UDC 623.4.02:519.8

Pismennaya V. A., Yakutin A. V.

Increasing the efficiency of solving the target allocation problem in aerospace defence systems

The article considers methods of solving the target allocation problem in surface-to-air missile systems for missile defence. Important specifics of the problem in question are rapid nature of anti-missile combat and the requirement for target illumination during homing. The control methods suggested make it possible to increase the efficiency of solving the target allocation problem by means of using prior data on target engagement probability and radar load estimation in the neighbourhood of future position points at the fire planning stage.

Keywords: target allocation, fire control, automated control system, linear programming, surface-to-air missile system.

Introduction

Automation of fire control tasks solving in surface-to-air missile complexes (SAMC) is required to ensure the effectiveness of anti-missile combat and reduce the decision making time during attack planning. The use of automated fire control algorithms allows to increase the number of engaged targets by ensuring the optimal firing sequence.

The target allocation algorithm is responsible for fire control in the SAMC. The main tasks of this algorithm are ranking targets according to the degree of their danger, calculating the coordinates of the impact points and assigning the SAMC information systems to the targets. The quality of solving the target allocation problem determines the SAMC effectiveness when defeating the strikes of attacking targets.

Problem statement

The enemy fires at the protected object with ballistic re-entry systems (targets) of the same type, which enter the SAMC facilities' action zone one by one. Estimated velocity vectors $\boldsymbol{V}_i$ and coordinates $\boldsymbol{R}_i$ for each of N detected targets of the strike are known, $i=\overline{1,N}$. Guiding the surface-to-air missile (SAM) on a target consists of two stages: command-inertial guidance based on information received from the radar, and homing carried out using a semi-active homing head. The amount of resources spent by the radar on the main tasks is limited and depends on the target range and the missile range, as well as on its guidance stage.

It is required to maximise the average number of engaged targets, taking into account the danger of each of target and the limitation on the radar resources. The following criterion [1] is used to characterise the performance of the accomplished firing

$$Q=\sum_{i=1}^{N} p_i k_i \to \max_{p_i}, \quad (1)$$

where p_i – the probability of hitting the i-th target;

k_i – the i-th target danger coefficient.

The probabilities p_i depend on the target vulnerability and the impact point coordinates. A priori data on target engagement probabilities were obtained from the paper [2]. In general, the probability p_i in the kill zone of the SAMC varies. Based on this, zones with high, medium and low damage probability can be distinguished. Therefore, the task of criterion (1) maximisation is in finding the optimal impact points (IP) with the maximum target engagement probability given the limitations on the radar resources.

It should be noted that the main tasks of target allocation can be reduced to the task of assigning optimal impact points. Task (1) will be solved given the assumption that the targets are already allocated between the information and measurement systems, the missiles are considered to be of the same type, and the difference in the geographic coordinates of launchers can be neglected.

Specifics of the target allocation task for antimissile defence SAMC

The main specific features of the target allocation task for the antimissile defence (AMD) SAMC include the following:

- rapid nature of anti-missile combat;
- the need in targets illumination during homing;
- uncertainty in the target impact area;
- reduction of the target kill zone depending on the type of target.

Due to the rapid nature of anti-missile combat, especially in conditions of massive target strike, solution of the target allocation task shall be automated, as well as accounting for limitations of the speed-of-response of the applied algorithms.

The radar loading increases abruptly during the missile homing stage due to the need to track the missile and target at a high rate and to provide for target illumination. Generally, to take this feature into account, the hit time is assigned in such a way that maximum one target would be detected at the homing stage. However, this significantly reduces the depth of the kill zone.

Uncertainty in the target impact point coordinates is associated with the initial errors in the estimated coordinates and velocity as well as with the lack of data on the ballistic coefficient. Therefore, the target danger for the protected object shall be evaluated using its defeat probability with account for the time to the target impact point.

When hitting small-scale high-speed targets, the depth of the detection zone and the kill zone reduces, which creates significant difficulties in solving the target allocation task. As a result, target acquisition failures may occur even if there are free fire and information facilities in the SAMC.

Target allocation task solution methods

Let us consider three methods of solving the target allocation task for AMD SAMC. The first method involves ranking targets by danger index based on multiple criteria and depending on the following parameters: time remaining for targets firing, target danger for the protected object, etc. Targets are ranked according to several criteria using the successive concessions method [3]. After that, the first possible impact point is assigned for the most dangerous target, and the IP for the next most dangerous target is assigned taking into account the hit time separation for different targets during the homing stage.

