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## Mathematical model of radio engineering ship system errors in the autoland problems studies

The paper describes the results of statistical processing of flight path errors recorded on board aircraft during aircraft carrier landing approaches using the S-band radar system and the microwave radio-beacon landing system. The analysis and statistical processing data averaging is carried out. A mathematical model of a random (noise) component of the measurement error of radio engineering landing systems used on aircraft carriers is built. The study introduces a mathematical model based on actual data of full-scale tests, which allows us to carry out a wide range of studies to determine the accuracy of various types aircraft landing approaches, including the presence of ship rocking and spatial evolution of the aircraft. The results of mathematical modelling are presented.

*Keywords:* aircraft, ship, landing system, statistical error handling.

### Introduction

The effectiveness of naval aviation to a large extent depends on the performance of instrument ship-landing systems. At present, precision approach radar (PAR) systems and microwave landing systems of the *MLS* type [1, 2] are widely used in Russia and abroad to ensure aircraft landing on aircraft carriers. Unlike the widespread VHF ground landing systems of ILS type and UHF systems of radio beacon landing aids, they have smaller dimensions and higher accuracy, which ensures their effectiveness when used in aircraft landing on board ships.

The behaviour pattern of guidance signal errors on different segments of approach glide slope in real operating conditions is of great interest to developers of airborne navigation and landing systems of naval aviation aircraft. The use of “rated” tactical and technical characteristics of systems when developing algorithms of complex data processing for airborne computing devices may lead to additional errors [3] and, as a consequence, to a decrease in the operational effectiveness of naval aviation application.

The purposes of the present paper are estimation and analysis of the statistical characteristics of the guidance signals for ship landing systems by processing of the digital values of

these signals registered on board aircraft for development of mathematical models and software complexes intended for carrying out research on the characteristics of automatic landing systems.

### Initial data

For estimating statistical characteristics of the guidance signals during aircraft landing on an aircraft carrier, this paper uses the results of full-scale tests obtained during approach training of two aircrafts equipped with PAR and *MLS* airborne equipment. To study the characteristics of PAR and *MLS* errors, ten instrument approaches from a distance of 8 km to the edge of the ship’s flight deck were selected. The approaches were made following a standard shipboard landing glide path with a slope angle of 4°, with pilots using flight director control mode.

The shipboard landing approaches were carried out in the daytime, in visual weather conditions, at ship’s speed up to 22 knots and aircraft ground speed of about 250 km/h.

The values of glide slope off-course signals obtained on board the aircraft were used in the systems of flight and navigation complex to provide data for the auto pilot and flight director control system and were recorded in special-purpose flight data recorder at a frequency of 3 Hz.

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**Test results processing methodology**

During the tests, no reference facilities and external trajectory measurements from more accurate systems were available, therefore, the statistical processing was aimed at estimation of the high-frequency components of the guidance signal errors, i.e. estimation of the noise component of *PFN* path following [2]. At the same time, considering fairly high inertia of the aircraft, it was decided to perform approximation of the recorded measurements by a 3rd order polynomial at relatively small time intervals (ranges) so as to preclude systematic errors of measurements and aircraft actual deviation from the selected trajectory. Such method is traditionally used in flight test processing technologies [4]. At the same time, spectral characteristics of landing system errors were not investigated, since data recording was performed with 3 Hz frequency, under the real rate of landing systems’ information update of 13...20 Hz.

Statistical processing of the recorded data from *PAR* and *MLS* systems was done with the use of *MATLAB* and *Excel* programs. The measurement arrays recorded during each approach were divided into segments, which were approximated by a 3rd order polynomial with coefficients calculated by the least-squares method. The polynomial approximant was taken for a mathematical

expectation, with differences generated between it and the initial data. Then a root-mean-square error (RMSE) of the measurements was calculated for range intervals of 0.5 km. The obtained RMSE values were averaged for all landing approaches using some or other system (*PAR* or *MLS*, and the characteristic curves for respective error magnitude for high-frequency guidance signal component vs. aircraft distance to the touchdown aim point (TDAP) were plotted.

