UDC 621.396.96

Yastrebov A. V.

Iterative inversion algorithm for an interference correlation matrix

The article considers efficiency of a multi-channel interference canceller as a function of the number of compensation channels used for a fixed number of jammers. The author suggests an iterative inversion algorithm for an interference correlation matrix, making it possible to achieve zero loss at a certain stage and decrease the computational complexity of determining weight factors for compensation channels.

Keywords: spatial signal processing, interference canceller, minimising computational complexity, inverse of a Hermitian matrix.

Introduction

Nowadays, we can see a rapid growth in the development of radar countermeasure systems such as noise jammers (NJ), which require more sophisticated hardware for target locators [1–3]. This trend is basically driven by more intensive use of unmanned aerial vehicles fitted with noise emitters, thus causing a continuous increase in a number of operating NJs. One of the methods intended to counter NJ is the application of the multi-channel interference canceller (MIC) as part of radar stations (RS). Besides, the number of compensation channels of the canceller shall not be less than the number of jammers [1, 3]. That is why, at the RS design stage, the number of compensation channels of the canceller is to be selected depending on the expected maximum possible number of NJs.

The paper analyses the possibility of adaptive use of MIC resources for NJ interference cancellation in order to minimize computational complexity of the spatial processing algorithm with the number of jammers being less than the number of the canceller's compensation channels. This allows to use released resources of a computing system for other tasks.

Multi-channel interference canceller

The MIC block diagram is given in Fig. 1. Fig. 2 shows typically used compensation channels, directional patterns (DP) of which $A(\varphi)$ (where A – normalized amplification of the antenna system, dB; φ – azimuth, deg) have a deep fall towards the maximum of the main channel DP.

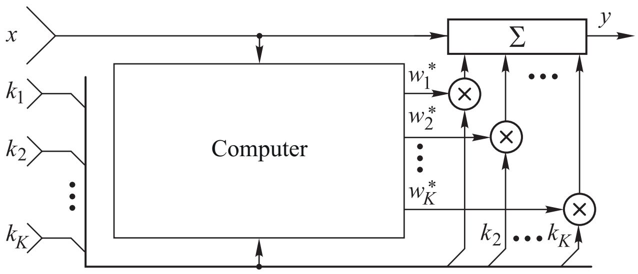


Fig. 1. MIC block diagram

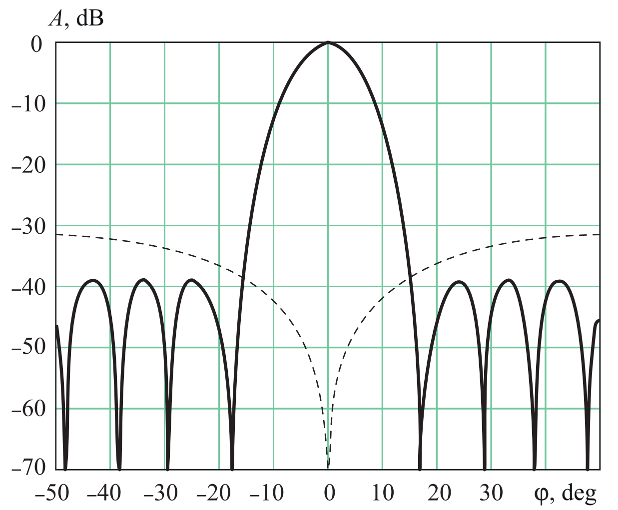


Fig. 2. Directional pattern of the main channel and compensation channels

Using **K**, we will designate the complex envelope (CE) vector of signals in MIC compensation channels; in this case, the process CE at the canceller output can be written as follows:

$$y = x + \mathbf{W}^H \mathbf{K}, \quad (1)$$

where x – signal CE in the MIC main channel;

$\mathbf{W}$ – weight factor vector;

$(\cdot)^H$ – sign of Hermitian conjugation.