The advantages of this method are high running speed and ease of implementation. The disadvantage is that the found solutions for index (1) are not optimal in the general case.

The second method for solving the target allocation task is based on development of an assignment matrix [4, 5] describing the range of decisions to be made to maximise criterion (1). The matrix is presented in the tabular form.

The size of the assignment matrix is equal to $n \times m$, where n is the number of targets tracked by the radar and not assigned to missiles; m is the number of IP times available for assignment taking into account their separation during the homing stage (Δt). Here also, t_1 and t_m are the earliest and latest hit times for all n targets, respectively.

Assignment matrix

Target number \ Hit time	t_1	t_2	…	t_m
1	P_{11}	P_{12}	…	P_{1m}
2	P_{21}	P_{22}	…	P_{2m}
⋮	⋮	⋮	…	⋮
n	P_{n1}	P_{n2}	…	P_{nm}

Times t_1 and t_m are calculated by integrating the equations of targets movement on the passive trajectory section [6] and solving the equation of the time balance for the target and the missile flight to the IP; $t_{j+1} = t_j + \Delta t,\ \ j = 1, ..., m-1.$

The components of the assignment matrix are calculated using the formula

$$P_{ij} = p_{ij} k_i,$$

where p_{ij} – the i-th target engagement probability at hit time t_j;

k_i – the i-th target danger coefficient.

Then the solution of task (1) is steadily related to the solution of the linear programming task:

$$W = \sum_{i=1}^{N} \sum_{j=1}^{m} P_{ij} x_{ij} \to \max_{x_{ij}}; \qquad (2)$$

$$\begin{cases} x_{ij} \in \{0; 1\}; \\ \sum_{j=1}^{m} x_{ij} \le 1, \quad i = 1, \dots, n; \\ \sum_{i=1}^{n} x_{ij} \le 1, \quad j = 1, \dots, m. \end{cases} \qquad (3)$$

Here, x_{ij} is the solution of the target allocation task, which can be written as

$$x_{ij} = \begin{cases} 1, & \text{if the impact point with the } i\text{-th target is assigned to time } t_j; \\ 0, & \text{otherwise.} \end{cases}$$

Due to limitations (3), each of the considered targets can be assigned to only one hit time and only one target can be assigned to each of the possible hit times. Task (2) under conditions (3) is proposed to be solved using the Hungarian algorithm [7] which solves a task in polynomial time.

The advantage of the algorithm based on generation of the assignment matrix is the ability to select the impact points for the most dangerous targets in the kill zones with the highest prior probability of destruction. The disadvantage of the algorithm is the need to use data on the targets engagement probability and the increase in the algorithm running time proportionally to the size of the assignment matrix.

The limitations of the radar resources in the two target allocation task solving methods described above were taken into account due to separation of the impact points during the homing stage. However, when the number of targets exceeds the number of possible hit times, $n > m$, some targets can be missed.

The third method for solving the target allocation task is a modified method based on generation of an assignment matrix. Its essence consists in assessing the radar loading for all time moments t_j and searching for the possibility of assigning several targets to one hit time. For this reason, the last limitation in system (3) is replaced by limitations of the radar resources, while the rest remain unchanged:

$$\begin{cases} x_{ij} \in \{0; 1\}; \\ \sum_{j=1}^{m} x_{ij} \le 1, \ i = 1, \dots, n; \\ \sum_{i=1}^{n} \varphi_{ij}(\boldsymbol{X}, \boldsymbol{R}_{ij}^{\text{ц}}, \boldsymbol{R}_{ij}^{\text{ЗУР}}) \le C, \ j = 1, \dots, m, \end{cases} \qquad (4)$$

where $\varphi_{ij}(\boldsymbol{X}, \boldsymbol{R}_{ij}^{\text{ц}}, \boldsymbol{R}_{ij}^{\text{ЗУР}})$ – linear function determining radar loading when hitting the i-th target at the time moment t_j;

$\boldsymbol{X} = (x_{ij})$ – the target allocation task solution;

$\boldsymbol{R}_{ij}^{\text{ц}}$, $\boldsymbol{R}_{ij}^{\text{ЗУР}}$ – position vectors of the i-th target and the missile assigned to the i-th target, at time moment t_j, respectively;

C – maximum loading of the radar.

Task (2) under conditions (4) cannot be solved using the Hungarian algorithm; therefore, the branch and bound method [8] and the simplex method were used to solve it. The advantage of the modified method based on the assignment matrix is the ability to assign several targets to the same hit time and, as a result, increase the number of fired targets. The disadvantage is much greater increase of the algorithm running time.

