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Methodology of determining design parameters for vibration damping in spacecraft structures

Modern spacecraft designs use elastic outrigger-type structural elements to accommodate equipment and devices. When the angular position of spacecraft changes and they stabilise in their orbits, these elements start to vibrate. In order to decrease the effect these vibrations have on the spacecraft dynamics, vibration damping delay time is introduced, amounting to several minutes. We present a methodology of determining design parameters for vibration damping systems that make it possible to damp vibrations in milliseconds.

Keywords: spacecraft, vibration damping system, design parameters, stress-strain state, elastic outrigger-type structural elements, vibrations, antiresonance.

Introduction

The present-day astronautics faces the following tasks [1]:

• exploration of the near-Earth space;

• sending missions to other planets of the Solar System;

• construction of habitable bases on the Moon and Mars.

To perform these tasks, companies of the aerospace industry of the Russian Federation build multi-functional spacecraft (SC) with long active service life, high reliability and high thrust-to-mass ratio. These spacecraft have complicated designs with numerous outboard elastic components (OEC).

Activation of SC engines causes oscillations of OEC that obstruct bringing the SC into oriented positions, stabilizing it before docking with modules of space stations, landing on celestial bodies, and keeping the required angular positions of SC while manoeuvring in space.

To decrease the effect of oscillations in OEC on travel of Earth-orbit automated SC, the time-out for damping oscillatory motions is reserved in flight cyclograms. This method can be efficient if there is a time margin of less than ten minutes for oscillation damping. But for SC used in exploration of deep space, in interplanetary missions, and in delivering payloads to planets of the Solar System such time expenditures for damping oscillations of OEC is not acceptable.

The aim of this paper is to propose the way to decrease the time for manoeuvring and changing angular positions of SC.

Stabilization of outboard elastic components of spacecraft

Papers [2–4] introduce the method for stabilization of OEC of SC structures, based on controlled damping of their millisecond oscillations. To damp oscillations of OEC, it is proposed to use oscillation damping systems (ODS) as part of SC travel control systems (TCS); the operating principle of ODS shall be based on generation and regulation of antiresonance forced oscillations of OEC. The control parts of ODS in SC shall be [3, 4] multi-mode OEC driving units. The layout of a controlled system for oscillation damping is shown in Figures 1 and 2.

Design parameters of oscillation damping controlled systems

The initial stages of designing the ODS are associated with determination and justification of their design parameters. In the paper [3], the following parameters are stated and justified:

• frequency ranges of OEC oscillations corresponding to frequency parameters of the sensors that obtain data on oscillations of OEC;

• frequencies of reading data on OEC oscillation parameters from the sensors located on OEC surfaces;

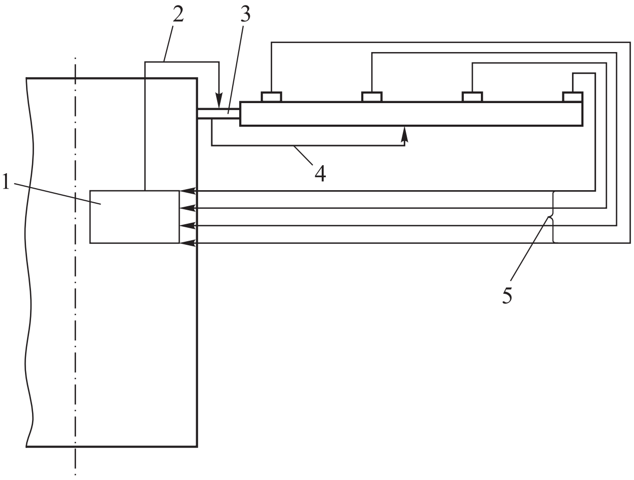

Fig. 1. Structural layout of an oscillation damping controlled system, main view:
1 – onboard computer; 2 – control signal;
3 – multi-mode driving unit; 4 – control dynamic influence that stabilizes the outboard component by means of antiresonance; 5 – data flows with parameters of OEC oscillations

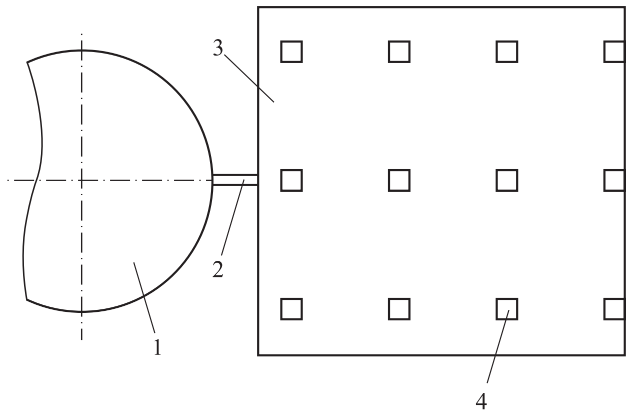

Fig. 2. Oscillation damping controlled system, top view:
1 – spacecraft body; 2 – multi-mode driving unit;
3 – outboard elastic component;
4 – sensors that register oscillation parameters of an outboard component

• ranges of angular speeds of rotors of the multi-purpose driving units ensuring frequencies of forced antiresonance oscillations that stabilize OEC.

The method of determining values of design parameters of controlled ODS is described in papers [3–5].

Method of determining values of design parameters of oscillation damping controlled systems

The method includes the mathematical complex for multi-parameter system simulation of stress-strain states (SSS) of outboard elastic components of spacecraft [3–5] and justified [2, 3] requirements for oscillation damping controlled systems. The mathematical complex for multi-parameter system simulation of SSS of SC OEC ensures prediction of their oscillation ranges. On the basis of this prediction and taking into account the justified requirements for controlled ODS, values of their design parameters can be defined exactly. The credibility of the developed mathematical complex is proved by the comparative analysis of frequencies of OEC oscillations in the existing space devices. Results of this analysis are obtained both in theoretical calculations and in experiments.

