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Angular superresolution of signals using virtual antenna arrays

We analysed existing publications concerning virtual antenna arrays and determined the limitations of using them in radar systems for the case of prior uncertainty regarding angular positions of signal sources. The paper shows that it is possible to increase angular coordinate resolution for the case of prior uncertainty regarding angular positions of signal sources by employing a virtual antenna array at typical signal-to-noise ratios used in radar signal processing. We provide results of simulating the signals numerically, which confirm our analytical calculations.

Keywords: angular superresolution of signals, virtual antenna arrays, spatial structure extrapolation

With the development of radio electronic information systems (REIS), demands on the quality of their basic performance characteristics are ever increasing [1, 2]. The validity of determining the composition of group targets, which depends primarily on REIS resolution capability by angular coordinates, speed, and range, is of particular importance for REIS functioning [3].

Until the late 1950s, accepted as the resolution capability boundary in various fields of natural sciences was the limit introduced by Rayleigh in 1888. This limit is conditioned by the characteristics of real instrument function $F(\theta)$ of the system, which determine the resolution capability by this parameter θ [4].

The angular resolution value is limited by the actual weight-and-dimensional characteristics of the antenna system. In this respect, to improve validity of determining the composition of group targets, the research of super-Rayleigh angular resolution of signals, i. e., such that goes beyond the Rayleigh resolution limit, has been carried out.

At present, the primary tasks of super-Rayleigh resolution are related to a new class of incorrectly set tasks [4].

To analyse the efficiency of the super-Rayleigh resolution methods being developed, various estimates have been offered, of which three parameters are basically used [2, 4–7]:

1) the first one determines a relative value of Rayleigh criterion excess by the analysed parameter;

2) the second – error of the parameter values found;

3) the third – signal-to-noise ratio (SNR) at which the specified value of Rayleigh criterion excess by the analysed parameter is achieved.

As compared with the Rayleigh resolution, the proposed methods of super-Rayleigh processing, as a new class of incorrectly set tasks, are less tolerant to various interference factors. For this reason the first and the third parameters are often combined into one, and the relative value of Rayleigh criterion excess is made contingent on SNR [2, 4–7].

Analysis of the efficiency of the known methods of angular superresolution, such as Berg's, Capon's, *MUSIC*, and other algorithms, demonstrates that a pair of equipotent sources distanced to the half-width $\Delta\Theta/2$ and one-third $\Delta\Theta/3$ of the radiation pattern (RP) are resolved if SNR for each of the sources equals to 17…22 dB and 25…32 dB, respectively [2, 5, 6].

Apart from the compulsory availability of a large SNR substantially exceeding the signal-to-noise ratio achieved in the radar systems, the known methods of angular superresolution imply other disadvantages as well. Thus, when using those methods, one will often observe not only

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poor conditioning of the correlation matrices of signals, which leads to computation procedure instability, but also the need for a priori knowledge of the number of sources whose bearings are taken. The latter quite often leads to biased estimates of the angular coordinates.

To overcome these disadvantages, various options for building virtual antenna arrays (VAA) are proposed [8–19].

Signals of the virtual elements of VAA are shaped within the boundaries of a virtual aperture, which lies beyond the aperture of real antenna array (RAA), through prolongation of the signals from the existing AE by various methods [8–19]. However, signals shaped analytically shall be adequate to real signals that could be received at the inputs of real aperture if it were equal to the virtual one.

It means that the use of signals of virtual elements in the VAA is equivalent to an increase of real antenna system aperture by the virtual aperture value.

It should also be noted that SNR at the outputs of RAA elements required for formation of the known VAA [8–14] is much higher than the SNR achieved at the antenna element outputs in the radar system.

As a result, the known VAA are applied in direction finding, since a great SNR at the RAA output, as required for implementation of the existing algorithms of super-Rayleigh angular resolution, really exists during direction finding [8–14].

The SNR at the outputs of radar system RAA antenna elements is much lower [1–3] than required for implementation of the existing VAA [8–17], therefore it is impossible to form VAA on the basis of radar antenna system using the known signal extrapolation methods [8–17].

In order to overcome this shortcoming of VAA formation, papers [15–17] propose to reduce the effect of random component on the received signal value by means of collective processing of signals [15, 17], which is supposed to ensure SNR of at least 10…14 dB at the outputs of RAA antenna elements.

However, the numerical results given in papers [15–17] demonstrate that the proposed method of VAA formation is only efficient in a special case when the position of signal sources is symmetrical relative to the expected angular direction of signal reception θ_{om}.

In case of arbitrary spatial arrangement of the signal sources, the proposed method of VAA formation leads to serious errors in determination of their angular coordinates.

For better understanding of the proposed processing and simplification of the obtained expressions, yet without loss of task solution commonality, let us consider real (RL) and virtual (VL) linear antenna arrays (LAA) of isotropic AE instead of phased antenna arrays.

Signal $s(t, \theta_m)$, received from the expected angular direction of signal reception θ_{om} and representing an additive mixture of signals from M-point objects located in a single resolvable volume at angular directions θ_m, can be written down [2, 3] as follows:

$$s(t,\theta_m)=\sum_{m=1}^{M} s_m(t-\tau_m,\theta_m),\quad m=\overline{1,M}. \qquad (1)$$

Here, $s_m(t-\tau_m, \theta_m)$ – signal received from the m-th object;

$\tau_m = R_m/c$ – delay time in arrival of the phase front of signal from the m-th object to the first AE;

R_m – radius-vector modulus connecting the first point of antenna receiving the plane wave front from the m-th point object and the object itself.