In order to maximize the output signal-to-noise ratio (SNR), weight vector **W** shall comply with the Wiener – Hopf equation [1, 3] and can be calculated by formula

$$\mathbf{W} = -\mathbf{M}^{-1}\mathbf{P}, \qquad (2)$$

where $\mathbf{M} = \langle \mathbf{K}\mathbf{K}^H \rangle$ – compensation channels interference correlation matrix (CM);

$\mathbf{P} = \langle \mathbf{K}x^* \rangle$ – vector of cross-correlation moments between the processes in the MIC main channel and compensation channels;

$\langle\cdot\rangle$ – sign of statistical averaging.

Fig. 3 shows the dependence of loss B in the MIC output SNR on the number k of used compensation channels at fixed number J of operating NJs relative to SNR at MIC output when all K compensation channels are used. The following simulation parameters are set: total number of compensation channels $K = 8$; number of jammers $J = 3$; similar self-noise power in all MIC channels, similar external interference power, with their total interference power relative to self-noise equal to 5000.

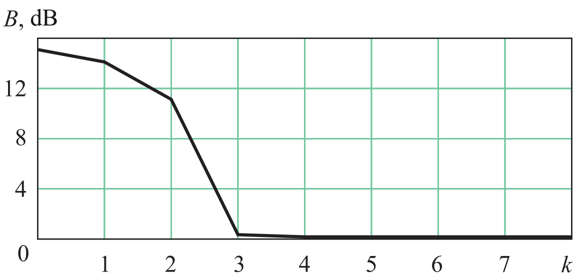


Fig. 3. Dependence of the SNR loss on the number of compensation channels

According to the data shown in Fig. 3, the SNR loss is actually equal to zero when the number of used compensation channels is equal to the number of NJs $(k = J)$. Determination of the MIC weight vector (2) includes the interference CM inversion procedure, the computational complexity of which is proportional to the third degree of the CM order (K^3). Therefore, using only some compensation channels allows to drastically reduce the amount of computations, if the number of NJs is smaller than the total number of compensation channels $(J < K)$, and also to avoid the SNR loss.

In both cases, there are deep falls formed in directions towards all jammers; however, if a smaller number of compensation channels is used, the level side lobes of the resultant DP will be a bit higher (Fig. 4).

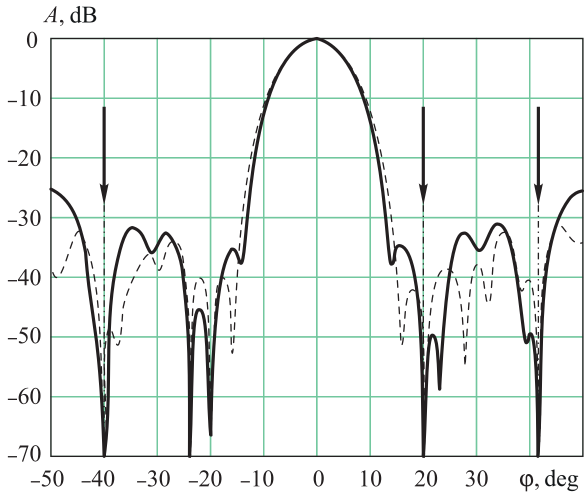


Fig. 4. Resultant MIC DP in use:
—— – $k = J = 3$ of compensation channels;
- - - – all $K = 8$ of compensation channels;
→ – directions towards NJ sources

The use of some compensation channels implies that matrix **M** of the interference CM is being used. For example, if the weight sum (1) contains only the first channel instead of the entire CM **M** and cross-correlation vector **P**, it is sufficient to take one element of both matrix and vector: m_{11} – element of matrix **M** located at the intersection of the first row and the first column, and p_1 – the first element of vector **P**, respectively.

The use of two compensation channels implies the inversion of the second-order matrix that is part of the initial interference CM **M** of the K-th order, and its multiplication by the cross-correlation vector between a signal in the main channel and signals in the first two compensation channels, etc.