The theoretic calculations are analytical, performed with the use of non-linear transcendental equation for oscillation frequencies of SC OEC:

$$\begin{aligned} &X_1\omega^8 \sin(\omega t + \varphi) + X_2\omega^7 \cos(\omega t + \varphi) + \\ &+X_3\omega^6 \sin(\omega t + \varphi) + X_4\omega^5 \cos(\omega t + \varphi) + \\ &+X_5\omega^4 \sin(\omega t + \varphi) + X_6\omega^3 \cos(\omega t + \varphi) + \\ &+X_7\omega^2 \sin(\omega t + \varphi) + X_8\omega \cos(\omega t + \varphi) + \\ &+X_9 \sin(\omega t + \varphi) = 0, \end{aligned} \quad (1)$$

where ω – OEC oscillation frequency;

t – mathematical time;

φ – initial phase of oscillations;

X_1–X_9 – factors in the equation that contain data on the OEC geometry, its viscoelastic, physical and mechanical parameters as well as data on anisotropy of these parameters.

Equation (1) is developed with the use of the symbolic method and is based on the formulas for a deformable solid, thermal conductivity and diffusion equations, taking into account complex effects of mechanical, thermal, radiation, and inertial loads in operational orbits of spacecraft as well as degrees of anisotropy of physical and mechanical and viscoelastic properties of OEC made of metal alloys and/or composite materials, conditions of end fixities (attachments) of orientable and non-controlled OEC into elastic bodies of SC, conditions of indeterminacy of distribution of particles of corpuscular cosmic rays, temperature fields, and elementary volumes of OEC across their masses [3, 4]. The equation coefficients (1) are calculated in a program developed with the use of *C++* programming language. OEC oscillation frequencies are determined graphically in *Gnuplot. Ver. 5.0* and with methods of *GNU Scientific Library (GSL). Ver. 1.8.*

The experimental data on OEC oscillation frequencies in the existing spacecraft are obtained in the course of their dynamic tests.

Mathematical complex of multi-parameter system simulation of stress-strain states of the outboard elastic components of spacecraft

The developed mathematical complex for multi-parameter system simulation includes eight groups of equations.

1. Equations of oscillatory motions of spacecraft OEC [3].

2. Equations of thermal conductivity of OEC and diffusion of elementary particles of cosmic rays in their materials:

$$\left(\frac{\lambda_{\alpha}+\lambda_{\beta\alpha}+\lambda_{\gamma\alpha}}{A_1^2}\right)\left(2h\frac{\partial^2 T_0}{\partial\alpha^2}+\sum_{\eta=2}^{\upsilon}\frac{\partial^2 T_{\eta}}{\partial\alpha^2}\int_{-h}^{h}\Omega_{\eta}d\gamma\right)+$$

$$+\left(\frac{\lambda_{\alpha}+\lambda_{\beta\alpha}+\lambda_{\gamma\alpha}}{A_1^2A_2}\frac{\partial A_2}{\partial\alpha}-\frac{\lambda_{\alpha}+\lambda_{\beta\alpha}+\lambda_{\gamma\alpha}}{A_1^3}\frac{\partial A_1}{\partial\alpha}\right)\times$$

$$\times\left(2h\frac{\partial T_0}{\partial\alpha}+\sum_{\eta=2}^{\upsilon}\frac{\partial T_{\eta}}{\partial\alpha}\int_{-h}^{h}\Omega_{\eta}d\gamma\right)+$$

$$+\left(\frac{\lambda_{\alpha\beta}+\lambda_{\beta}+\lambda_{\gamma\beta}}{A_2^2}\right)\left(2h\frac{\partial^2 T_0}{\partial\beta^2}+\sum_{\eta=2}^{\upsilon}\frac{\partial^2 T_{\eta}}{\partial\beta^2}\int_{-h}^{h}\Omega_{\eta}d\gamma\right)+$$

$$+\left(\frac{\lambda_{\alpha\beta}+\lambda_{\beta}+\lambda_{\gamma\beta}}{A_2^2A_1}\frac{\partial A_1}{\partial\beta}-\frac{\lambda_{\alpha\beta}+\lambda_{\beta}+\lambda_{\gamma\beta}}{A_2^3}\frac{\partial A_2}{\partial\beta}\right)\times$$

$$\times\left(2h\frac{\partial T_0}{\partial\beta}+\sum_{\eta=2}^{\upsilon}\frac{\partial T_{\eta}}{\partial\beta}\int_{-h}^{h}\Omega_{\eta}d\gamma\right)+\left(\lambda_{\alpha\gamma}+\lambda_{\beta\gamma}+\lambda_{\gamma}\right)\times$$

$$\times\left(2hT_1+\sum_{\eta=2}^{\upsilon}T_{\eta}\left[\Omega_{\eta}(h)-\Omega_{\eta}(-h)\right]\right)=$$

$$=\rho c\left(2h\frac{\partial T_0}{\partial t}+\sum_{\eta=2}^{\upsilon}\frac{\partial T_{\eta}}{\partial t}\int_{-h}^{h}\Omega_{\eta}d\gamma\right);$$

$$\left(\frac{\lambda_{\alpha}+\lambda_{\beta\alpha}+\lambda_{\gamma\alpha}}{A_1^2}\right)\times \tag{2}$$

$$\times\left(\frac{2h^3}{3}\frac{\partial^2 T_1}{\partial\alpha^2}+\sum_{\eta=2}^{\upsilon}\frac{\partial^2 T_{\eta}}{\partial\alpha^2}\int_{-h}^{h}\gamma\Omega_{\eta}d\gamma\right)+$$