Results obtained

To assess the effectiveness of the presented methods of target allocation task solution, they were simulated in conditions of a strike by eight ballistic targets of the same type. Danger coefficient k_i for all targets was taken equal to one. Figure 1 shows the simulation results in case of target allocation problem solving by the method based on targets ranking.

In Fig. 1, the X axis shows the time since the launch of the first missile, and the Y axis shows the time the missiles are in flight. The inclined lines represent the missile flight stages. The dotted line shows the radar loading level. There is a sudden change in the radar loading level here, at the moment the first target is illuminated. After that, loading decreases as missiles reach the impact points (see Fig. 1). The results of simulation in case of solving the target allocation task by the method based on the assignment matrix qualitatively repeat the data presented in Fig. 1. The average number of engaged targets was 3.05 and 3.31

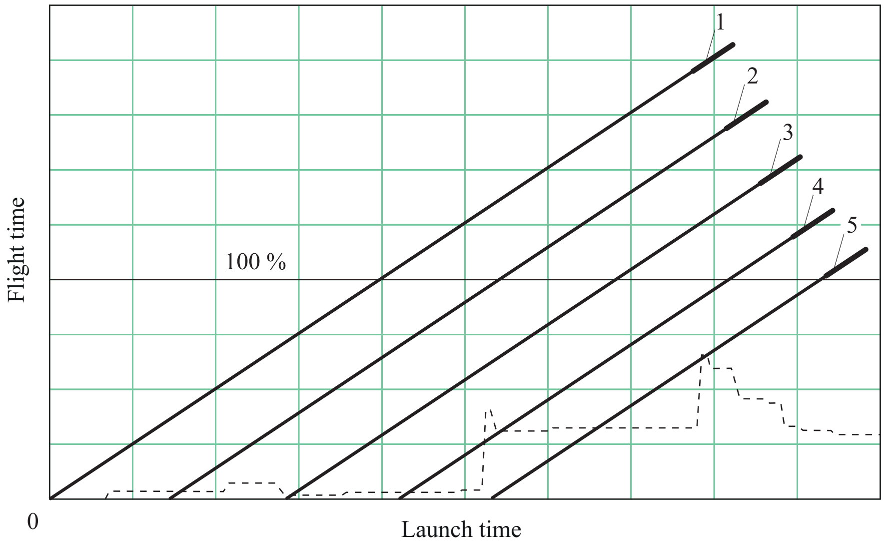

Fig. 1. The target allocation problem solving results based on targets (1–5) ranking by the composite danger index: —— – command-inertial guidance stage; ━━ – homing guidance stage; ----- – the radar loading level

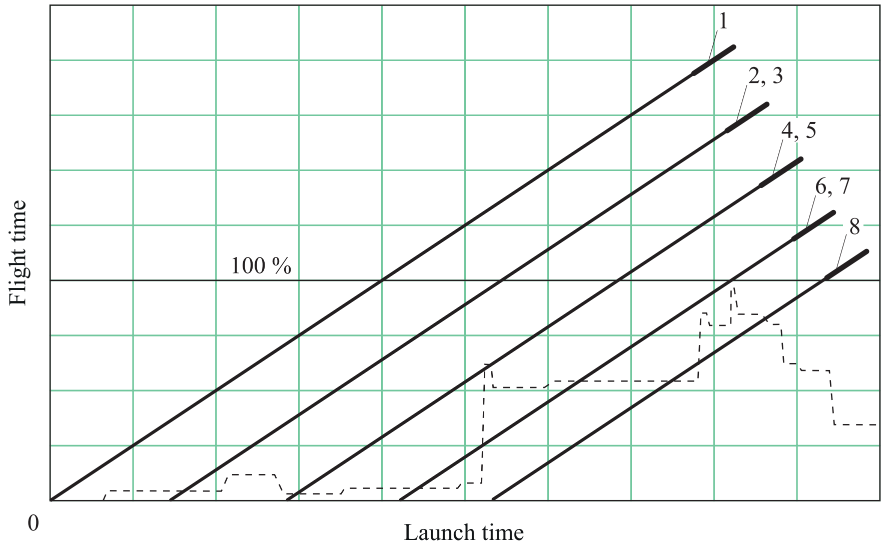

Fig. 2. The target allocation problem solving results based on the radar resources limitations: —— – command-inertial guidance stage; ━━ – homing guidance stage; ----- – the radar loading level

for the first and second methods, respectively. The impact points were assigned to five targets out of eight; as a result, three targets were not hit.

