has flatter tails for the MCMF and (1)–(2). The PFA is determined by the integral taken from the solving statistics distribution law in the noise environment, with lower and upper integration limits equal to the amplitude threshold of detection and to the infinity, respectively. Therefore, in order to maintain the same nominal value of PFA, the detection threshold for the methods of MCMF multiplexing (1)–(2) with flatter tails of histograms in comparison to the single-channel MF shall be increased. This will result in an increase in the signal-to-noise threshold ratio when analysing the characteristics of the detector with MCMF.

Another signal being more sensitive to the Doppler frequency shift is a noise-like signal, which can be substituted for a signal with

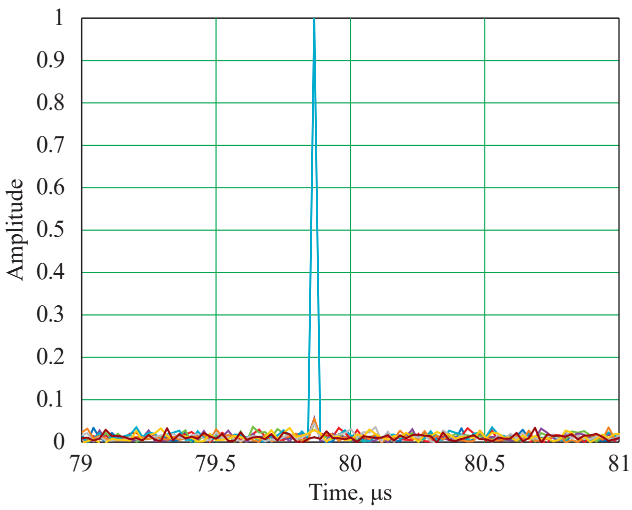

Fig. 11. Signal envelope
– 1st channel: –160 kHz; – 2nd channel: –120 kHz; – 3rd channel: –80 kHz; – 4th channel: –40 kHz; – 5th channel: 0 kHz; – 6th channel: 40 kHz; – 7th channel: 80 kHz; – 8th channel: 120 kHz; – 9th channel: 160 kHz

phase noise modulation (NM) [4] or a signal with M-sequence phase-shift keying [5].

Further, for the sake of definiteness, the signal with phase NM was used as a sounding signal [4]. The signal with NM on the receiver side is represented as follows:

$$s_k = e^{j(2\times\pi\times r_k \pm 2\times\pi\times F_d\times t_k}, \ k = 1 \ldots N_s,$$

where r_k – random numbers uniformly distributed on segment [0, 1], t_k – discrete values of time variable, N_s – number of signal samples.

We should note that the NM signal with a relatively high value of base B is an interesting case in terms of practical application. Such a signal has a low integral level of side lobes (SLL) by range ($\sim 1/\sqrt{B}$), which is close to the SLL of the LFM signal weighted with the help of the Hamming window. In comparison with NLMF signals, high sensitivity of signals with phase NM to the Doppler frequency shift needs a larger amount of Doppler signals of the MCMF.

Figure 11 shows signal envelopes in the channels of the NM signal MCMF (duration T – 80 µs, spectrum width W – 50 MHz). For clarity, the figure shows 9 MCMF channels with the Doppler frequency of 0 Hz.

According to Figure 11, the global correlation maximum is observed in one channel only, while in other MCMF channels that do not match with the received signal, correlation peaks have considerably lower levels and remain non-shifted. The main lobe of the compressed NM signal is actually represented as a single sample according to the Nyquist sampling criterion. These results allow to get rid of the analysis window $-N_w \ldots N_w$ and to enable MCMF multiplexing according to (1).

Figures 12 and 13 show an example of signal envelops after matched filtering and multiplexing of Doppler channels of the MCMF according to (1) and (2) for time-dependent overlapping signals with phase NM that are reflected from short-range targets moving at different radial velocities

(Doppler frequencies – 12 and 60 kHz). For definiteness, the size of analysis window N_w in (2) corresponded to the case with NLFM. To ensure a uniform and low SLL and admissible losses less than 0.5 dB in the dips of the AFR of the NM signal MCMF within the Doppler frequency range of –160…160 kHz, the number of channels was equal to 65. The variants with different channel spacing by 17 and 39 discrete values are considered, N_w – 26 discrete values.