$$+\left(\frac{\lambda_{\alpha}+\lambda_{\beta\alpha}+\lambda_{\gamma\alpha}}{A_1^2A_2}\frac{\partial A_2}{\partial\alpha}-\frac{\lambda_{\alpha}+\lambda_{\beta\alpha}+\lambda_{\gamma\alpha}}{A_1^3}\frac{\partial A_1}{\partial\alpha}\right)\times$$

$$\times\left(\frac{2h^3}{3}\frac{\partial T_1}{\partial\alpha}+\sum_{\eta=2}^{\upsilon}\frac{\partial T_{\eta}}{\partial\alpha}\int_{-h}^{h}\gamma\Omega_{\eta}d\gamma\right)+$$

$$+\left(\frac{\lambda_{\alpha\beta}+\lambda_{\beta}+\lambda_{\gamma\beta}}{A_2^2}\right)\times$$

$$\times\left(\frac{2h^3}{3}\frac{\partial^2 T_1}{\partial\beta^2}+\sum_{\eta=2}^{\upsilon}\frac{\partial^2 T_{\eta}}{\partial\beta^2}\int_{-h}^{h}\gamma\Omega_{\eta}d\gamma\right)+$$

$$+\left(\frac{\lambda_{\alpha\beta}+\lambda_{\beta}+\lambda_{\gamma\beta}}{A_2^2A_1}\frac{\partial A_1}{\partial^2}-\frac{\lambda_{\alpha\beta}+\lambda_{\beta}+\lambda_{\gamma\beta}}{A_2^3}\frac{\partial A_2}{\partial\beta}\right)\times$$

$$\times\left(\frac{2h^3}{3}\frac{\partial T_1}{\partial^2}+\sum_{\eta=2}^{\upsilon}\frac{\partial T_{\eta}}{\partial^2}\int_{-h}^{h}\gamma\Omega_{\eta}d\gamma\right)+$$

$$+\left(\lambda_{\alpha\gamma}+\lambda_{\beta\gamma}+\lambda_{\gamma}\right)\times$$

$$\times\sum_{\eta=2}^{\upsilon}T_{\eta}\left(h\left[\Omega_{\eta}(h)+\Omega_{\eta}(-h)\right]-\int_{-h}^{h}\Omega_{\eta}d\gamma\right)=$$

$$=\rho c\left(\frac{2h^3}{3}\frac{\partial T_1}{\partial t}+\sum_{\eta=2}^{\upsilon}\frac{\partial T_{\eta}}{\partial t}\int_{-h}^{h}\gamma\Omega_{\eta}d\gamma\right);$$

$$\left(\frac{D_{\alpha}+D_{\beta\alpha}+D_{\gamma\alpha}}{A_1^2}\right)\left(2h\frac{\partial^2 I_0}{\partial\alpha^2}+\sum_{\zeta=2}^{\vartheta}\frac{\partial^2 I_{\zeta}}{\partial\alpha^2}\int_{-h}^{h}\Lambda_{\zeta}d\gamma\right)+$$

$$+\left(\frac{D_\alpha+D_{\beta\alpha}+D_{\gamma\alpha}}{A_1^2A_2}\frac{\partial A_2}{\partial\alpha}-\frac{D_\alpha+D_{\beta\alpha}+D_{\gamma\alpha}}{A_1^3}\frac{\partial A_1}{\partial\alpha}\right)\times$$

$$\times\left(2h\frac{\partial I_0}{\partial\alpha}+\sum_{\zeta=2}^{\vartheta}\frac{\partial I_\zeta}{\partial\alpha}\int_{-h}^{h}\Lambda_\zeta d\gamma\right)+$$

$$+\left(\frac{D_{\alpha\beta}+D_\beta+D_{\gamma\beta}}{A_2^2}\right)\left(2h\frac{\partial^2 I_0}{\partial\beta^2}+\sum_{\zeta=2}^{\vartheta}\frac{\partial^2 I_\zeta}{\partial\beta^2}\int_{-h}^{h}\Lambda_\zeta d\gamma\right)+$$

$$+\left(\frac{D_{\alpha\beta}+D_\beta+D_{\gamma\beta}}{A_2^2A_1}\frac{\partial A_1}{\partial\beta}-\frac{D_{\alpha\beta}+D_\beta+D_{\gamma\beta}}{A_2^3}\frac{\partial A_2}{\partial\beta}\right)\times$$

$$\times\left(2h\frac{\partial I_0}{\partial\beta}+\sum_{\zeta=2}^{\vartheta}\frac{\partial I_\zeta}{\partial\beta}\int_{-h}^{h}\Lambda_\zeta d\gamma\right)+\left(D_{\alpha\gamma}+D_{\beta\gamma}+D_\gamma\right)\times$$

$$\times\left(2hI_1+\sum_{\zeta=2}^{\vartheta}I_\zeta\left[\Lambda_\zeta(h)-\Lambda_\zeta(-h)\right]\right)=$$

$$=2h\frac{\partial I_0}{\partial t}+\sum_{\zeta=2}^{\vartheta}\frac{\partial I_\zeta}{\partial t}\int_{-h}^{h}\Lambda_\zeta d\gamma;$$

$$\left(\frac{D_\alpha+D_{\beta\alpha}+D_{\gamma\alpha}}{A_1^2}\right)\times$$

$$\times\left(\frac{2h^3}{3}\frac{\partial^2 I_1}{\partial\alpha^2}+\sum_{\zeta=2}^{\vartheta}\frac{\partial^2 I_\zeta}{\partial\alpha^2}\int_{-h}^{h}\gamma\Lambda_\zeta d\gamma\right)+$$