Iterative algorithm

Below we will analyse the iterative algorithm of the interference CM, which allows to compute inverse CM at each k-th step for k compensation channels being used. The proposed algorithm is based on the F. G. Frobenius formula of block matrix inversion [4]:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{B}\mathbf{H}^{-1}\mathbf{C}\mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{B}\mathbf{H}^{-1} \\ -\mathbf{H}^{-1}\mathbf{C}\mathbf{A}^{-1} & \mathbf{H}^{-1} \end{bmatrix}, \quad (3)$$

where $\mathbf{H} = \mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B}$.

Let us introduce designation $\mathbf{A}_k$ for the interference CM in the first k compensation channels ($k \le K$) or, similarly, for the matrix composed of elements that are located in the first k rows and columns of the initial CM $\mathbf{M}$ of order K (Fig. 5). Using $\mathbf{P}_k$, we designate the cross-correlation vector between a signal in the main channel and signals in the first k compensation channels.

Matrix $\mathbf{M}$ is a Hermitian matrix, therefore, with account for the data shown in Fig. 5,

$$\mathbf{B}_k = \mathbf{C}_k^H, \quad \mathbf{C}_k = \mathbf{B}_k^H. \quad (4)$$

The first step corresponds to the use of a single compensation channel, weight factor w_1 of which is calculated by formula

$$w_1 = -\mathbf{A}_1^{-1}\mathbf{P}_1. \quad (5)$$

Here, vector $\mathbf{P}_1$ consists of a single compensation element.

Since matrix $\mathbf{M}$ is a Hermitian matrix, matrix $\mathbf{A}_1 = m_{11}$ (Fig. 5, a) is a real number, therefore, its inverse matrix is also a real number:

$$\mathbf{A}_1^{-1} = 1/m_{11}. \quad (6)$$

The second step corresponds to the use of two MIC compensation channels. Let us determine the matrix inverse to matrix $\mathbf{A}_2$ (Fig. 5, b). For this purpose we will represent formula (3) as follows:

$$\mathbf{A}_2^{-1} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}_1^{-1} + \mathbf{A}_1^{-1}\mathbf{B}_1\mathbf{H}_1^{-1}\mathbf{C}_1\mathbf{A}_1^{-1} & -\mathbf{A}_1^{-1}\mathbf{B}_1\mathbf{H}_1^{-1} \\ -\mathbf{H}_1^{-1}\mathbf{C}_1\mathbf{A}_1^{-1} & \mathbf{H}_1^{-1} \end{bmatrix}, \quad (7)$$

where $\mathbf{H}_1 = \mathbf{D}_1 - \mathbf{C}_1\mathbf{A}_1^{-1}\mathbf{B}_1$.

Since the CM $\mathbf{M}$ is a Hermitian matrix, matrix $\mathbf{D}_1$ in formula (7) is a real number. Value $\mathbf{C}_1\mathbf{A}_1^{-1}\mathbf{B}_1$ with account for (4) can be written as $\mathbf{B}_1^H\mathbf{A}_1^{-1}\mathbf{B}_1$. The resulting expression is a quadratic form and a real number. Therefore, matrix $\mathbf{H}_1$ is a real number, that is why the matrix inverse to matrix $\mathbf{H}_1$ is also a real number equal to $1/\mathbf{H}_1$. Taking the above into consideration, we can represent expression (7) as follows:

$$\mathbf{A}_2^{-1} = \begin{bmatrix} \mathbf{A}_1^{-1} + \mathbf{E}_1(\mathbf{E}_1/\mathbf{H}_1)^H & -\mathbf{E}_1/\mathbf{H}_1 \\ -(\mathbf{E}_1/\mathbf{H}_1)^H & 1/\mathbf{H}_1 \end{bmatrix}, \quad (8)$$

where, with account for (4), the following designation is introduced

$$\mathbf{E}_1 = \mathbf{A}_1^{-1}\mathbf{B}_1. \quad (9)$$

By continuing the iterative procedure, we can determine the matrix inverse to matrix $\mathbf{A}_3$ in a similar way (Fig. 5, c).