According to Figures 12 and 13, the method of MCMF multiplexing (1) allows to avoid losses when observing signals from short-range targets moving at different radial velocities, while there is a slight increase in the SLL by range. As for the method (2), losses are eliminated as well through a larger signal spacing (similarly to the example with NLFM). In both cases, there is no signal ML expansion while for the NLFM signal (1) a considerable signal expansion will take place (Figures 6 and 7).

For the analysed broadband NM signal, Figures 14 and 15 show maximum and average SLLs estimated by range.

According to Figures 14 and 15, multiplexing of the NLFM signal MCMF according to (1) gives a slight increase in the average SLL by range (for the analysed signal, its is higher by 4 dB on average as compared with the method (2)), while the maximum SLL is practically the same.

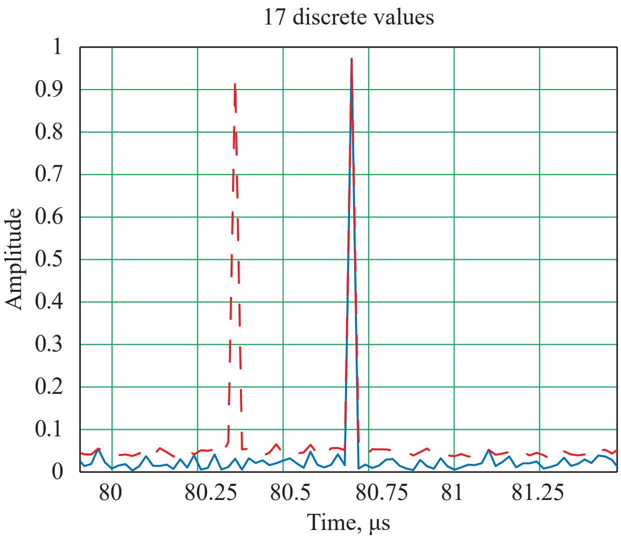

Fig. 12. Signal envelope
– (2); – (1)

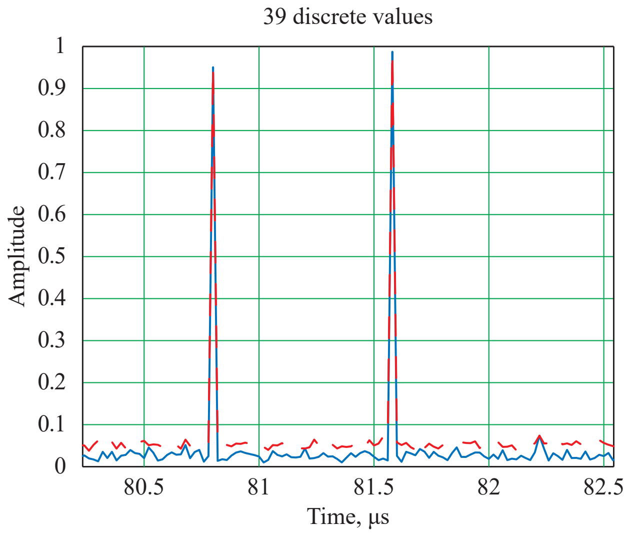

Fig. 13. Signal envelope
– (2); – (1)

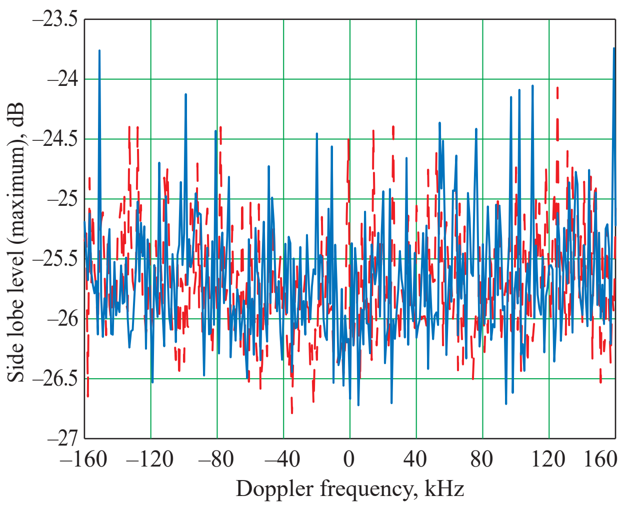

Fig. 14. Maximum side lobe level
– (2); – (1)