$$+\left(\frac{D_\alpha+D_{\beta\alpha}+D_{\gamma\alpha}}{A_1^2A_2}\frac{\partial A_2}{\partial\alpha}-\frac{D_\alpha+D_{\beta\alpha}+D_{\gamma\alpha}}{A_1^3}\frac{\partial A_1}{\partial\alpha}\right)\times$$

$$\times\left(\frac{2h^3}{3}\frac{\partial I_1}{\partial\alpha}+\sum_{\zeta=2}^{\vartheta}\frac{\partial I_\zeta}{\partial\alpha}\int_{-h}^{h}\gamma\Lambda_\zeta d\gamma\right)+$$

$$+\left(\frac{D_{\alpha\beta}+D_\beta+D_{\gamma\beta}}{A_2^2}\right)\times$$

$$\times\left(\frac{2h^3}{3}\frac{\partial^2 I_1}{\partial\beta^2}+\sum_{\zeta=2}^{\vartheta}\frac{\partial^2 I_\zeta}{\partial\beta^2}\int_{-h}^{h}\gamma\Lambda_\zeta d\gamma\right)+$$

$$+\left(\frac{D_{\alpha\beta}+D_\beta+D_{\gamma\beta}}{A_2^2A_1}\frac{\partial A_1}{\partial\beta}-\frac{D_{\alpha\beta}+D_\beta+D_{\gamma\beta}}{A_2^3}\frac{\partial A_2}{\partial\beta}\right)\times$$

$$\times\left(\frac{2h^3}{3}\frac{\partial I_1}{\partial\beta}+\sum_{\zeta=2}^{\vartheta}\frac{\partial I_\zeta}{\partial\beta}\int_{-h}^{h}\gamma\Lambda_\zeta d\gamma\right)+$$

$$+\left(D_{\alpha\gamma}+D_{\beta\gamma}+D_\gamma\right)\times$$

$$\times\sum_{\zeta=2}^{\vartheta}I_\zeta\left(h\left[\Lambda_\zeta(h)+\Lambda_\zeta(-h)\right]-\int_{-h}^{h}\Lambda_\zeta d\gamma\right)=$$

$$=\frac{2h^3}{3}\frac{\partial I_1}{\partial t}+\sum_{\zeta=2}^{\vartheta}\frac{\partial I_\P}{\partial t}\int_{-h}^{h}\gamma\Lambda_\zeta d\gamma.$$

Here, ρ – density of OEC;

c – specific heat capacity of OEC at constant strain tensor;

A_1, A_2 – coefficients of the first quadratic form of the median surface of OEC;

$2h$ – OEC thickness value;

λ_α, λ_β, λ_γ, $\lambda_{\alpha\beta}$, $\lambda_{\beta\alpha}$, $\lambda_{\alpha\gamma}$, $\lambda_{\gamma\alpha}$, $\lambda_{\beta\gamma}$, $\lambda_{\gamma\beta}$ – coefficients of OEC thermal conductivity;

D_α, D_β, D_γ, $D_{\alpha\beta}$, $D_{\alpha\gamma}$, $D_{\beta\alpha}$, $D_{\beta\gamma}$, $D_{\gamma\alpha}$, $D_{\gamma\beta}$ – coefficients of diffusion of elementary particles from corpuscular cosmic radiation in the OEC material;

T_0, T_1, T_η – functions of the law of OEC thermal field distribution across its volume $T(\alpha,\ \beta,\ \gamma,\ t)=T_0(\alpha,\ \beta,\ t)+\gamma T_1(\alpha,\ \beta,\ t)+$ $+\sum_{\eta=2}^{\upsilon}\Omega_\eta(\gamma)T_\eta(\alpha,\ \beta,\ t)$;

I_0, I_1, I_ζ – functions of the law of distribution of elementary particles from corpuscular cosmic radiation across the OEC volume $I(\alpha,\ \beta,\ \gamma,\ t)=I_0(\alpha,\ \beta,\ t)+\gamma I_1(\alpha,\ \beta,\ t)+$ $+\sum_{\zeta=2}^{\vartheta}\Lambda_\zeta(\gamma)I_\zeta(\alpha,\ \beta,\ t)$;

Ω_η – ηth function of a general type that defines the non-linearity of OEC temperature (T) measurement across its thickness;

Λ_ζ – ζth function of a general type, which defines the non-linearity of measurement of concentration (I) of elementary particles from corpuscular cosmic radiation across the OEC thickness;

α, β, γ – orthogonal curvilinear coordinates.

3. Equations derived with the use of boundary conditions developed for facial surfaces of SC OEC [3].

4. Equations derived with the use of initial conditions developed for determining SSS of elastic components at the initial (reference) moment

$$
\begin{aligned}
&u_0(\alpha,\ \beta,\ t=0)+\gamma\psi_0(\alpha,\ \beta,\ t=0)+\\
&+\sum_{i=1}^{n} f_i(\gamma)\psi_i(\alpha,\ \beta,\ t=0)=0;\\
&v_0(\alpha,\ \beta,\ t=0)+\gamma\Phi_0(\alpha,\ \beta,\ t=0)+\\
&+\sum_{j=1}^{m} F_j(\gamma)\Phi_j(\alpha,\ \beta,\ t=0)=0;\\
&\mathbf{w}_0(\alpha,\ \beta,\ t=0)+\gamma\boldsymbol{\delta}_0(\alpha,\ \beta,\ t=0)+\\
&+\sum_{k=1}^{p} R_k(\gamma)\delta_k(\alpha,\ \beta,\ t=0)=0,
\end{aligned}
\tag{3}
$$

where $u_0(\alpha,\ \beta,\ t=0)$, $v_0(\alpha,\ \beta,\ t=0)$ – components of the vector of travel of a random elementary volume of OEC median surface in the plane of this surface at the reference moment;