The increment to determine the interference CM for the number of compensation channels $k > 1$ can be represented as follows:

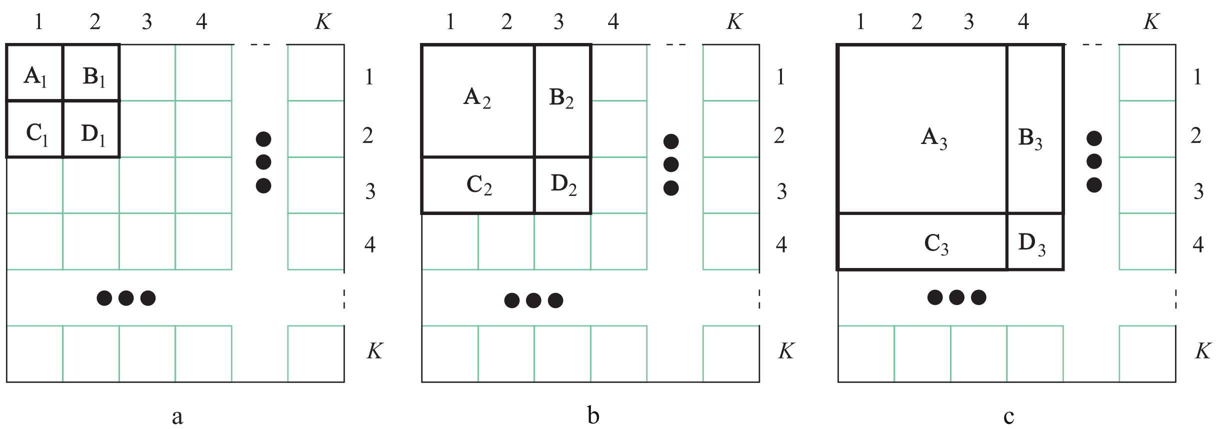
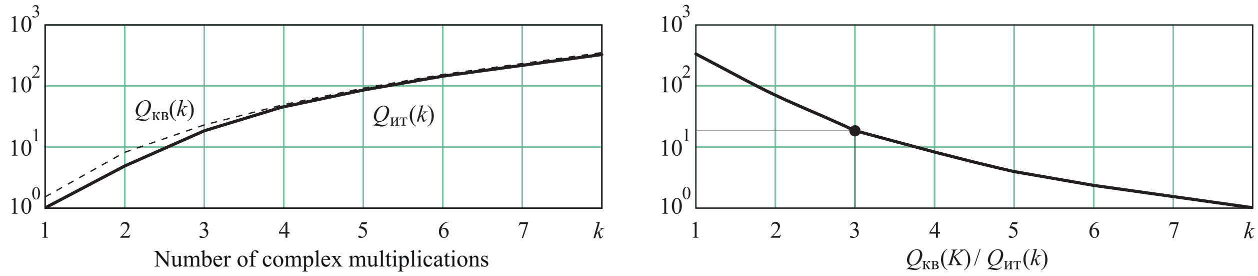


Fig. 5. Iterations of CM inversion algorithm:
a – matrix $\mathbf{A}_1$; b – matrix $\mathbf{A}_2$; c – matrix $\mathbf{A}_3$



a b

Fig. 6. Graphs showing the dependence of the square root method’s computational complexity $Q_{кв}$ (- - -) and the proposed method’s computational complexity $Q_{ит}$ (——) on the number of compensation channels k (a) and the ratio of the square root method’s computational complexity $Q_{кв}$ at the number of compensation channels $K = 8$ to the proposed method’s computational complexity $Q_{ит}$, depending on the number of compensation channels $k = 1 \div K$ (b)

$$\mathbf{A}_k^{-1} = $$
$$= \begin{bmatrix} \mathbf{A}_{k-1}^{-1} + \mathbf{E}_{k-1}\left(\mathbf{E}_{k-1}/\mathbf{H}_{k-1}\right)^H & -\mathbf{E}_{k-1}/\mathbf{H}_{k-1} \\ -\left(\mathbf{E}_{k-1}/\mathbf{H}_{k-1}\right)^H & 1/\mathbf{H}_{k-1} \end{bmatrix}, \quad (10)$$

where

$$\mathbf{E}_{k-1} = \mathbf{A}_{k-1}^{-1}\mathbf{B}_{k-1}; \quad (11)$$

$$\mathbf{H}_{k-1} = \mathbf{D}_{k-1} - \mathbf{B}_{k-1}^H\mathbf{E}_{k-1}. \quad (12)$$

Computational complexity of the algorithm

Lets us determine computational resources needed to execute the k-th step of the iterative procedure.