The results of simulation in case of solving the target allocation task by the modified method based on the assignment matrix are shown in Fig. 2.

It can be seen that two targets are assigned to three hit times (see Fig. 2). The average number of engaged targets was 5.42. The radar loading level in Fig. 2 is significantly higher than the radar loading level in Fig. 1, but does not exceed the maximum allowable value.

Conclusion

The main specifics of the target allocation task for the SAMC have been reviewed, and three methods for the task solution have been described. When considering the methods, improvement of the effectiveness of the obtained solutions was emphasised.

Two tools have been proposed to increase the effectiveness of the target allocation task solution: using prior data on target engagement probability and radar load estimation in the neighbourhood of impact points at the fire planning stage. Consideration of prior data on target engagement probability allows to assign impact points to targets, ensuring higher engagement probability. Consideration of the radar load allows to assign multiple targets to one hit time.

The results of running the algorithms when defeating the strikes of ballistic targets of the same type have been presented herein. The results showed the effectiveness of the proposed methods for solving the target allocation task.

Bibliography

1. *Balagansky I. A., Merzhievsky L. A.* Deystviye sredstv porazheniya i boyepripasov. Novosibirsk: Izd-vo NGTU, 2004. 408 s. (Russian)

2. *Yakutin A. V.* A Model for Surface-to-Air Missile Efficiency Estimation // Antenny. M.: Radiotekhnika. 2013. No. 1 (188). P. 30–32. (Russian)

3. *Sobol I. M ., Statnikov R. B.* Vybor optimalnykh parametrov v zadachakh so mnogimi kriteriyami. M.: Drofa, 2006. 175 s. (Russian)

4. *Pismennaya V. A., Yakutin A. V.* Target Assignment Problem Solution with Using of Hungarian Algorithm // Uspekhi sovremennoy radioelektroniki. 2016. No. 2. P. 32–36. (Russian)

5. *Voronin V. V., Solovey R. V., Gritsyna N. T.* Decision of the Problem Distribution Integer when Fire Governing in Many-Server Anti-Aircraft Missile Complex // Sistemi ozbroyennya i viyskova tekhnika. 2014. No. 1 (37). P. 16–19.

6. *Nikitina A. A., Gritsyk P. A.* Tracking of the Maneuvering target // Antenny. M.: Radiotekhnika. 2013. No. 1 (188). P. 24–29. (Russian)

7. *Kuhn H.* The Hungarian method for the assignment problem. Naval Research Logistics, Quarterly. 1955. Vol. 2. P. 83–97.

8. *Panteleyev A. V., Letova T. A.* Metody optimizatsii v primerakh i zadachakh. M.: Vysshaya shkola, 2008. 544 s. (Russian)

Submitted on 23.01.2017

Pismennaya Viktoriya Aleksandrovna – Engineer of the first rank, Public Joint-Stock Company Scientific and Production Association “ALMAZ R&P Corp.”, Moscow.
Science research interests: automated control systems.

Yakutin Aleksandr Vladimirovich – Candidate of Engineering Sciences, Head of Sector, Public Joint-Stock Company Scientific and Production Association “ALMAZ R&P Corp.”, Moscow.
Science research interests: target allocation problem, tertiary radar data processing algorithms.

Повышение эффективности решения задачи целераспределения в системах воздушно-космической обороны

Рассмотрены методы решения задачи целераспределения для зенитных ракетных комплексов противоракетной обороны. Существенными особенностями рассматриваемой задачи являются скоротечность противоракетного боя и необходимость осуществлять подсвет целей на этапе самонаведения. Предложенные методы позволят повысить эффективность решения задачи целераспределения за счет использования априорных данных по вероятностям поражения целей и оценки загрузки локатора в окрестности точек встречи на этапе планирования обстрела.

Ключевые слова: целераспределение, управление огнем, автоматизированная система управления, линейное программирование, зенитный ракетный комплекс.

Письменная Виктория Александровна – инженер первой категории публичного акционерного общества «Научно-производственное объединение “Алмаз” имени академика А. А. Расплетина», г. Москва.
Область научных интересов: автоматизированные системы управления.

Якутин Александр Владимирович – кандидат технических наук, начальник сектора публичного акционерного общества «Научно-производственное объединение “Алмаз” имени академика А. А. Расплетина», г. Москва.
Область научных интересов: задача целераспределения, алгоритмы третичной обработки.