$\psi_0(\alpha,\ \beta,\ t=0)$, $\Phi_0(\alpha,\ \beta,\ t=0)$ – components of a rotation vector of a random cross-section of OEC relative to its median surface at the reference moment;

$\mathbf{w}_0(\alpha,\ \beta,\ t=0)$ – the vector of transverse (perpendicular to the OEC median surface) travel of a random elementary volume of the OEC median surface at the initial moment;

$\boldsymbol{\delta}_0(\alpha,\ \beta,\ t=0)$ – rotation vector of a random elementary area of the OEC median surface relative to this surface at the initial moment;

$f_i(\gamma)$, $F_j(\gamma)$, $R_k(\gamma)$ – i-th, j-th and k-th functions of a general type that define the non-linearity of travel of a random elementary OEC volume in the direction perpendicular to its median surface;

$\psi_i(\alpha,\ \beta,\ t=0)$, $\Phi_j(\alpha,\ \beta,\ t=0)$, $\delta_k(\alpha,\ \beta,\ t=0)$ – i-th, j-th and k-th required functions at the reference moment.

5. Equations developed for ensuring statistical identifiability of the mathematical complex for multi-parameter system simulation of SSS of outboard elastic components of spacecraft:

$$
\begin{aligned}
&\int_{-h}^{h_i}\Bigg(\rho\frac{\partial^2 U}{\partial t^2}-\sum_{g=1}^{r}F_g^{\alpha}-\frac{1}{A_1}\frac{\partial\sigma_\alpha}{\partial\alpha}-\frac{1}{A_2}\frac{\partial\tau_{\beta\alpha}}{\partial\beta}-\\
&-\frac{1}{A_1A_2}\left[\frac{\partial A_2}{\partial\alpha}(\sigma_\alpha-\sigma_\beta)+2\frac{\partial A_1}{\partial\beta}\tau_{\beta\alpha}\right]\Bigg)d\gamma=\\
&=\Bigg[B_{33}\Bigg(\frac{1}{A_1}\left[\frac{\partial u_0}{\partial\alpha}+\gamma\frac{\partial\psi_0}{\partial\alpha}+\sum_{i=1}^{n}f_i\frac{\partial\psi_i}{\partial\alpha}\right]+\\
&+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\beta}\left[v_0+\gamma\Phi_0+\sum_{j=1}^{m}F_j\Phi_j\right]\Bigg)\Bigg]\Bigg|_{-h}^{h_i}+\\
&+\Bigg[B_{34}\Bigg(\frac{1}{A_2}\left[\frac{\partial v_0}{\partial\beta}+\gamma\frac{\partial\Phi_0}{\partial\beta}+\sum_{j=1}^{m}F_j\frac{\partial\Phi_j}{\partial\beta}\right]+\\
&+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha}\left[u_0+\gamma\psi_0+\sum_{i=1}^{n}f_i\psi_i\right]\Bigg)\Bigg]\Bigg|_{-h}^{h_i}+\\
&+\Bigg[B_{35}\Bigg(\boldsymbol{\delta}_0+\sum_{k=1}^{p}R_k'\delta_k\Bigg)\Bigg]\Bigg|_{-h}^{h_i}+\\
&+\Bigg[B_{36}\Bigg(\frac{1}{A_2}\left[\frac{\partial u_0}{\partial\beta}+\gamma\frac{\partial\psi_0}{\partial\beta}+\sum_{i=1}^{n}f_i\frac{\partial\psi_i}{\partial\beta}\right]+\\
&+\frac{1}{A_1}\left[\frac{\partial v_0}{\partial\alpha}+\gamma\frac{\partial\Phi_0}{\partial\alpha}+\sum_{j=1}^{m}F_j\frac{\partial\Phi_j}{\partial\alpha}\right]-\\
&-\frac{1}{A_1A_2}\Bigg[\frac{\partial A_1}{\partial\beta}\Bigg(u_0+\gamma\psi_0+\sum_{i=1}^{n}f_i\psi_i\Bigg)+\\
&+\frac{\partial A_2}{\partial\alpha}\Bigg(v_0+\gamma\Phi_0+\sum_{j=1}^{m}F_j\Phi_j\Bigg)\Bigg]\Bigg)\Bigg]\Bigg|_{-h}^{h_i}+\\
&+\Bigg[B_{37}\Bigg(\psi_0+\sum_{i=1}^{n}f_i'\psi_i+\frac{1}{A_1}\times\\
&\times\left[\frac{\partial\mathbf{w}_0}{\partial\alpha}+\gamma\frac{\partial\boldsymbol{\delta}_0}{\partial\alpha}+\sum_{k=1}^{p}R_k\frac{\partial\delta_k}{\partial\alpha}\right]\Bigg)\Bigg]\Bigg|_{-h}^{h_i}+
\end{aligned}
\tag{4}
$$