To calculate vector $\mathbf{E}_{k-1}$ by formula (11), $(k-1)^2$ complex multiplications (CMult) will be required.

Calculation of $\mathbf{H}_{k-1}$ by formula (12) will require $(k-1)$ of CMult.

In formula (10), matrix $\mathbf{A}_{k-1}^{-1}$ is already known at the previous step. Since $\mathbf{H}_{k-1}$ is a real number, $2(k-1)$ of real multiplications (RMult) shall be done in order to calculate value $\mathbf{E}_{k-1}/\mathbf{H}_{k-1}$.

In order to multiply the complex column vector by the normalized Hermitian conjugated vector $\mathbf{E}_{k-1}\left(\mathbf{E}_{k-1}/\mathbf{H}_{k-1}\right)^H$, $2(k-1)$ of RMult will be required to determine diagonal elements, and $2(k-1)(k-2)$ of RMult to determine elements located below the main diagonal.

Taking into account that one complex multiplication consists of four real multiplications, the total computational complexity $Q_{ит}$ of the algorithm for computing the inverse CM of size $k \times k$, expressed in CMult is as follows

$$Q_{ит}(k) = 1 + \frac{1}{4}\sum_{n=2}^{k}(6n^2 + 3n - 12). \quad (13)$$

In practice, in order to determine matrix M^{-1} of size $K \times K$ inverse to the interference CM, the Cholesky method or the so-called square root method is often applied [5]. Its computational complexity is $Q_{кв} = K^3/2 + K^2$ of CMult [1]. According to Fig. 6, a, the number of required operations for both methods is almost the same. Fig. 6, b shows the graph illustrating the ratio of the square root method’s computational complexity $Q_{кв}$ at $K = 8$ to computational complexity $Q_{ит}(k)$ of each step in the proposed method at $k = 1, 2, ..., K$.

In the example analysed above, with NJ $J = 3$ and the number of compensation channels $K = 8$, we can interrupt the iterative procedure at $k \geq 3$ and use matrix $\mathbf{A}_k^{-1}$ to determine the weight vector of the canceller. The computational complexity of the proposed method, for example, for $k = 3$, is around 20 times lower than that of the square root method (see Fig. 6, b).

Iterative procedure interruption algorithm

The process power (1) at the canceller output with the optimal weight vector $\mathbf{W}$ (2) is equal to

$$\sigma_y^2 = \left\langle \left|x + \mathbf{W}^H\mathbf{K}\right|^2 \right\rangle = $$
$$= \left\langle |x|^2 \right\rangle + \mathbf{P}^H\mathbf{W} = \sigma_x^2 + \mathbf{P}^H\mathbf{W}, \quad (14)$$

where σ_x^2 – total power of interference and self-noise (SN) in the main channel.

Assuming that SNs are independent in all the receiving channels, we can represent the expression for the total SN power at the canceller output as follows:

$$\sigma^2_{\text{СШ}} = \sigma^2_0 + \sigma^2_{\text{k}} \mathbf{W}^H \mathbf{W}, \qquad (15)$$

where σ^2_0 – SN power in the main channel;

σ^2_{k} – SN power in each compensation channel.

Assume that SN power values are a priori known.

The weight factor vector shall be calculated at each k-th step of the iterative procedure for matrix inversion

$$\mathbf{W}_k = -\mathbf{A}_k^{-1} \mathbf{P}_k. \qquad (16)$$

Here, $\mathbf{P}_k$ – vector consisting of first k-th elements of cross-correlation vector $\mathbf{P}$.