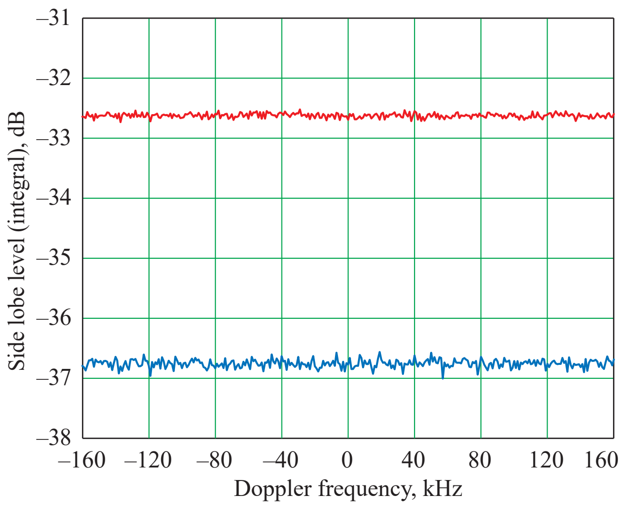

Fig. 15. Average side lobe level
– (2); – (1)

Thus, based on the joint analysis of all results regarding the SSL value, ML width, amplitudes of signals from short-range targets with different Doppler frequencies, we recommend the method (1) for multiplexing of the Doppler channels of the NLFM signal MCMF.

As a whole, BBSs with phase NM (in comparison with FM signals (LFM, NLFM)) are more preferable not only in terms of the range-related SLL value and additional capabilities for radar protection against interference (e.g. against repeater pulse jamming, angel echo), but also in terms of efficiency when used in systems that require multi-channel processing in a wide range of radial velocities of targets being detected (by Doppler frequency, pulse migration) [1, 2]. Due to degraded resolution of targets with different motion parameters, the use of FM signals is likely to be a reasonable choice for application in a narrower range of radial velocities of targets being detected.

Conclusions

A comparative analysis of the methods of MCMF multiplexing for signals with NLFM and phase NM sensitive to the Doppler shift is conducted. Analysis results prove that application of various methods for multiplexing is more efficient for signals with the intra-pulse modulation laws considered.

1. For NLFM signals, the most efficient solution is the adaptive method of MCFM multiplexing (2) that allows to avoid unwanted signal expansion with formation of spurious side lobes. This method proves efficient in a wide range of radial velocities of targets being detected. For efficient resolution of signals from short-range targets moving at considerably different radial velocities, it is necessary that the signal spacing be equal to the amount of discrete values that exceeds the maximum shift of the range correlation peak by range in all MCMF channels.

2. If NM signals are used, in order to avoid losses in case of signal overlapping with different Doppler frequencies, we recommend MCMF multiplexing at each discrete value of range using the maximum selection pattern (1). In comparison with the method involving protection from signal expansion (2), there is an increase in the average SLL (in the example considered herein, for a practically interesting case of BBS with NM – up to 4 dB). As a whole, such an increase in the SLL is insignificant. The average SLL of the NM signal with relatively large bases is close to the SLL of the LFM signal weighted by means of the Hamming window.

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Science research interests: signal detection in radar systems, digital signal processing, simulation modelling.

Сравнительный анализ способов объединения каналов многоканального согласованного фильтра

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В статье на примере сигналов с нелинейной частотной модуляцией и с шумовой модуляцией фазы, чувствительных к смещению по частоте Доплера, внутрипериодная обработка которых для уменьшения потерь требует многоканального по данному параметру построения, проведен сравнительный анализ способов объединения доплеровских каналов фильтра сжатия при различном разносе по дальности и радиальной скорости разрешаемых целей. Результаты имитационного моделирования показали, что для сигналов с рассмотренными законами внутриимпульсной модуляции более эффективно применение разных способов объединения каналов.

Ключевые слова: многоканальный согласованный фильтр, нелинейная частотная модуляция, шумовая модуляция фазы, объединение каналов, расширение сигнала

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