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$$+\left[B_{38}\left(\Phi_0+\sum_{j=1}^{m}F'_j\Phi_j+\frac{1}{A_2}\times\right.\right.$$

$$\left.\left.\left.\times\left[\frac{\partial \mathbf{w}_0}{\partial\beta}+\gamma\frac{\partial \boldsymbol{\delta}_0}{\partial\beta}+\sum_{k=1}^{p}R_k\frac{\partial\delta_k}{\partial\beta}\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{39}\left(T_0+\gamma T_1+\sum_{\eta=2}^{\upsilon}\Omega_\eta T_\eta\right)+\right.$$

$$\left.\left.+B_{40}\left(I_0+\gamma I_1+\sum_{\zeta=2}^{\vartheta}\Lambda_\zeta I_\zeta\right)\right]\right|_{-h}^{h_i};$$

$$\int_{-h}^{h_i}\left(\rho\frac{\partial^2 V}{\partial t^2}-\sum_{g=1}^{r}F_g^{\beta}-\frac{1}{A_1}\frac{\partial\tau_{\alpha\beta}}{\partial\alpha}-\frac{1}{A_2}\frac{\partial\sigma_\beta}{\partial\beta}-\right.$$

$$\left.-\frac{1}{A_1A_2}\left[\frac{\partial A_1}{\partial\beta}(\sigma_\beta-\sigma_\alpha)+2\frac{\partial A_2}{\partial\alpha}\tau_{\alpha\beta}\right]\right)d\gamma=$$

$$=\left[B_{41}\left(\frac{1}{A_1}\left[\frac{\partial u_0}{\partial\alpha}+\gamma\frac{\partial\psi_0}{\partial\alpha}+\sum_{i=1}^{n}f_i\frac{\partial\psi_i}{\partial\alpha}\right]+\right.\right.$$

$$\left.\left.\left.+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\beta}\left[v_0+\gamma\Phi_0+\sum_{j=1}^{m}F_j\Phi_j\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{42}\left(\frac{1}{A_2}\left[\frac{\partial v_0}{\partial\beta}+\gamma\frac{\partial\Phi_0}{\partial\beta}+\sum_{j=1}^{m}F_j\frac{\partial\Phi_j}{\partial\beta}\right]+\right.\right.$$

$$\left.\left.\left.+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha}\left[u_0+\gamma\psi_0+\sum_{i=1}^{n}f_i\psi_i\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left.\left[B_{43}\left(\boldsymbol{\delta}_0+\sum_{k=1}^{p}R'_k\delta_k\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{44}\left(\frac{1}{A_2}\left[\frac{\partial u_0}{\partial\beta}+\gamma\frac{\partial\psi_0}{\partial\beta}+\sum_{i=1}^{n}f_i\frac{\partial\psi_i}{\partial\beta}\right]+\right.\right.$$

$$+\frac{1}{A_1}\left[\frac{\partial v_0}{\partial\alpha}+\gamma\frac{\partial\Phi_0}{\partial\alpha}+\sum_{j=1}^{m}F_j\frac{\partial\Phi_j}{\partial\alpha}\right]-$$

$$-\frac{1}{A_1A_2}\left[\frac{\partial A_1}{\partial\beta}\left(u_0+\gamma\psi_0+\sum_{i=1}^{n}f_i\psi_i\right)+\right.$$

$$\left.\left.\left.\left.+\frac{\partial A_2}{\partial\alpha}\left(v_0+\gamma\Phi_0+\sum_{j=1}^{m}F_j\Phi_j\right)\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{45}\left(\psi_0+\sum_{i=1}^{n}f'_i\psi_i+\frac{1}{A_1}\times\right.\right.$$

$$\left.\left.\left.\times\left[\frac{\partial \mathbf{w}_0}{\partial\alpha}+\gamma\frac{\partial \boldsymbol{\delta}_0}{\partial\alpha}+\sum_{k=1}^{p}R_k\frac{\partial\delta_k}{\partial\alpha}\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{46}\left(\Phi_0+\sum_{j=1}^{m}F'_j\Phi_j+\frac{1}{A_2}\times\right.\right.$$

$$\left.\left.\left.\times\left[\frac{\partial \mathbf{w}_0}{\partial\beta}+\gamma\frac{\partial \boldsymbol{\delta}_0}{\partial\beta}+\sum_{k=1}^{p}R_k\frac{\partial\delta_k}{\partial\beta}\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{47}\left(T_0+\gamma T_1+\sum_{\eta=2}^{\upsilon}\Omega_\eta T_\eta\right)+\right.$$

$$\left.\left.+B_{48}\left(I_0+\gamma I_1+\sum_{\zeta=2}^{\vartheta}\Lambda_\zeta I_\zeta\right)\right]\right|_{-h}^{h_i};$$

$$\int_{-h}^{h_i}\left(\rho\frac{\partial^2 W}{\partial t^2}-\sum_{g=1}^{r}F_g^{\gamma}-\frac{1}{A_1}\frac{\partial\tau_{\alpha\gamma}}{\partial\alpha}-\frac{1}{A_2}\frac{\partial\tau_{\beta\gamma}}{\partial\beta}-\right.$$

$$\left.-\frac{1}{A_1A_2}\left[\frac{\partial A_2}{\partial\alpha}\tau_{\alpha\gamma}+\frac{\partial A_1}{\partial\beta}\tau_{\beta\gamma}\right]\right)d\gamma=$$

$$=\left[B_{17}\left(\frac{1}{A_1}\left[\frac{\partial u_0}{\partial\alpha}+\gamma\frac{\partial\psi_0}{\partial\alpha}+\sum_{i=1}^{n}f_i\frac{\partial\psi_i}{\partial\alpha}\right]+\right.\right.$$

$$\left.\left.\left.+\frac{1}{A_1A_2}\frac{\partial A_1}{\partial\beta}\left[v_0+\gamma\Phi_0+\sum_{j=1}^{m}F_j\Phi_j\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{18}\left(\frac{1}{A_2}\left[\frac{\partial v_0}{\partial\beta}+\gamma\frac{\partial\Phi_0}{\partial\beta}+\sum_{j=1}^{m}F_j\frac{\partial\Phi_j}{\partial\beta}\right]+\right.\right.$$

$$\left.\left.\left.+\frac{1}{A_1A_2}\frac{\partial A_2}{\partial\alpha}\left[u_0+\gamma\psi_0+\sum_{i=1}^{n}f_i\psi_i\right]\right)\right]\right|_{-h}^{h_i}+$$