In most cases spatio-temporal processing is implemented at separable (factorable) stages of processing, with the latter being possible if the processed signals are narrow-band in the spatio-temporal sense [2, 3, 20–22].

With account of the factorisation provision, a signal from the m-th object received by the n-th AE of LAA can be written down [2, 3, 20–22] as follows:

$$s_{mn}(t-\tau_m-\tau_{mn}) \approx \mathrm{Re}\{S_m(t-\tau_m)e^{j2\pi f_0 t}\, S_{\theta mn}\}, \qquad (2)$$

where $S_{mn}(t - \tau_m - \tau_{mn})e^{j2\pi f0t} \approx S_m(t - \tau_m)e^{j2\pi f0t}$, $S_{\theta mn} = e^{j\varphi n(\theta m)}$ – complex temporal and spatial components of a signal which is narrow-band in the spatio-temporal sense;

$$n = \overline{1, N};$$

$\tau_{mn} = [(n - 1)d\sin\theta_m]/c$ – delay time in arrival of the phase front of signal from the m-th object, relative to the first AE of LAA, to the n-th RL antenna element with AE spacing d;

$S_m(t - \tau_m)$ – complex signal envelope;

f_0 – carrier frequency;

$\varphi_n(\theta_m) = [-2\pi(n - 1)d\sin\theta_m]/\lambda$;

λ – wavelength.

It is known that complex decision statistics (CDS) Z_m for taking a decision on detection of a desired deterministic signal from one m-th object against the background of additive equivalent spatio-temporal white noise can be written down [2, 3, 20–22] as follows:

$$Z_m = \mathbf{Y}^T \mathbf{K}_{\theta t}^{-1*} \mathbf{S}^*_{o0tm} = \tag{3.1}$$

$$= \sigma^{-2} \mathbf{Y}^T \left(\mathbf{S}^*_{o0m} \otimes \mathbf{I}_t\right)\left(1 \otimes \mathbf{S}^*_{otm}\right) = \tag{3.2}$$

$$= \sigma^{-2} \mathbf{Y}^T \left(\mathbf{I}_\theta \otimes \mathbf{S}^*_{otm}\right)\left(\mathbf{S}^*_{o0m} \otimes 1\right). \tag{3.3}$$

Here, $\mathbf{Y} = S_{\theta tm} + \mathbf{N}$ – column vector of the complex envelope of signal received from the m-th object;

$\mathbf{S}_{\theta tm} = \mathbf{S}_{\theta m} \otimes \mathbf{S}_{ot}$, $\mathbf{S}_{o\theta tm} = \mathbf{S}_{o\theta m} \otimes \mathbf{S}_{ot}$ – column vectors of the complex envelopes of desired signals received and expected from the m-th object with spatial $\mathbf{S}_{\theta m} = \|S_{\theta mn}\|$, $\mathbf{S}_{o\theta m} = \|S_{o\theta mn}\|$, $S_{\theta mn} = e^{j\varphi n(\theta mn)}$, $S_{o\theta mn} = e^{j\varphi n(\theta omn)}$, $n = \overline{1, N}$ and temporal $\mathbf{S}_{ot} = \|S_{otk}\|$, $S_{otk} = S_o(t_k)$, $k = \overline{1, K}$ structures obtained through spatial and temporal discretisation;

θ_{om} – angular direction to the m-th signal source;

$\mathbf{N} = \|\mathbf{N}_n\|$, $n = \overline{1, N}$ – block column vector of spatio-temporal white noise;

$\mathbf{N}_n = \|N_{nk}\|$, $N_{nk} = N_n(t_k)$, $k = \overline{1, K}$ – column vector of noise temporal structure of the n-th spatial channel.

Correlation function $\mathbf{K}_{\theta t}$ of spatio-temporal white noise at equipotent intrinsic noises of LAA AE is determined in expression (3.1) by the following equation [20]:

$$\mathbf{K}_{\theta t} = \mathbf{K}_\theta \otimes \mathbf{K}_t = \sigma^2 \mathbf{I}_\theta \otimes \mathbf{I}_t, \tag{4}$$

where $\mathbf{K}_t = M\left[\mathbf{N}_n \mathbf{N}_n^{*T}\right] = \sigma^2 \mathbf{I}_t$ – correlation matrix of the white noise temporal structure;

$M[\bullet]$ – mathematical expectation;

$\sigma^2 = M\left[N_{nk} N^*_{nk}\right]$ – noise dispersion;

$\mathbf{I}_\theta$ – spatial structure of correlation matrix $\mathbf{K}_{\theta t}$ characterising independence of noise of different spatial channels;

$\mathbf{I}_\theta$, $\mathbf{I}_t$ – unity matrices with dimensionality N and K, respectively;

$\otimes$ – operation of Kronecker multiplication of matrices;

*, ^{T} – upper indices of complex conjugation and transposition of matrices.

Relationship (3.2) determines an algorithm in which first spatial and then temporal processing is performed. Expression (3.3) is an algorithm in which first témporal processing after each AE is performed, and then spatial processing [2, 3, 20–22].