The canceller output power at the k-th step of the iterative procedure for interference CM inversion can be calculated by substituting expression (16) in equation (14):

$$\sigma^2_y\left(\mathbf{W}_k\right) = \sigma^2_x + \mathbf{P}_k^H \mathbf{W}_k. \qquad (17)$$

The total SN power at the canceller output with account for expressions (15) and (16) at the k-th step of the iterative procedure can be calculated as follows:

$$\sigma^2_{\text{СШ}}\left(\mathbf{W}_k\right) = \sigma^2_0 + \sigma^2_k \mathbf{W}_k^H \mathbf{W}_k. \qquad (18)$$

With the optimal weight vector $\mathbf{W}$ (2), the process at the canceller output (1) does not correlate with active input interference, so the output power (14) with the optimal vector is equal to the total SN power (15).

Therefore, for interrupting the iterative inversion procedure and calculating the required number of compensation channels $k_{\text{опт}}$ to suppress active NJs, if instead of accurate interference CM $\mathbf{M}$ and cross-correlation vector $\mathbf{P}$ their maximum reliable estimates $\widehat{\mathbf{M}}$ and $\widehat{\mathbf{P}}$ are used, the condition is represented as follows:

$$\sigma^2_y\left(\widehat{\mathbf{W}}_k\right) \le \gamma \sigma^2_{\text{СШ}}\left(\widehat{\mathbf{W}}_k\right), \qquad (19)$$

where $\widehat{\mathbf{W}}_k$ – weight factor vector determined at the k-th step of the iterative procedure if estimates $\widehat{\mathbf{M}}$ and $\widehat{\mathbf{P}}$ are used;

γ – factor depending on the accuracy of power estimates (17) and (18).

Conclusion

Using the algorithm described above, the iterative procedure for interference CM inversion can be interrupted. As a result, this allows to reduce the time and computational resources for implementing spacial signal processing in RS with practically zero SNR loss, along with a minor increase in the level of side lobes of the resultant DP.

Bibliography

1. *Monzingo R. A., Miller T. W.* Introduction to Adaptive Arrays. Transl. from Eng. Ed. by *V. A. Leksachenko*. M.: Radio i svyaz', 1986. 448 s.

2. *Richards M. L., Scheer J. A., Holm W. A.* Principles of Modern Radar. In 2 vols. Vol. 1. Basic Principles. NJ.: SciTech Publishing, Edison, 2010. 924 p.

3. *Melvin W. L., Scheer J. A.* Principles of Modern Radar. In 2 vols. Vol. 2. Advanced Techniques. NJ.: SciTech Publishing, Edison, 2013. 846 p.

4. *Gantmacher F. R.* The Theory of Matrices. M.: Nauka, Gl. red. fiz.-mat. lit., 1988. 552 p.

5. *Faddeeva V. N.* Computational methods of linear algebra. M.: Gostekhizdat, 1950. 240 p. (Russian)

Submitted on 06.02.2017

Yastrebov Andrey Viktorovich – Leading Engineer, Nizhny Novgorod State Technical University named after R. E. Alekseev, Nizhny Novgorod.

Science research interests: radiolocation, digital signal processing.

Итерационный алгоритм обращения корреляционной матрицы помех

Рассмотрена эффективность многоканального компенсатора помех в зависимости от числа используемых компенсационных каналов при фиксированном числе внешних помехопостановщиков. Предложен итерационный алгоритм обращения корреляционной матрицы помех, позволяющий на определенном шаге достичь нулевых потерь на выходе компенсатора и уменьшить вычислительную сложность нахождения весовых коэффициентов компенсационных каналов.

Ключевые слова: пространственная обработка сигналов, компенсатор помехи, минимизация вычислительной сложности, обращение эрмитовой матрицы.

Ястребов Андрей Викторович – ведущий инженер ФГБОУ ВО «Нижегородский государственный технический университет им. Р. Е. Алексеева», г. Нижний Новгород.
Область научных интересов: радиолокация, цифровая обработка сигналов.