$$+\left.\left[B_{19}\left(\boldsymbol{\delta}_0+\sum_{k=1}^{p}R'_k\delta_k\right)\right]\right|_{-h}^{h_i}+$$

$$+\left[B_{20}\left(\frac{1}{A_2}\left[\frac{\partial u_0}{\partial\beta}+\gamma\frac{\partial\psi_0}{\partial\beta}+\sum_{i=1}^{n}f_i\frac{\partial\psi_i}{\partial\beta}\right]+\right.\right.$$

$$+\frac{1}{A_1}\left[\frac{\partial v_0}{\partial\alpha}+\gamma\frac{\partial\Phi_0}{\partial\alpha}+\sum_{j=1}^{m}F_j\frac{\partial\Phi_j}{\partial\alpha}\right]-$$

$$-\frac{1}{A_1A_2}\left[\frac{\partial A_1}{\partial\beta}\left(u_0+\gamma\psi_0+\sum_{i=1}^{n}f_i\psi_i\right)+\right.$$

$$\left.\left.\left.+\frac{\partial A_2}{\partial\alpha}\left(v_0+\gamma\Phi_0+\sum_{j=1}^{m}F_j\Phi_j\right)\right]\right)\right]\Bigg|_{-h}^{h_i}+$$

$$+\left[B_{21}\left(\psi_0+\sum_{i=1}^{n}f_i'\psi_i+\frac{1}{A_1}\times\right.\right.$$

$$\left.\left.\times\left[\frac{\partial\mathbf{w}_0}{\partial\alpha}+\gamma\frac{\partial\boldsymbol{\delta}_0}{\partial\alpha}+\sum_{k=1}^{p}R_k\frac{\partial\delta_k}{\partial\alpha}\right]\right)\right]\Bigg|_{-h}^{h_i}+$$

$$+\left[B_{22}\left(\Phi_0+\sum_{j=1}^{m}F_j'\Phi_j+\frac{1}{A_2}\times\right.\right.$$

$$\left.\left.\times\left[\frac{\partial\mathbf{w}_0}{\partial\beta}+\gamma\frac{\partial\boldsymbol{\delta}_0}{\partial\beta}+\sum_{k=1}^{p}R_k\frac{\partial\delta_k}{\partial\beta}\right]\right)\right]\Bigg|_{-h}^{h_i}+$$

$$+\left[B_{23}\left(T_0+\gamma T_1+\sum_{\eta=2}^{\upsilon}\Omega_\eta T_\eta\right)+\right.$$

$$\left.+B_{24}\left(I_0+\gamma I_1+\sum_{\zeta=2}^{\vartheta}\Lambda_\zeta I_\zeta\right)\right]\Bigg|_{-h}^{h_i}.$$

Here, h_i – the coordinate of the OEC median surface or of the surface equidistant to it (coordinate $h_i = 0$ corresponds to the OEC median surface);

U, V, W – components of the vector of travel of a random OEC elementary volume;

$B_i (i = 1, 2, \ldots, 48)$ – elasticity coefficients of SC OEC;

$\sigma_\alpha, \sigma_\beta, \sigma_\gamma$ – normal stresses;

$\tau_{\beta\alpha} = \tau_{\alpha\beta}, \tau_{\gamma\alpha} = \tau_{\alpha\gamma}, \tau_{\gamma\beta} = \tau_{\beta\gamma}$ – tangential stresses;

$F_g^\alpha, F_g^\beta, F_g^\gamma$ – components of the vector of the g-th external mass (volume) effect.

6. The equations that impose boundary conditions for butt surfaces of SC OEC [3]. These are developed with the use of data on conditions of fixities of oriented and non-controlled OEC to elastic bodies of SC and on conditions of joining butt surfaces of OEC with the equipment or instruments of special-purpose or service systems.

7. Equations that impose boundary conditions and are developed for the lateral surfaces of SC bodies, taking into account the components of transverse frameworks (butt and intermediate frames) in their structures:

$$\left.\begin{array}{l}\mathbf{w}_0(\alpha=\alpha_w,\ \beta=\beta_w,\ t)=0;\\ \boldsymbol{\delta}_0(\alpha=\alpha_w,\ \beta=\beta_w,\ t)=0;\\ \delta_k(\alpha=\alpha_w,\ \beta=\beta_w,\ t)=0;\end{array}\right\}\Rightarrow$$
$$\Rightarrow W(\alpha=\alpha_w,\ \beta=\beta_w,\ \gamma,\ t)=0, \quad (5)$$
$$w=1,\ \ldots,\ r,$$

where α_w, β_w – coordinates of the w-th zone of the lateral SC surface reinforced with a component of the transverse framework;

r – number of components of the transverse framework in the SC structure.

8. Equations for fixities (attachments) of the orientable and non-controlled OEC to elastic bodies of SC designed with taking into account the mutual effects of oscillations of body elastic shells (BES) and of OEC that influence the parameters of their oscillatory motions:

$$\mathbf{w}_{\text{ВУЭ}}(\alpha=0,\ \beta=\beta_m,\ t)=$$
$$=\sin\Omega_1\cos\Omega_2 u_{\text{УОК}}(\alpha=\alpha_n,\ \beta=\beta_n,\ t);$$

$$\mathbf{w}_{\text{ВУЭ}}(\alpha=0,\ \beta=\beta_m,\ t)=$$
$$=\cos\Omega_2\,\psi_{\text{УОК}}(\alpha=\alpha_n,\ \beta=\beta_n,\ t);$$

$$\mathbf{w}_{\text{ВУЭ}}(\alpha=0,\ \beta=\beta_m,\ t)=$$
$$=\sin\Omega_2\,\nu_{\text{УОК}}(\alpha=\alpha_n,\ \beta=\beta_n,\ t); \quad (6)$$