When receiving signals from M number of signal sources, with signal duration τ_c and spectrum width ΔF_c, unresolvable by range $(\tau_{m\,\max} - \tau_{m\,\min}) << \tau_c$ and frequency $(F_{д\,\max} - F_{д\,\min}) << \Delta F_c$, using expression (3.1), signal component of the CDS Z_m can be represented [2, 3, 20–22] as follows:

$$M[Z_m] = \sigma^{-2} M\left[\sum_{m=1}^{M} \left(\mathbf{S}_{\theta m} \otimes \mathbf{S}_{ot} + \mathbf{N}\right)^T \left(\mathbf{S}_{o\theta t} \otimes \mathbf{S}_{ot}\right)^*\right] =$$

$$= q_o^2 N \sum_{m=1}^{M} \rho\left(\theta_{om}, \theta_{om}\right),$$

$$m = \overline{1, M}. \tag{5}$$

Here, $N\rho(\theta_m, \theta_{om}) = \mathbf{S}^T_{\theta m} \mathbf{S}^*_{o\theta m} = N\,\mathrm{sinc}(\pi N d_m \vartheta_m/\lambda)$ $e^{-j(\pi dm(N-1)\vartheta m/\lambda}$ – function of angular discrepancy (DF) of the spatial structures of signals received from true θ_m and expected θ_{om} directions of signal arrival;

$d_m = d\cos\theta_m$ – RL spacing projection on the plane phase front of a wave arriving at expected angle θ_{om};

$$\vartheta_m = |\theta_m - \theta_{om}| \le \Delta\Theta;$$

$q_o^2 = \mathbf{S}^T_{ot} \mathbf{S}^*_{ot} / \sigma^2$ – SNR of the m-th object after temporal coordinated processing in one AE.

It follows from expression (4) that normalised DF (NDF) $\rho(\theta_m, \theta_{om})$ coincides with the known expression of normalised RP of the LAA under uniform amplitude and linear phase illuminations.

To illustrate the principle of the proposed angular superresolution under conditions of a priori uncertainty regarding angular positions of signal sources, we shall confine ourselves to simulation of the processing which is defined by expression (3.3).

Selected as an example was a RL LAA with 64 AE, the RP width of which is $\Delta\Theta \approx 1.2°$. Based on this RL LAA, a virtual LAA was formed, with $N = 64$ real and $N_в = 128$ virtual AE, their total number making $(N + N_в) = 192$.

For the purpose of comparing angular resolution, virtual LAA and LAA with equal number of AE (192) were considered, with a priori uncertain arrangement of the sources of signals unresolvable by RL LAA with 64 AE.

Non-symmetrical angular positions of the first $\theta_1 = 11.7°$, the second $\theta_2 = 12.1°$, and the third $\theta_3 = 12.6°$ unresolvable signal sources were arbitrarily selected, offset relative to the expected angular direction of signal arrival $\theta_{om} = 12°$.

Given in Fig. 1 is the result of simulation with the use of three NDF of the RL radiation pattern with 64 AE by power, noise-free, describing reception of signals from three unresolvable signal sources depending on angular detuning $\theta = \theta_{om} - \theta_m$. Moreover, to understand the principles not only of the known (3) but also of the proposed processing, distribution in the RL antenna elements of the real component of the received signals’ spatial structure (RCD) $\mathrm{Re}\sum_{m=1}^{M}\left(\mathbf{S}_{\theta mn}\mathbf{S}^{*}_{o\theta mn}\right)$ and distribution of the values of the real component phases (CPD) were simulated

$$\arg\sum_{m=1}^{M}\left(\mathbf{S}_{\theta mn}\mathbf{S}^{*}_{o\theta mn}\right),\ n=\overline{1,N}.$$

The results of RCD and CPD simulation in the absence of noise, with three unresolvable signals received by RL LAA with 64 AE, are given in Fig. 2.

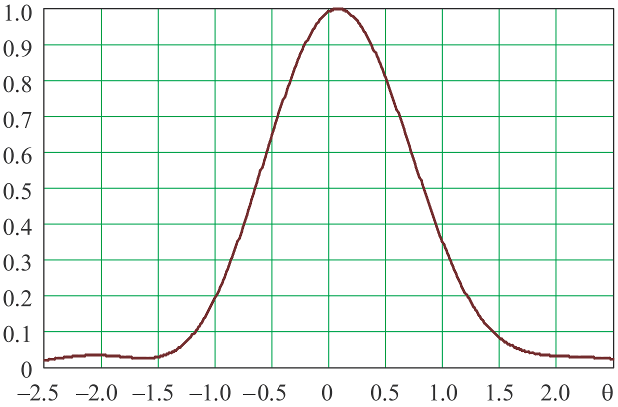


Fig. 1. RP of RL LAA with 64 AE under processing of three unresolvable signal sources

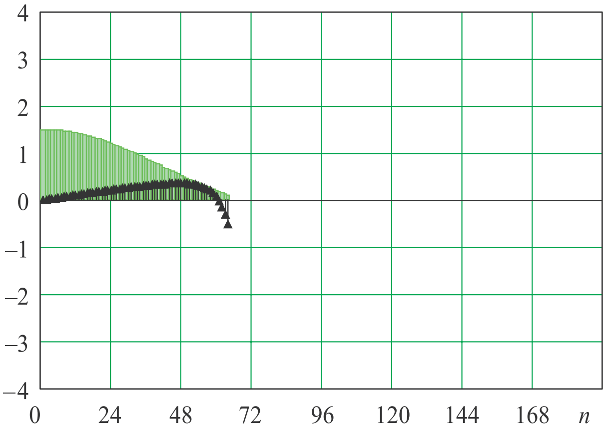


Fig. 2. RCD (▮) and CPD (▲) of RL LAA with 64 AE under processing of three unresolvable signal sources

Given in Fig. 3 are the results of RCD and CPD simulation, noise-free, in RL LAA with 192 AE during phasing, respectively, to the first $\theta_{om} = \theta 1$, the second $\theta_{om} = \theta_2$, and the third $\theta_{om} = \theta_3$ signal sources.