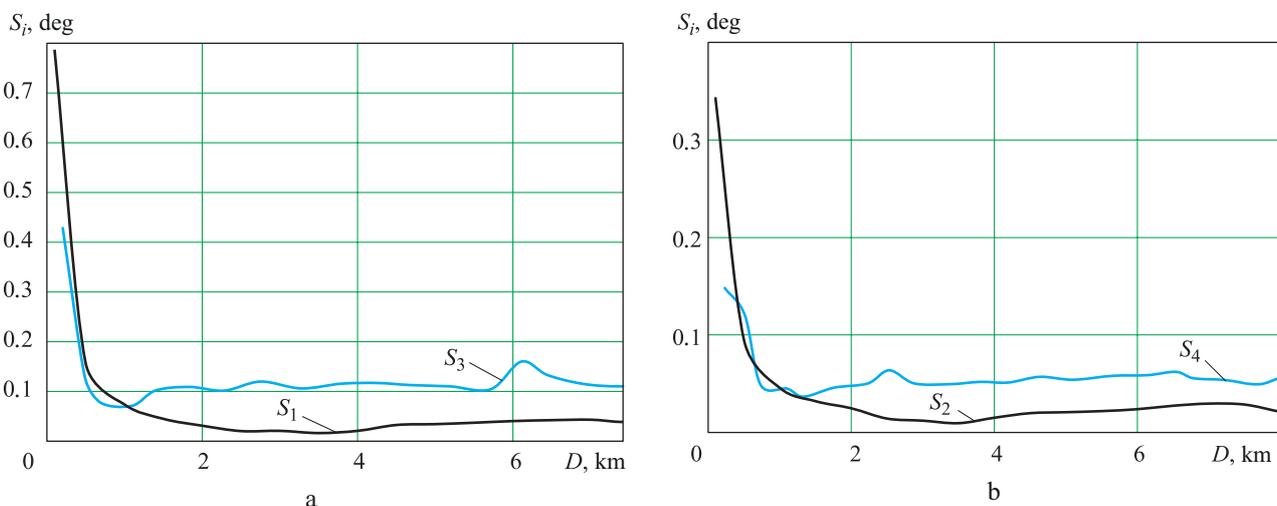
Hereinafter, the following notations will be used for the obtained error estimates:

- $S_1$  – *PAR* error in the yaw channel;
- $S_2$  – *PAR* error in the glide slope channel;
- $S_3$  – *MLS* error in the yaw channel;
- $S_4$  – *MLS* error in the glide slope channel.

The estimates of  $S_i$  errors given below represent a doubled value of the calculated RMSE, which is basically equivalent to a probability of 0.95.

**Statistical data processing results**

Fig. 1 shows guidance signal errors  $S_i$  (with 0.95 probability) as a function of the distance between aircraft and TDAP during shipboard landing approaches when using the guidance signals from *PAR* and *MLS*, obtained as a result of data processing according to the methodology described above. The presented data partially account for the earlier obtained results [5].



**Fig. 1.** Guidance signal errors for *PAR* and *MLS* systems, depending on the distance between aircraft and estimated landing point:  
 a – yaw channel; b – glide slope channel



It follows from the results of statistical processing of the guidance signals recorded during landing approaches, as shown in Fig. 1, that:

1) at a distance over 2 km from TDAP, the errors of yaw  $S_1$  and glide slope  $S_2$  guidance signals for the PAR system virtually do not change, whereas drawing closer to TDAP, they increase at first smoothly (up to 1 km away) and then drastically (6–9-fold);

2) at a distance over 1 km from TDAP, the errors of yaw  $S_3$  and glide slope  $S_4$  guidance signals for the *MLS* system virtually do not change, whereas drawing closer to TDAP, they increase drastically (3–5-fold);