$$\mathbf{w}_{\text{ВУЭ}}(\alpha=0,\ \beta=\beta_m,\ t)=$$
$$=\sin\Omega_1\sin\Omega_2\,\Phi_{\text{УОК}}(\alpha=\alpha_n,\ \beta=\beta_n,\ t);$$

$$\mathbf{w}_{\text{ВУЭ}}(\alpha=0,\ \beta=\beta_m,\ t)=$$
$$=\cos\Omega_1\cos\Omega_2\mathbf{w}_{\text{УОК}}(\alpha=\alpha_n,\ \beta=\beta_n,\ t);$$

$$\mathbf{w}_{ВУЭ}(\alpha = 0,\ \beta = \beta_m,\ t) =$$

$$= \boldsymbol{\delta}_{УОК}(\alpha = \alpha_n,\ \beta = \beta_n,\ t);$$

$$k = n = m = 1,\ \ldots,\ p,$$

where $\mathbf{w}_{ВУЭ}$ – vector of transverse travel of a random elementary volume of the median surface of the k-th OEC;

$u_{УОК}$, $v_{УОК}$ – components of the travel vector of a random elementary volume of the BES median surface of in the plane of this surface;

$\psi_{УОК}$, $\Phi_{УОК}$ – components of the rotation vector of a random cross-section of an SC body elastic shell relative to its median surface;

$\mathbf{w}_{УОК}$ – travel vector of a random elementary volume of the BES median surface perpendicular to this surface;

$\boldsymbol{\delta}_{УОК}$ – rotation vector of a random elementary area of the median surface of SC elastic shell relative to this surface;

α_n, β_n – coordinates of the lateral surface of SC, to which the n-th zone of attachment of the k-th outboard elastic component corresponds;

β_m – coordinate of the zero butt surface of the k-th OEC (the surface with which the zero of the O system of orthogonal curvilinear coordinates $O\alpha\beta\gamma$ is aligned) calculated by the OEC width to which the m-th zone attached to the elastic body of the spacecraft corresponds;

Ω_1 – the turning angle of the k-th OEC relative to the spacecraft body (the angle between the longitudinal axis of the k-th OEC and the line of the lateral surface of the spacecraft body);

Ω_2 – turning angle of the k-th OEC relative to its longitudinal axis;

p – number of outboard elastic components in the SC structure.

The mathematical complex for multi-parameter system simulation of stress-strain states of the outboard elastic components of spacecraft ensures predicting of ranges of their oscillation frequencies.

Credibility of the mathematical simulation complex and requirements for oscillation damping controlled systems

The credibility of the developed equations has been proved by the results of a comparative analysis of frequency values of OEC oscillations in the existing spacecraft obtained in theoretical calculations and in experiments. The difference between theoretical and experimental values of oscillation frequencies is within 5 %. Results of theoretical and experimental investigations of oscillation frequencies of narrow, broad, and rod plate OEC are given it the Table.

Papers [2, 3] justify the requirements for controlled ODS of elastic components of spacecraft. Below are their definitions.

1. Control circuits of ODS and travel control systems of spacecraft shall function in mutual coordination.

2. Sensors located on surfaces of OEC shall record the parameters of their oscillatory motions within the ranges of oscillatory motions that appear during flight missions.

3. Frequencies of reading data from sensors shall exceed those of the outboard elastic component oscillations.

4. Technical characteristics of driving units shall ensure such angular speeds of output shafts of rotors that correspond to frequencies of forced antiresonance oscillations which stabilize SC OEC.

5. The driving units shall be of multi-mode type and shall generate forced antiresonance oscillations of the outboard components with parameters that would be measured in accordance with control signals of onboard computers.

Using the requirements for controlled ODS and OEC oscillation frequency ranges, it is possible to determine exactly the design parameters of oscillation damping systems employed for damping oscillations in SC OEC.

Results of theoretical and experimental investigations of frequencies of oscillations in outboard elastic components (OEC)

Parameter	Sequence number of the oscillation frequency in the range	Oscillation frequency value obtained in theoretical calculation, Hz	Oscillation frequency value obtained in experiments, Hz
Narrow OEC	1 2 3 4 5 20	0.820 0.950 1.910 4.780 7.320 69.41	0.84 0.99 2.00 5.00 7.00 72.0
Broad OEC	1	0.315	0.32
	2	0.352	0.34
	3	0.367	0.38
	17	0.655	0.65
	24	0.796	0.76
	97	5.817	5.76
Rod OEC	1	0.830	0.86
	2	0.850	0.87
	20	49.78	52.0
	52	149.73	146

Conclusion

The developed method for determining the design parameters of controlled oscillation damping systems of SC OEC ensures efficiency at the initial stages of design of these systems.

OEC oscillation damping with the use of controlled ODS are performed in millisecond time spans.

The performed calculations show that the use of oscillation damping controlled systems as part of SC travel control systems helps to decrease the time for manoeuvring and changing angular positions of SC in their flight missions by 20–27 %.

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Методика определения проектных параметров систем гашения колебаний конструкций космических аппаратов

В конструкциях современных космических аппаратов используются выносные упругие элементы для размещения аппаратуры и устройств, которые при изменении угловых положений аппаратов и их стабилизации на орбитах приходят в колебательное движение. Для снижения влияния колебаний на динамику аппаратов резервируют время ожидания их успокоения, составляющее единицы минут. Представлена методика определения проектных параметров систем гашения колебаний, обеспечивающих успокоение колебательных движений в миллисекундных диапазонах времени.

Ключевые слова: космический аппарат, система гашения колебаний, проектные параметры, напряженно-деформированное состояние, выносные упругие элементы, колебания, антирезонанс.

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