It is known [14] that in the theory of prediction no prediction will be close to reality unless the same regularities are in effect within the prediction interval as within the pre-history interval.

In the meantime, comparison between RCD of a real LAA with 64 AE (see Fig. 2) and RCD of a real LAA with 192 AE (see Fig. 3) shows that even in the absence of noise, RCD regularities observed in the RL LAA with 64 AE are not encountered in the RCD after the 65th AE and in the RL LAA with the 192nd AE.

The task becomes more complicated in the presence of noise. During simulation, after temporal processing in one spatial channel of the

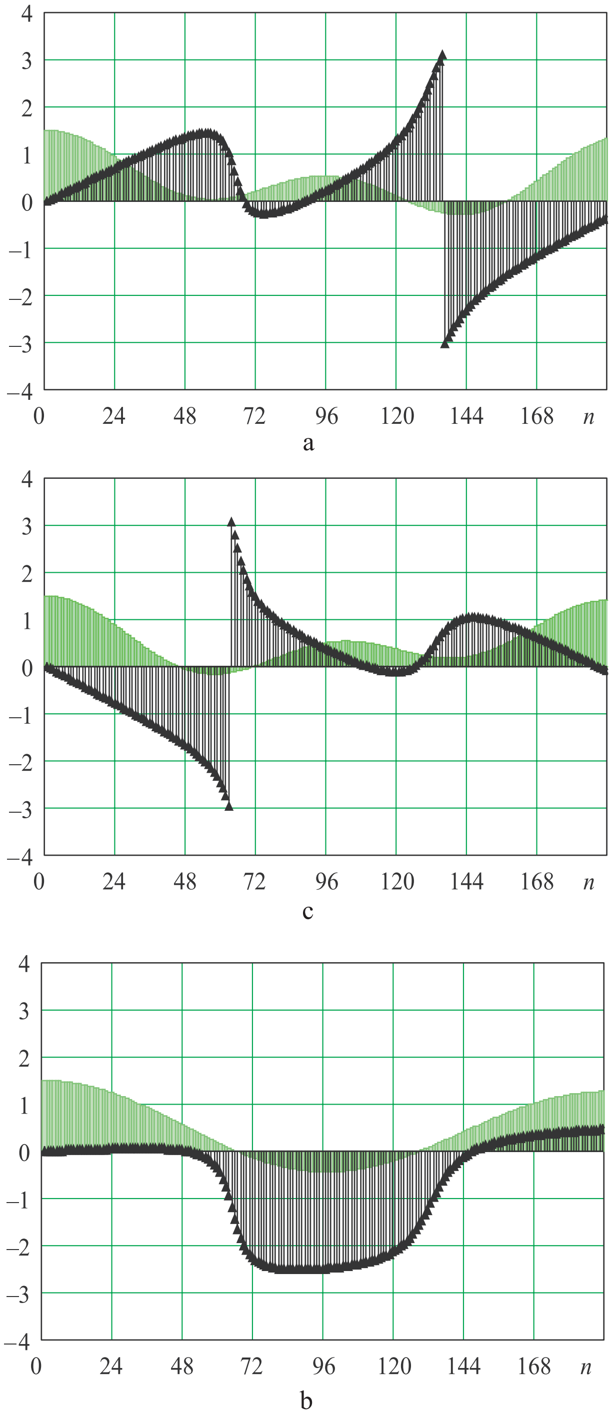


Fig. 3. RCD (🟩) and CPD (▲) w/o noise in RL LAA with 192 AE when phasing to resolvable signal sources at:
a – θ_1; b – θ_2; c – θ_3

RL LAA with 64 AE, SNR was selected equal to –4.5 dB. Selection of such SNR after AE is conditioned by the region of standard indices of radar detection quality, which are determined by the SNR value after spatio-temporal processing. Thus, after spatio-temporal processing, the SNR, as determined by the region of standard indices of radar detection quality, was selected equal to 13.3 dB. In this case the SNR required for obtaining the standard indices of radar detection quality on the RL LAA antenna elements amounts to a value which is much smaller than required for formation of previously proposed VAA [14, 15, 17], and after total processing – a value which is smaller than required for implementation of the known angular superresolution algorithms, such as Berg’s, Capon’s, *MUSIC* algorithms, and others [2, 5, 6].

For consistency of the analysis of noise effect on the RCD of real LAA when resolving different signal sources, the same implementation of the spatio-temporal noise was simulated.

Given in Fig. 4 are RCD and CPD for RL LAA with 192 AE when receiving signals with noise, when the RL LAA is phased, respectively, to the first $\theta_{om} = \theta_1$, the second $\theta_{om} = \theta_2$, and the third $\theta_{om} = \theta_3$ resolvable signal sources.

It follows from the simulation results that, with noise prevailing, the RCD of real LAA with 192 AE differ from one another insignificantly (see Fig. 4). Such RCD levelling, in spite of the fact that RCD dependences for various expected angular directions of noise-free signal reception differ substantially (see Fig. 3), confirms low probability of obtaining accurate signal extrapolation by the known methods under small SNR at the RL LAA antenna elements [17].

Thus, with typical quality indices of signal detection by radar facilities under conditions of a priori uncertainty regarding angular positions of signal sources, solving the task of angular superresolution using virtual antenna arrays presents not only scientific but also practical interest.