3) at a distance over 2 km from TDAP, the errors of yaw and glide slope guidance signals for the PAR system are 3–4 times as small as for the *MLS* system, whereas at a distance of about 1 km from TDAP, the errors of both systems are virtually equal;

4) during approach with the use of PAR at distances over 2 km from TDAP, the high-frequency components of the errors of yaw  $S_1$  and glide slope  $S_2$  guidance signals are effectively equal to about  $0.03^\circ$  and  $0.02^\circ$ , respectively;

5) during approach with the use of *MLS* at distances over 1 km from TDAP, the high-frequency components of the errors of yaw  $S_3$  and glide slope  $S_4$  guidance signals are effectively equal to about  $0.11^\circ$  and  $0.06^\circ$ , respectively.

#### **Analysis of statistical data processing results**

The behaviour pattern of the RMSE of the guidance signals from the PAR and *MLS* systems, as given in Fig. 1, can be explained as follows.

When using primary S-band and M-band radars to form guidance signals, there emerges an issue of an additional error in defining the glide slope off-course descent by aircraft versus the preset value, as caused by geometric dimensions of the aircraft [1]. When the aircraft is at relatively large distances from the radar, this error is determined mainly by the value of the aircraft radar cross-section the radar emitting power, however, when the aircraft approaches the radar antenna

installation point, multiple re-reflections from the elements of aircraft fuselage and tail unit are essential. When the angular size of the radio-frequency signal reflection area becomes equal to or exceeding the values of the allowable angular measurement errors preset for the radar, this error may exceed the preset requirements.

The installation location of radar is determined by technical possibilities of its arrangement on board the ship. When making a landing approach, the aircraft is always drawing closer to the radar installation point, and its geometric dimensions may significantly influence the measurement error. The problem of decreasing measurement error of angular deflections in the PAR systems can be solved by installing angle reflectors on the aircraft or applying the principles of a secondary radar [1, 6]. The problem under consideration is particularly essential for radar landing systems used for unmanned aerial vehicles landing in the automatic mode [7].

When used for generation of guidance signals in radio beacon landing systems of *MLS* type, the error of the high-frequency component of the guidance signals increases as the aircraft approaches TDAP, as noted above. A similar effect is observed in many radio beacon systems [8] when the aircraft approaches the radio beacons. It is conditioned by the interaction of airborne antenna patterns and the radio navigation field generated by the radio beacon antenna system under rapid changes of signal level at the airborne receiver input. This leads to additional errors in the operation of the airborne receiver radio paths tracking systems.

#### **Mathematical model of errors**

When developing the algorithms of complex processing of data for airborne navigation and landing systems, a model of the guidance signal errors at different segments of the glide-slope trajectory, depending on the distance to TDAP, is of special interest. The use of such model allows to develop optimal algorithms that essentially increase the effectiveness of control signals generation,



including those for aircraft AP/FD control system. The results of statistical data processing given in Fig. 1 make it possible to propose the following models of high-frequency components of the *PFN* guidance signal errors of *PAR* and *MLS* shipboard landing systems.

For the *PAR* and *MLS* systems, the following dependencies can be taken as a mathematical model for the high-frequency components of measurement errors when making shipboard landing approaches:

$$S_i = \begin{cases} k_i, & \text{at } D > 1 \text{ km;} \\ k_i + f_i(D), & \text{at } D \leq 1 \text{ km.} \end{cases}$$

Here,  $S_i$  – respective measurement errors (at  $i = 1$  in yaw for *PAR*, at  $i = 2$  in glide slope for *PAR*, at  $i = 3$  in yaw for *MLS*, at  $i = 4$  in glide slope for *MLS*);

$k_i$  – parameters that are constants, taking the following values:  $k_1 = 0.03 \dots 0.05^\circ$ ;  $k_2 = 0.03 \dots 0.03^\circ$ ;  $k_3 = 0.10 \dots 0.13^\circ$ ;  $k_4 = 0.04 \dots 0.06^\circ$ ;  $f_i(D)$  – function linearly decreasing with distance.