The disclosed results of RCD simulation confirm the known conclusion that under conditions of a priori uncertainty regarding angular positions of signal sources, when SNR at the antenna elements is less (much less) than unity, the task of extrapolation of the spatial structure of a cumulative signal (1) shaped by *M*-point objects

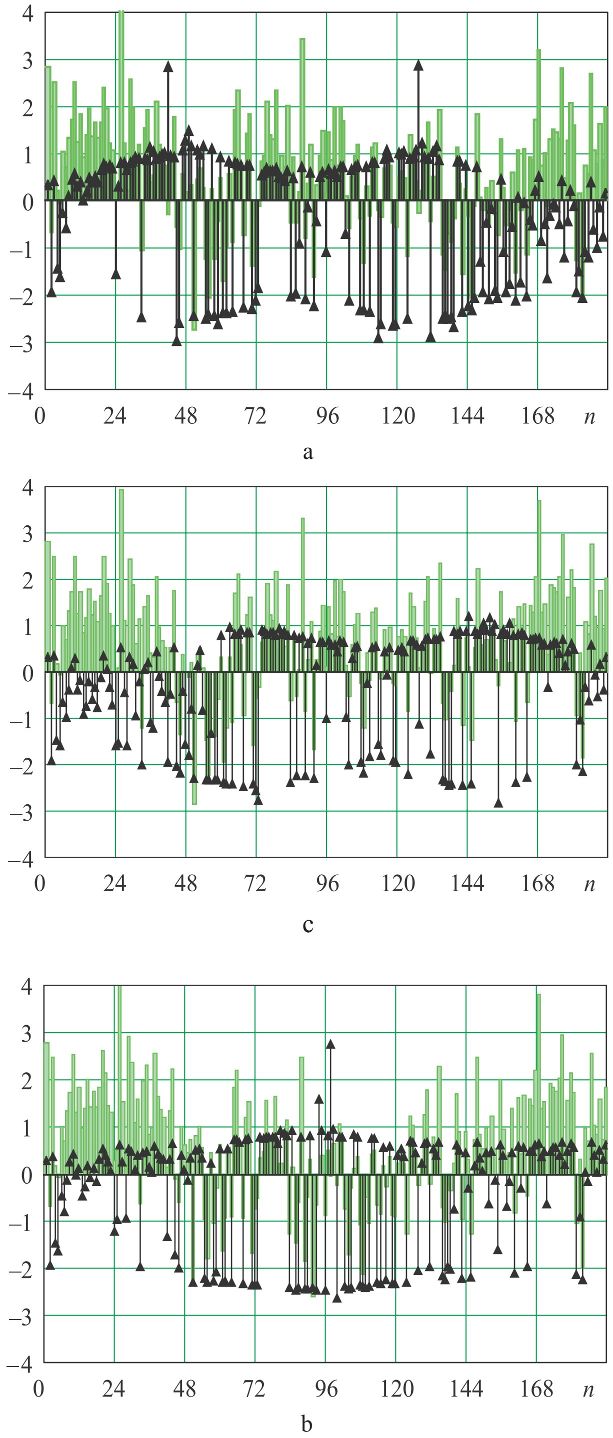


Fig. 4. RCD (🟩) and CPD (▲) with noise in RL LAA with 192 AE when phasing to resolvable signal sources: a – to the first; b – to the second; c – to the third

located in a single resolvable volume cannot be solved by the known methods.

When receiving the signal from a remote zone, the phase structure of its spatial component for any $n_\text{в}$-th virtual AE, same as for the real one, can be written down as follows:

$$S_{\theta m n_\text{в}} = e^{j\varphi n_\text{в}(\theta m)} = e^{-j2\pi(N+n_\text{в}-1)d\sin\theta m/\lambda}.$$

Since in a number of cases researchers aim to have VL aperture exceeding RL aperture G-fold, the number of AE in the VL LAA will amount to

$$N_\text{в} = GN,$$

where G – the number of RL signal iterations necessary for formation of required number $N_\text{в}$.

Then the expected phase incursion of signal from the m-th object in the $n_\text{в}$-th VL antenna element $\varphi_{n_\text{в}g}(\theta_m)$ under the g-th iteration will be determined by the relationship

$$\varphi_{n_\text{в}g}(\theta_m) = \varphi_{N_{\text{в}g}-1}(\theta_m) + \varphi_{n_\text{в}g}(\theta_m).$$

Here, $\varphi_{n_\text{в}g}(\theta_m) = -2\pi(n_{\text{в}g} - 1)d\sin\theta_m]/\lambda$ – phase incursion in the $n_{\text{в}g}$-th VL antenna element under g-th iteration;

$n_{\text{в}g} = \overline{(g-1)N+1-(g-1)N,(g+1)N-gN} = \overline{1,N}$ – local number of VL antenna element at the g-th iteration;

$g = \overline{1,G}$;

$\varphi_{N_{\text{в}g}-1}(\theta_m) = -2\pi(g-1)Nd\sin\theta_m/\lambda$ – group phase incursion of all VL antenna elements for the g-th iteration.

Since extrapolation of the received signal $\mathbf{S}_{\theta tm}$ can only be performed in the presence of additive noise, the accepted implementation $\mathbf{Y}$ is extrapolated:

$$\mathbf{Y}_G = \|S_{\theta omg}\mathbf{Y}\| = \mathbf{S}_{\theta omG} \otimes \Psi,\ g = \overline{1,G}.$$

Here, $\mathbf{S}_{\theta omG} = \|\ \mathbf{S}_{\theta omg}\ \| = \|e^{-2\pi(g-1)Nd\sin\theta om/\lambda}\|$,
$g = \overline{1,G}$.

Then, after G number of iterations of the received RL signals, complex decision statistics Z_{mG} for taking a decision on detection of a resolvable desired deterministic signal from the m-th object against the background of additive equivalent spatio-temporal white noise $\mathbf{N}$, similar to expressions (3.1)–(3.3), can be written down as follows:

$$Z_{mG} = \mathbf{Y}_G^T \mathbf{K}_{\theta tG}^{-1*} \mathbf{S}_{o\theta tmG}^* = \quad (5.1)$$

$$= \sigma^{-2}\mathbf{Y}_G^T\left(\mathbf{S}_{o\theta mG}^* \otimes \mathbf{I}_t\right)\left(1 \otimes \mathbf{S}_{otm}^*\right) = \quad (5.2)$$

$$= \sigma^{-2}\mathbf{Y}_G^T\left(\mathbf{I}_{\theta G} \otimes \mathbf{S}_{otm}^*\right)\left(\mathbf{S}_{o\theta mG}^* \otimes 1\right), \quad (5.3)$$

where $\mathrm{K}_{\theta tG} = M[\mathbf{N}_G \mathbf{N}_G^{*T}]$;

$g = \overline{1, G}$;

$\mathbf{N}_G = \|\mathrm{S}_{\theta omg}\mathbf{N}\| = \mathbf{S}_{\theta omG} \otimes \mathbf{N}$,

$\mathbf{S}_{o\theta mG} = \|\mathrm{S}_{\theta omg}\ \mathbf{S}_{o\theta m}\|$;

$\mathrm{S}_{o\theta tmG} = \mathbf{S}_{o\theta mG} \otimes \mathbf{S}_{otm}$ – expected desired signal with account of G iterations;

$\mathbf{I}_{\theta G}$ – unity matrix with dimensionality $N + N_{\text{в}}$.

Then, taking into account expression (5.1), signal component of the CDS Z_{mG} can be represented as the formula

$$M[Z_{mG}] = \mathbf{S}_{\theta tmG}^{T}\mathbf{K}_{\theta tG}^{-1*}\mathbf{S}_{o\theta tmG}^{*} + \sum_{r=1, r\neq m}^{M} \mathbf{S}_{\theta trG}^{T}\mathbf{K}_{\theta tG}^{-1*}\mathbf{S}_{o\theta tmG}^{*}, \quad (6)$$

where $\mathbf{S}_{\theta tmG} = \mathbf{S}_{\theta mG} \otimes \mathbf{S}_{otm}$;

$\mathbf{S}_{\theta mG} = \|\mathbf{S}_{\theta omG}\mathbf{S}_{\theta m}\|$;

$g = \overline{1, G}$;

$\mathbf{S}_{\theta trG} = \mathbf{S}_{\theta mrG} \otimes \mathbf{S}_{otr}$;

$\mathbf{S}_{\theta mrG} = \mathbf{S}_{\theta mG} \otimes \mathbf{S}_{\theta tr}$;

$\mathbf{S}_{\theta r} = \mathbf{S}_{\theta r} \otimes \mathbf{S}_{otr}$;

$\mathbf{S}_{\theta r} = \|\mathbf{S}_{\theta rn}\|$;

$S_{\theta rn} = e^{-j2\pi(n-1)d\sin\theta r/\lambda}$;

$r \neq m,\ r = \overline{1, M}$.

It appears infeasible to obtain analytical expression of the estimate of true angular direction to the m-th signal source $\hat{\theta}_m$, therefore minimisation of detuning $\nu_m = (\theta_{mo} - \theta_m)$ of the expected angular direction of VL reception θ_{moi} and true direction to the m-th signal source θ_m will be achieved under maximisation of the decision statistics [3, 21]:

$$\max_{\theta_{moi}}\left(\operatorname{Re} M[Z_{mG}(\theta_{moi})]\right),\ i = \overline{1, I}.$$

The number of points $I = \Delta\Theta/\Delta\theta_{mo\text{в}}$ will be determined by the required accuracy and possible sample spacing of the expected angular direction of signal arrival $\Delta\theta_{mo\text{в}}$ in VL LAA within the limits of RL RP width $\Delta\Theta$.

Resulting from this, the maximum value of VL RP phased to the expected angular direction θ_{moi} will ensure extraction of signals from angular direction θ_m.

To confirm feasibility of processing by means of VL, as proposed by the authors of this paper in the patent [18], mathematical simulation was undertaken, limited to two iterations of RL signals. To illustrate angular superresolution under conditions of a priori uncertainty regarding angular positions of signal sources, temporal coordinated signal processing in each spatial channel was carried out, followed by spatial processing using VL LAA.

Given in Fig. 5 are the results of simulation both without noise and with spatio-temporal RCD and CPD noise of a virtual LAA formed by two iterations, with $N = 64$ of real and $N_{\text{в}} = 128$ virtual AE under maximisation of the decision statistics $\max_{\theta_{moi}}(\operatorname{Re}M[Z_{mG}(\theta_{moi})])$, $i = i = \overline{1, I}$ when phasing, respectively, to the first θ_{o1}, the second θ_{o2}, and the third θ_{o3} signal sources.

Simulation of an ideal case with no noise is provided for the purpose of clarification of the proposed principle of VL LAA formation.

Given in Fig. 6 are the results of simulation of the VL LAA radiation pattern by power with $(N + N_{\text{в}}) = 192$ AE in the presence of spatio-temporal noise, resolving, respectively, the first θ_{o1}, the second θ_{o2}, and the third θ_{o3} signal sources that are not resolved by RL LAA with $N = 64$ AE (see Fig. 1), under maximisation of the decision statistics

$$\max_{\theta_{moi}}\left(\operatorname{Re} M[Z_{mG}(\theta_{moi})]\right),\ i = \overline{1, I}.$$

Same as before, the SNR during simulation, after temporal processing in one RL spatial channel with 64 AE, was equal to –4.8 dB, and at the RL LAA output it made 13.3 dB.