In this case,  $f_i(D) = 0$  at  $D = 1$  km; the maximum value of  $f_i(D)$  at  $D = 0$  km (in TDAP) is determined by the landing system type and the measurement channel:  $f_1(0) = 0.79^\circ$ ,  $f_2(0) = 0.35^\circ$ ;  $f_3(0) = 0.45^\circ$ ;  $f_4(0) = 0.15^\circ$ .

The use of a range of values for coefficient  $k_i$  is conditioned by the insufficient volume of statistical material; further refinement by increasing the amount of the initial data is needed.

### Modelling shipboard landing system errors

The error model suggested above was used in the development and debugging of data generating algorithms in the aircraft onboard navigation and landing systems for investigation of the characteristics of aircraft carrier landing systems in the presence of ship rocking and spatial evolution of the aircraft.

Given in Fig. 2 as an example are the results of digital modelling of errors occurring in the yaw (Fig. 2, a) and glide slope (Fig. 2, b) channels of the radar instrument landing system in the

presence of spatial evolutions of the aircraft and pitching, rolling, and heaving motions of the ship with characteristics presented in Fig. 2, c, d, and e, respectively.

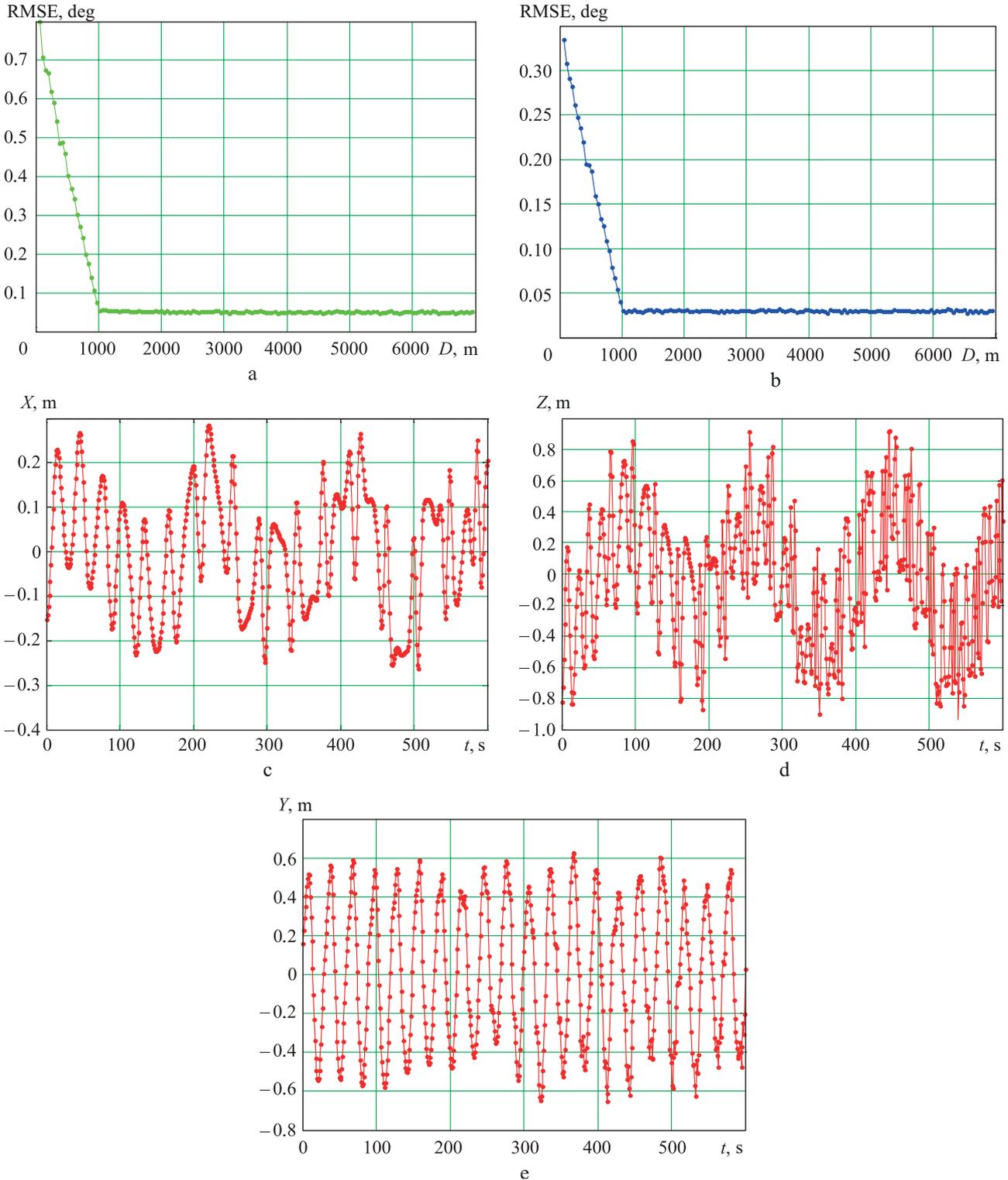
The modelling was carried out in the *MATLAB* environment. The ship's motions were simulated using regular ship rocking equations [9].