Let us summarise the simulation results:

1. The presence of prevailing noise in the receiving channels of radar systems excludes a possibility of VAA formation through extrapolation of signals from the outputs of RAA antenna elements using the known methods.

2. For radar angular superresolution at SNR typical for target detection, it should be expedient to implement the technique proposed by the authors of this paper in patent [18]. When applying it after a required number of iterations of RL signals

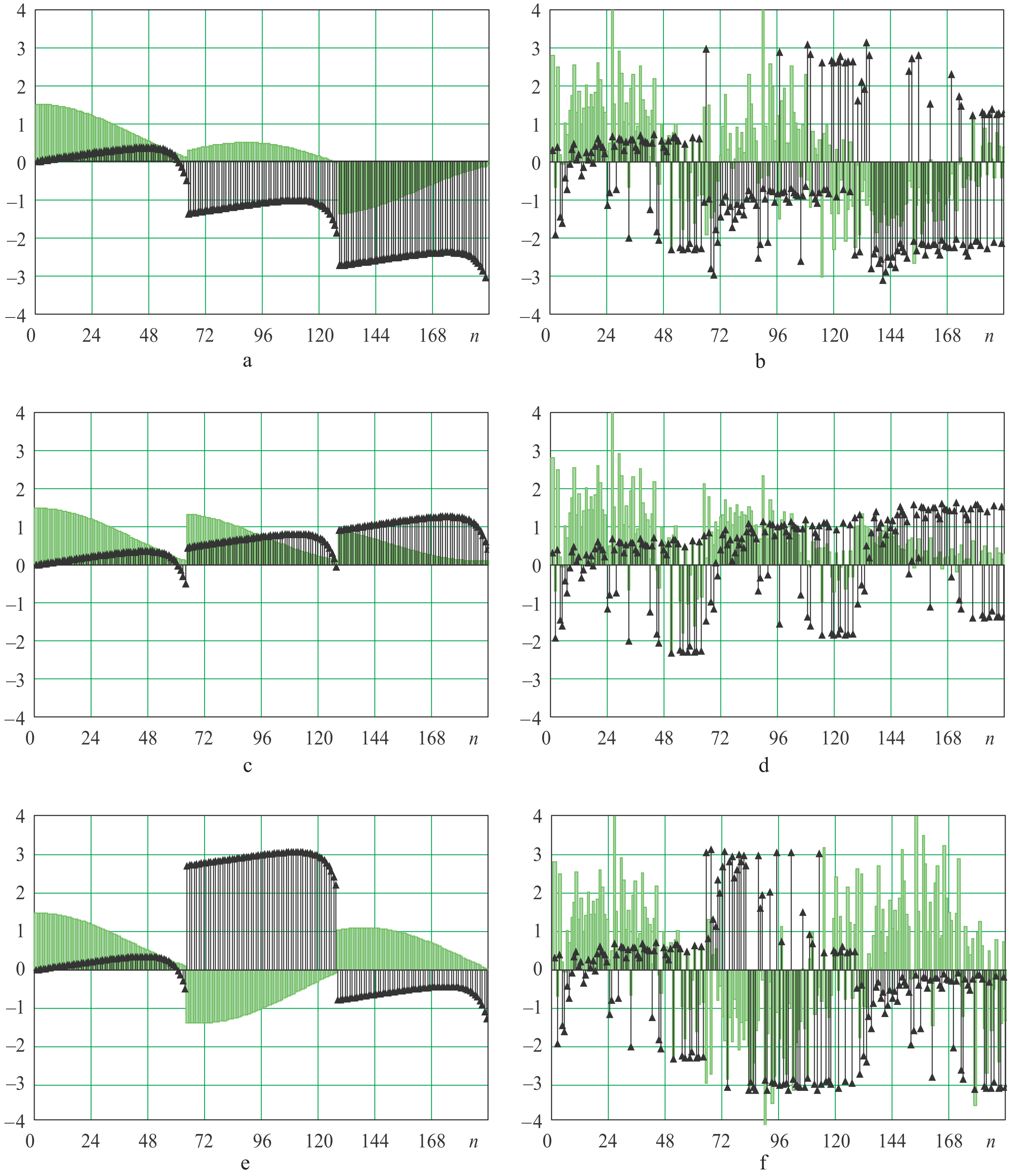
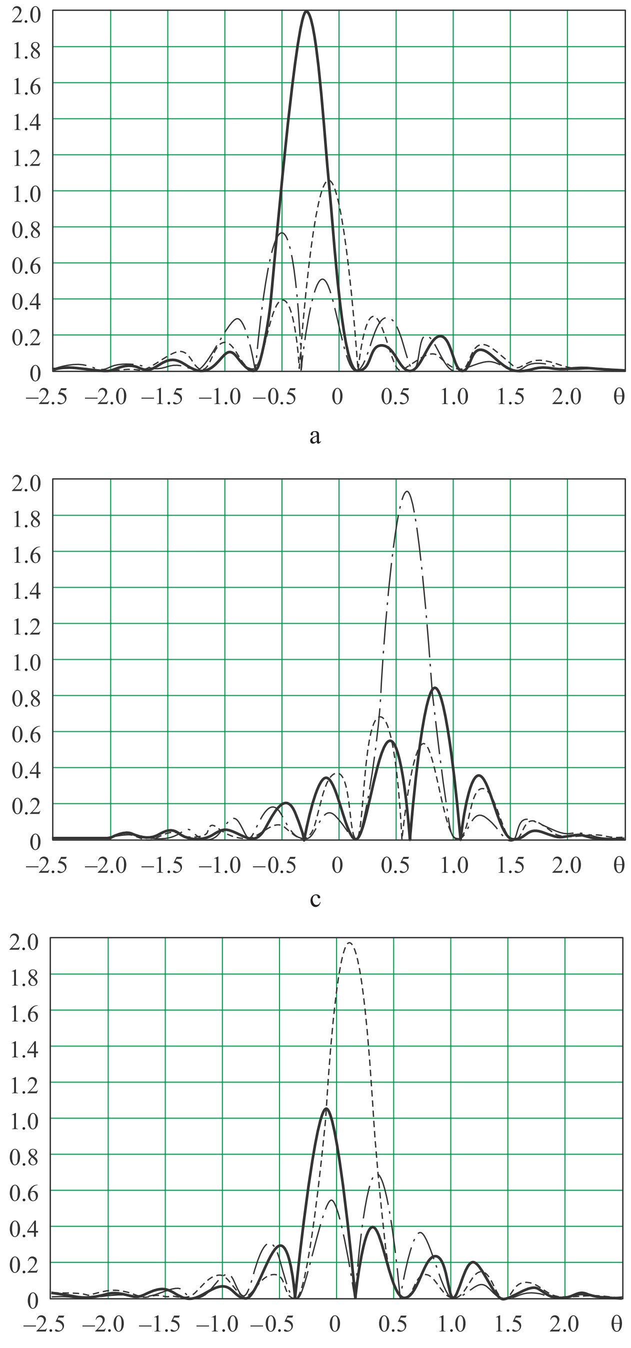