Based on the analysis results of data obtained through modelling in the software package, it can be concluded that the generated measurement error values of the guidance signals from the instrument landing systems are on the whole consistent with the results obtained in full-scale tests. However, the accepted error model will be more optimistic at a distance of 2...1 km than the results obtained from statistical processing of the test results.

### Conclusion

The statistical characteristics of guidance signals during instrument approach using radar (*PAR*) and radio-beacon (*MLS*) landing systems have been evaluated. The experimentally obtained dependencies of the estimates of errors in determining guidance signals in yaw and glide slope from the estimated landing point can be fairly accurately approximated by using relatively simple mathematical expressions. Based on the results of statistical processing of the full-scale tests data, a mathematical model was developed, generating measurement errors of instrument landing systems used on aircraft carriers. The model also accounts for the real characteristics and parameters of ship's motions and spatial evolution of the aircraft.

The performed modelling of aircraft approaches and the errors of guidance signals obtained in the process demonstrate a general consistency between the guidance signal errors generated in the software system and the available full-scale tests data. It should be appropriate to use the developed software package in conducting research on the characteristics of systems of automatic landing on aircraft carriers. It is also expedient to run a comparison between the theoretical



**Fig. 2.** Results of digital modelling of errors occurring in yew and glide slope channels of radar instrument landing system

calculations and the obtained data for clarifying and correcting the proposed mathematical model, and perform modelling of approaches under different parameters of ship's motions and evolutions of the aircraft. To improve reliability of the

guidance signal errors estimations, it is necessary to increase the initial data amount.

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## Математическая модель ошибок радиотехнических корабельных систем для исследования проблем автоматической посадки

Приведены результаты статистической обработки зарегистрированных на борту летательных аппаратов отклонений от заданной траектории при выполнении заходов на посадку на авианесущий корабль при использовании радиолокационной системы сантиметрового диапазона радиоволн и микроволновой радиомаячной системы посадки. Проведен анализ и выполнено осреднение результатов статистической обработки. Построена математическая модель случайной (шумовой) составляющей ошибки измерений радиотехнических систем посадки, используемых на авианесущих кораблях. Предложена математическая модель, основанная на реальных данных натуральных испытаний, которая позволяет проводить исследования широкого спектра по определению точностных характеристик выполнения захода на посадку на корабли различного типа, в том числе при наличии качки корабля и пространственных эволюций летательного аппарата. Приведены результаты математического моделирования. **Ключевые слова:** летательный аппарат, корабль, система посадки, статистическая обработка ошибок.



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