Fig. 5. RCD (🟩) and CPD (▲) w/o and with noise of virtual LAA with 64 real and 128 virtual AE when resolving the first (a, b), the second – (c, d), the third – (e, f) sources



a

c

b

Fig. 6. EP of VL LAA with double iteration of RL LAA with 64 real AE when phasing to the first (a), the second (b), and the third (c) signal sources

spatial structure, taking into account the expected angular directions of signal reception, VL LAA phasing to a respective signal source will be determined by maximisation of the decision statistics.

3. The proposed method of angular superresolution under conditions of a priori uncertainty regarding angular positions of signal sources has enabled resolution of VL signals with SNR. After temporal processing in one spatial channel of RL LAA with 64 AE, it amounted to –4.8 dB, which is much less than the results obtained by using the known signal extrapolation methods. In this case SNR at the RL LAA output is equal to 13.3 dB, which is less than required for implementation of the known algorithms after spatio-temporal processing. The simulated SNR value lies within the limits of typical quality indices of signal detection by radar facilities.

4. The proposed method of VAA formation enables to resolve in the RP main lobe the signal sources arranged randomly relative to the expected angular direction of signal reception (see Fig. 6) that are unresolvable by RAA (see Fig. 2).

5. When implementing the proposed method and algorithm of angular superresolution under conditions of a priori uncertainty regarding angular positions of signal sources, we have revealed presence of the following features which allow to relate this task to a new class of incorrectly set tasks:

- the need for development of new techniques for solving the task of angular superresolution not only in the theory of antennas and spatio-temporal detection of signals, but also when building algorithms of spatial processing of signals by antenna arrays;
- formation of VAA is feasible with the use of new regularities that have not been applied before in the classical theory not only of antennas, but of the spatio-temporal detection of signals as well;
- the need for building new algorithms of angular superresolution enabling the use of non-standard a priori information on the spatial structure of VAA signal sources.

6. Double iteration of the RCD of a real LAA with 64 AE has been implemented; at each step, the iteration takes into account changes in the expected phase structures of the iterated signals necessary for VL LAA formation (see Fig. 5, a, c, e). It has been established that the structures

of the iterated signals with different expected angular directions of their reception θ_{o1}, θ_{o2} and θ_{o3} differ from one another in spite of the same initial signals of RL LAA with 64 AE (see Fig. 2).

7. The RCD and CPD of a real (see Fig. 3, a, b, c) and virtual (see Fig. 5, a, c, e) LAA with equal number of AE substantially differ from one another in the presence and the absence of noise.

8. Presence of several resolvable targets leads to a certain offset of the VL RP maximum (see Fig. 6). It is caused by the proposed method for solving a new class of incorrectly set tasks.

Summing up all of the above, the proposed method ensures angular superresolution by virtual antenna arrays of randomly arranged signal sources in the radar facilities, with signal-to-noise ratio determined by the standard indices of radar detection quality.

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Угловое сверхразрешение сигналов с использованием «виртуальных» антенных решеток

Проведен анализ опубликованных материалов по «виртуальным» антенным решеткам, выявлены ограничения по их использованию в радиолокационных средствах в условиях априорной неопределенности угловых положений источников сигналов. Показана возможность повышения разрешающей способности по угловым координатам в условиях априорной неопределенности угловых положений источников сигналов с использованием «виртуальной» антенной решетки при типовых отношениях сигнал/шум, применяемых при обработке радиолокационных сигналов. Приведены результаты цифрового моделирования сигналов, подтверждающие аналитические выкладки.

Ключевые слова: угловое сверхразрешение сигналов, «виртуальные» антенные решетки, экстраполяция пространственной структуры.

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